ECE 341 - Homework #4

Binomial and Uniform Distributions. Summer 2023

Binomial Distribution

Assume you roll a 10-sided die. You win on a 1 or 2, lose on 3..10 (p = 0.2)

$$X(z) = \left(\frac{0.8z+0.2}{z}\right)$$

1) Determine the probability of winning 4 times out of 10 rolls

$$p = \binom{n}{m} p^m (1-p)^{n-m}$$

$$p = \binom{10}{4} (0.2)^4 (0.8)^6$$

$$p = 0.08808$$

2) Determine the probability distribution when rolling this die 10 times

Option 1: Use the binomial formula for m=0..10

$$p = \binom{10}{m} p^m (1-p)^{10-m}$$

Option 2: Use convolution in Matlab

```
>> d1 = [0.8,0.2];
>> d2 = conv(d1,d1);
>> d4 = conv(d2,d2);
>> d8 = conv(d4,d4);
>> d10 = conv(d8,d2)
```

0	1	2	3	**4**	5	6
0.1074	0.2684	0.3020	0.2013	**0.0881**	0.0264	0.0055

7	8	9	10
0.0008	0.0001	0.0000	0.0000

Check: The probability of 4 heads matches up with problem #1

NOAA has been keeping track of world weather for the past 142 years. 27 of last 30 years have been in the 30 hottest years on record.

3a) What is the probability of any given year being one of the 30 hottest on record (i.e. what is p?)

$$p = \left(\frac{30}{142}\right) \qquad \textit{odds of a random year being in the top 30}$$

$$1 - p = \left(\frac{112}{142}\right) \qquad \textit{odds of a random year not being in the top 30}$$

3b) What is the probability of 27 of the last 30 years being the hottest on record?

$$p = \binom{30}{27}\left(\frac{30}{142}\right)^{27}\left(\frac{112}{142}\right)^{3}$$

$$p = 1.7175 \cdot 10^{-13}$$

$$p = 0.000\ 000\ 000\ 001\ 715$$

*There **is** a chance this is just due to random variations in the weather. It's a pretty small probability though.*

Uniform Distribution

Assume a fair six-sided die:

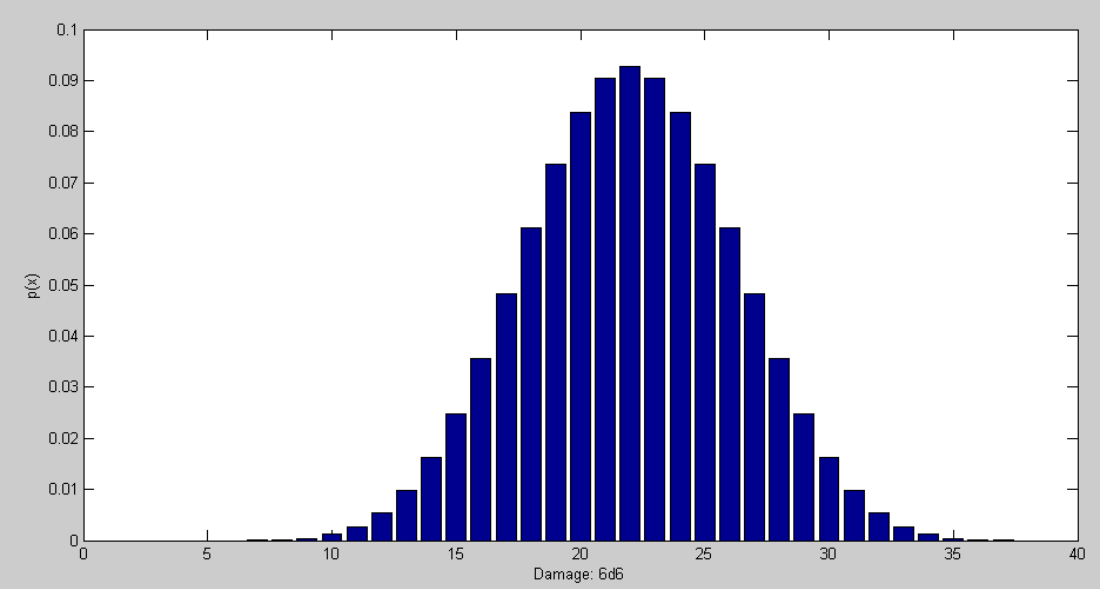
$$Y(z) = \left(\frac{1}{6}\right)\left(\frac{z^5+z^4+z^3+z^2+z+1}{z^6}\right)$$

