

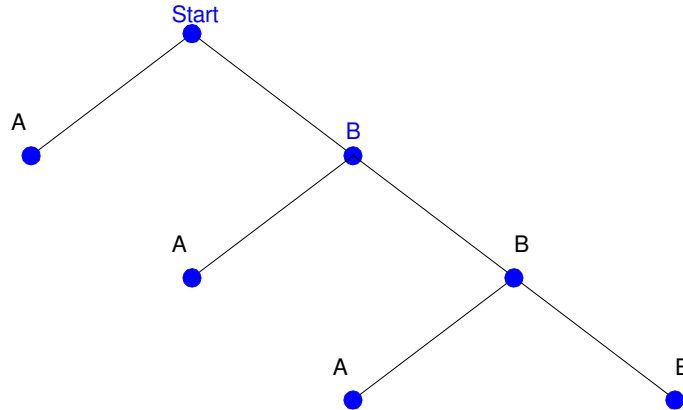
# ECE 341 - Homework #1 Solution

Tree Diagrams and Enumeration. Summer 2023

1) Two teams, A and B, are playing a best of 5 game series.

- The series is over once one team wins 3 games.
- A starts with +2 points (odds)

Draw the tree diagram for all possible outcomes of the series.



2) List all possible combinations of rolling two 5-sided dice (d5) (enumeration).

(1,1) 1	(1,2) 2	(1,3) 3	(1,4) 4	(1,5) 5
(2,1) 2	(2,2) 2	(2,3) 3	(2,4) 4	(2,5) 5
(3,1) 3	(3,2) 3	(3,3) 3	(3,4) 4	(3,5) 5
(4,1) 4	(4,2) 4	(4,3) 4	(4,4) 4	(4,5) 5
(5,1) 5	(5,2) 5	(5,3) 5	(5,4) 5	(5,5) 5

The odds are

- 1:  $1/25$
- 2:  $3/25$
- 3:  $5/25$
- 4:  $7/25$
- 5:  $9/25$

Two players, A and B, are playing a game of dice.

- Player A rolls two 5-sided dice (2d5) the largest of the two numbers (i.e. problem #2)
- Player B rolls an 8-sided die (d8).

Player A wins on ties.

3) What is the conditional probability

- Player A wins given B's score is 3 (B rolled a 3)

A wins if A gets {3, 4, 5} points.

The odds are:

$$3: \quad 5/25$$

$$4: \quad 7/25$$

$$5: \quad 9/25$$

Total:  $21/25$

**The probability that A wins given that B rolled a 3 is  $21/25$**

4) What is the probability that player A will win any given game?

This is a conditional probability

- $p(A|B=1)p(B=1) = (25/25)(1/8)$
- $p(A|B=2)p(B=2) = (24/25)(1/8)$
- $p(A|B=3)p(B=3) = (21/25)(1/8)$
- $p(A|B=4)p(B=4) = (16/25)(1/8)$
- $p(A|B=5)p(B=5) = (9/25)(1/8)$
- $p(A|B=6)p(B=6) = (0)(1/8)$
- $p(A|B=7)p(B=7) = (0)(1/8)$
- $p(A|B=8)p(B=8) = (0)(1/8)$

Adding them all up

$$p(A) = 95/200 = 0.4750$$

**A has a 47.5% chance of winning this game**

## Enumeration & Farkle

Write a Matlab program to go through every combination of 6d6 and determine...

5) The odds of rolling 4-of-a-kind =  $1800 / 46656 = 3.86\%$

6) The odds of rolling three doubles =  $1800 / 46656 = 3.86\%$

```
Problem #5 & #6: Farkle
      xxxx ab      aa bb cc      Total
      1800          1800          46656
```

Elapsed time is 1.390626 seconds.

### Matlab Code:

```
tic
Pair4 = 0;
Pair222 = 0
Total = 0;

for d1 = 1:6
    for d2 = 1:6
        for d3 = 1:6
            for d4 = 1:6
                for d5 = 1:6
                    for d6 = 1:6
                        Total = Total + 1;
                        Dice = [d1,d2,d3,d4,d5,d6];
                        % check for pairs
                        N = zeros(1,6);
                        for i=1:6
                            for j=1:6
                                if(Dice(j) == i)
                                    N(i) = N(i) + 1;
                                end
                            end
                        end
                        [N,b] = sort(N, 'descend');
                        if ( (N(1) == 4) & (N(2) < 2) )
                            Pair4 = Pair4 + 1;
                        end
                        if((N(1)==2) & (N(2)==2) & (N(3)==2) )
                            Pair222 = Pair222 + 1;
                        end
                    end
                end
            end
        end
    end
end

clc
disp('Problem #5 & #6: Farkle')
disp('      xxxx ab      aa bb cc      Total');
disp([Pair4, Pair222, Total]);

toc
```

## Enumeration in 6-card Poker

7) In 6-card poker, you're dealt 6 cards and keep the best 5. Determine using enumeration the odds of being dealt 2-pair

hand = xx yy ab      a and b different than x and y (could be the same though)

In 6-Card Poker, there are 2,532,816 ways to get 2-pair

There are 20,358,520 different hands

The odds are

$$p = \left( \frac{2,532,816}{20,358,520} \right) = 0.1244$$

**There is a 12.44% chance of getting 2-pair with 6-card poker**

8) Determine using enumeration the odds of being dealt one-pair

hand = xx a b c d      a, b, c, d x all different

In 6-card poker, there are 9,884,160 ways to get one pair

There are 20,358,520 different hands

The odds are

$$p = \left( \frac{9,884,160}{20,358,520} \right) = 0.4855$$

**There is a 48.55% chance of getting a pair in 6-card poker**

## Code:

```
% 6-Card Stud
% Probability of 2-pair & 2 of a kind

tic
Pair22 = 0;    % 2-pair
Pair2 = 0;    % pair
H = 0;        % total number of hands

for c1=1:47
    for c2 = c1+1:48
        for c3 = c2+1:49
            clc
            disp([c1,c2, c3])
            for c4 = c3+1:50
                for c5 = c4+1:51
                    for c6 = c5+1:52
                        H = H + 1
                        Hand = [c1,c2,c3,c4,c5,c6];
                        Value = mod(Hand,13) + 1;
                        Suit = floor(Hand/13) + 1;

                        N = zeros(1,13);
                        for n=1:13
                            N(n) = sum(Value == n);
                        end

                        [N,a] = sort(N, 'descend');
                        if ((N(1) == 2)*(N(2) == 2)) Pair22 = Pair22 + 1; end
                        if ((N(1) == 2)*(N(2) == 1)) Pair2 = Pair2 + 1; end
                    end
                end
            end
        end
    end
end

end
end
end

[H, Pair22, Pair2]
toc
```

note: on my computer, this code took 10 minutes to run

- Enumeration works, but it can be really slow
- A better method might be useful (combinatorics)