

ECE 341 - Homework #3

Dice Games and z-Transform - Summer 2023

Yahtzee (5 dice)

In the game of Yahtzee, you roll five dice.

- You can then keep whichever dice you like and re-roll the rest.
- You can then do this a second time.

Whatever the results are after three rolls scores points. A Yahtzee is when you roll 5-of-a-kind.

1) Compute the odds of rolling a full-house when rolling 5 dice one time

dice = xxx yy

x and y have different frequency, so do these separate (11122 is different from 22211)

$M = ({}^6C_1 \text{ for } x)({}^5C_3 \text{ for } x, \text{ choose } 3)({}^5C_1 \text{ for } y)({}^2C_2 \text{ for } y, \text{ choose } 2)$

$$M = \binom{6}{1} \binom{5}{3} \binom{5}{1} \binom{2}{2} = 300$$

There are 150 ways to roll a full-house

$$N = 6^5$$

$$p = \left(\frac{M}{N}\right) = 3.848\%$$

2) Compute the odds of rolling four-of-a-kind when rolling 5 dice one time

dice = xxxx y

$M = (6 \text{ numbers choose } 1 \text{ for } x)(5 \text{ spots choose } 4 \text{ for } x)(5 \text{ remaining numbers choose } 1 \text{ for } y)(1 \text{ spot for } y)$

$$M = \binom{6}{1} \binom{5}{4} \binom{5}{1} \binom{1}{1} = 150$$

$$p = \left(\frac{M}{N}\right) = 1.929\%$$

3) Compute the odds of rolling three-of-a-kind when rolling 5 dice one time

dice = xxx y z

M = (6 numbers choose 1 for x)(5 spots for x, choose 3)(5 other numbers choose 2 for xy)(2 spots for y choose 1)

$$M = \binom{6}{1} \binom{5}{3} \binom{5}{2} \binom{2}{1} = 1200$$

$$M = \binom{6}{1} \binom{5}{3} \binom{5}{1} \binom{4}{1} = 1200$$

$$p = \left(\frac{M}{N}\right) = 15.432\%$$

4) Write a Matlab program which computes the number of ways to roll five dice and

- Get a full house
- Get 4-of-a-kind, and
- Get 3-of-a-kind

Compare your answers to what you computed using combinatorics

Code: Using enumeration (exact answer)

```
% Yahtzee

Pair4 = 0;
Pair3 = 0;
Pair32 = 0;

for d1=1:6
    for d2=1:6
        for d3=1:6
            for d4=1:6
                for d5=1:6
                    Dice = [d1,d2,d3,d4,d5];

                    N = zeros(1,6);
                    for i=1:6
                        N(i) = sum(Dice == i);
                    end
                    N = sort(N, 'descend');
                    if(N(1) == 4) Pair4 = Pair4 + 1; end
                    if((N(1) == 3)*(N(2)==2)) Pair32 = Pair32 + 1; end
                    if((N(1) == 3)*(N(2)==1)) Pair3 = Pair3 + 1; end
                end
            end
        end
    end
end
end
[Pair4, Pair32, Pair3]
```

Result:

	4ok	FH	3OK
ans =	150	300	1200

These match my calculations

z-Transforms

5) Find the inverse z-transform

$$X = \left(\frac{0.01z^2}{(z-0.98)(z-0.96)(z-0.9)} \right)$$

$$X = \left(\frac{0.01z}{(z-0.98)(z-0.96)(z-0.9)} \right) z$$

$$X = \left(\left(\frac{6.125}{z-0.98} \right) + \left(\frac{-8}{z-0.96} \right) + \left(\frac{1.875}{z-0.9} \right) \right) z$$

$$X = \left(\frac{6.125z}{z-0.98} \right) + \left(\frac{-8z}{z-0.96} \right) + \left(\frac{1.875z}{z-0.9} \right)$$

$$x(k) = 6.125(0.98)^k - 8(0.96)^k + 1.875(0.9)^k \quad k \geq 0$$

6) Find the inverse z-transform

$$X = \left(\frac{0.1(z-0.9)}{(z-1)(z-0.95)(z-0.8)} \right)$$

$$zX = \left(\frac{0.1(z-0.9)}{(z-1)(z-0.95)(z-0.8)} \right) z$$

$$zX = \left(\left(\frac{1}{z-1} \right) + \left(\frac{-0.6667}{z-0.95} \right) + \left(\frac{-0.3333}{z-0.8} \right) \right) z$$

$$zX = \left(\frac{z}{z-1} \right) + \left(\frac{-0.6667z}{z-0.95} \right) + \left(\frac{-0.3333z}{z-0.8} \right)$$

$$zx(k) = \left(1 - 0.6667(0.95)^k - 0.3333(0.8)^k \right) u(k)$$

$$x(k) = \left(1 - 0.6667(0.95)^{k-1} - 0.3333(0.8)^{k-1} \right) u(k-1)$$

7) A new Ford F150-Lightning costs \$72,314 from Cars.com. If you take out a 60-month loan at 4.74% interest, what is your monthly payment? Solve using z-transforms.

Let $x(k)$ be how much you owe the bank at month k

- Borrow \$ L at month 0
- Monthly payments of p starting month #1
- $a = 0.0474/12$ (monthly interest rate)

$$x(k+1) = (1+a)x(k) + L \cdot \delta(k) + p \cdot u(k-1)$$

Convert to z-transforms

$$zX = (1+a)X + L + p\left(\frac{1}{z-1}\right)$$

$$(z - (1+a))X = L + p\left(\frac{1}{z-1}\right)$$

$$X = \left(\frac{1}{z-(1+a)}\right)L + \left(\frac{1}{(z-1)(z-(1+a))}\right)p$$

Take partial fraction expansion

$$X = \left(\frac{1}{z-(1+a)}\right)L + \left(\left(\frac{-1/a}{z-1}\right) + \left(\frac{1/a}{z-(1+a)}\right)\right)p$$

Take the inverse-z transform

$$zX = \left(\frac{z}{z-(1+a)}\right)L + \left(\frac{1}{a}\right)\left(\left(\frac{-z}{z-1}\right) + \left(\frac{z}{z-(1+a)}\right)\right)p$$

$$zx(k) = \left(\left(1+a\right)^k \cdot L + \left(\frac{p}{a}\right)\left(\left(1+a\right)^k - 1\right)\right)u(k)$$

$$x(k) = \left(\left(1+a\right)^{k-1} \cdot L + \left(\frac{p}{a}\right)\left(\left(1+a\right)^{k-1} - 1\right)\right)u(k-1)$$

At $k=60$, the loan balance is zero

$$x(60) = 0 = \left(\left(1+a\right)^{59} \cdot L + \left(\frac{p}{a}\right)\left(\left(1+a\right)^{59} - 1\right)\right)$$

$$p = \left(\frac{a(1+a)^{59}}{(1+a)^{59}-1}\right)L$$

Plugging in numbers

```
>> a = 0.0474/12;
>> L = 72314;
>> p = a * (1+a)^59 / ((1+a)^59 - 1) * L
```

```
p = 1376.4
```

Your monthly payments will be \$1376.4