4) Assume you roll 6 dice (6d6 or a level-6 fireball). Determine the

- pdf
- mean and standard deviation
- The probability of doing 30 or more damage with a level-6 fireball

Using convolution in Matlab

```
>> d1 = [0,1,1,1,1,1,1] / 6
>> d2 = conv(d1,d1);
>> d4 = conv(d2,d2);
>> d6 = conv(d4,d2);
>> N = [0:36];
>> bar(N,d6)
>> xlabel('Damage: 6d6');
>> ylabel('p(x)')
```



To find the mean and variance

$$\mu = \sum (p_i \cdot x_i)$$

$$\sigma^2 = \sum \left(p_i \cdot (x_i - \mu)^2 \right)$$

In Matlab

```
>> size(d6)

     1    37

>> N = [0:36];
>> x = sum(N .* d6)

x =   21.0000              mean

>> s2 = sum(d6 .* (N-x).^2 )

s2 =   17.5000             variance

>> s = sqrt(s2)

s =    4.1833              standard deviation
```

The probability of doing 30 damage:

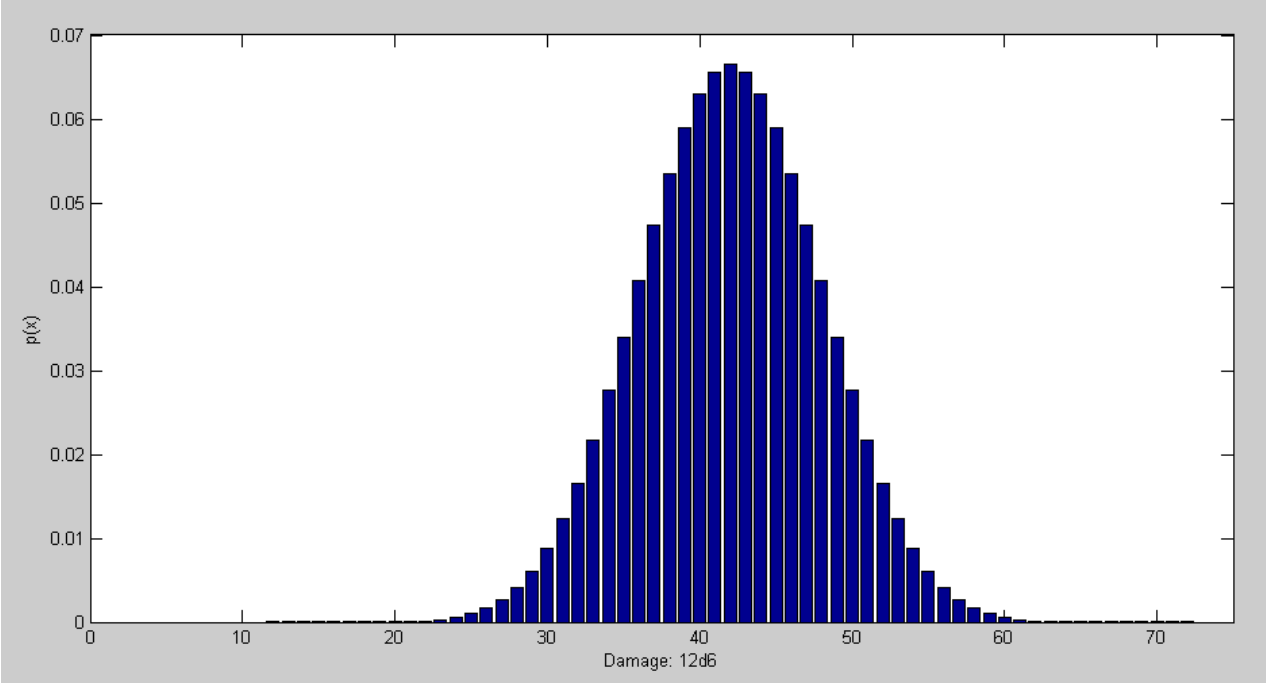
- note: the index is offset by one. d6(1) is for a score of 0 points

```
>> d6(31)

ans =    0.0098            the probbility of doing 30  damage


>> sum(d6(31:37))

ans =    0.0197            the probabilty of doing 30+ damage


>> sum(d6)

ans =    1.0000            all probabilies add to one
```

5) Assume you roll 12 dice (12d6 or a level-12 fireball). Determine the

- pdf
- mean and standard deviation
- The probability of doing 50 or more damage with a level-12 fireball

```
>> d12 = conv(d6,d6);
>> N = [0:72];
>> bar(N,d12_
>> bar(N,d12)
>> xlim([0,75])
>> xlabel('Damage: 12d6');
>> ylabel('p(x)')
```



Mean and variance:

```
>> N = [0:72];
>> x = sum(N .* d12)

x =    42                          mean

>> s2 = sum(d12 .* (N-x).^2)

s2 =   35.0000                     variance

>> s = sqrt(s2)

s =    5.9161                      standard deviation
```

Doing 50+ damage

- Indicies are offset by one again...

```
>> d12(51)
ans =    0.0276            probability of doing 50 damage
>> sum(d12(51:73))
ans =    0.1036            probability of doing 50+ damage
>> sum(d12)
ans =    1.0000            all probabilities must add to 1.000
```

6) Assme you roll two 4-sided dice, three 6-sided dice, and four 8-sided dice and take the sum:

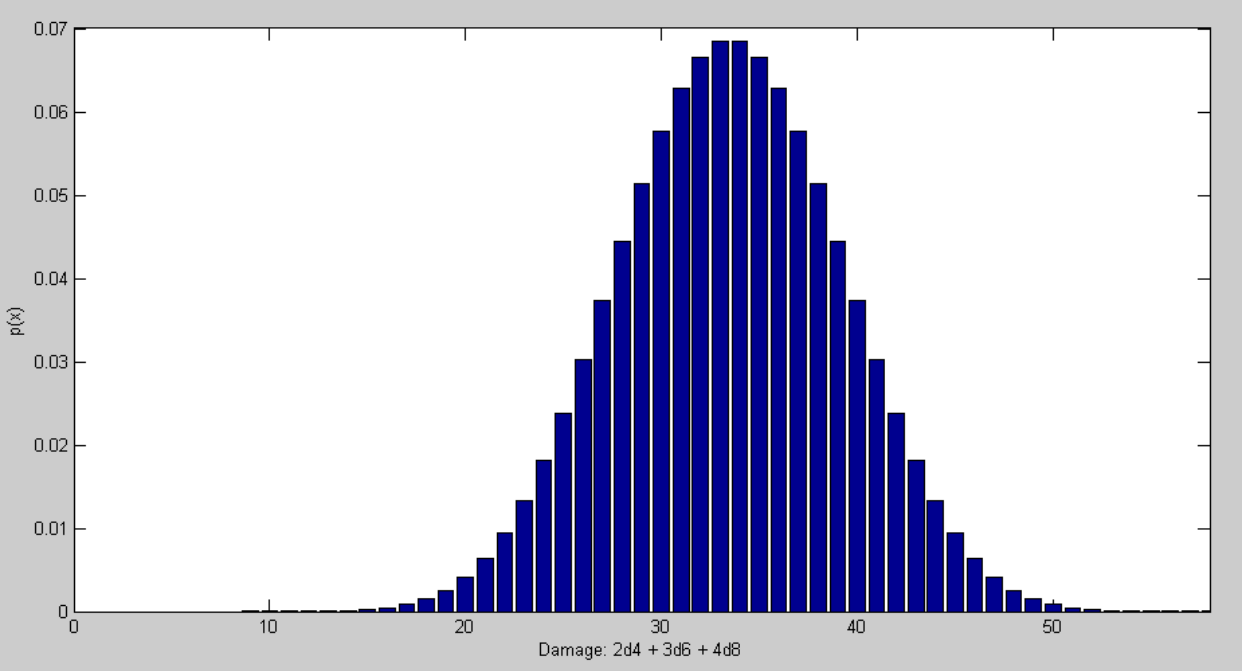
Y = 2d4 + 3d6 + 4d8

Determine

- The pdf for Y
- The mean and standard deviation of Y, and
- The probability that the sum is 40 or more

Using convolution in Matlab

```
>> d4 = [0,1,1,1,1] / 4;
>> d6 = [0,1,1,1,1,1,1] / 6;
>> d8 = [0,1,1,1,1,1,1,1,1] / 8;
>> d4x2 = conv(d4,d4);
>> d6x2 = conv(d6,d6);
>> d6x3 = conv(d6x2,d6);
>> d8x2 = conv(d8,d8);
>> d8x4 = conv(d8x2,d8x2);
>> d4d6 = conv(d4x2,d6x3);
>> d4d6d8 = conv(d4d6,d8x4);
>> N = [0:58];
>> bar(N,d4d6d8)
>> xlabel('Damage: 2d4 + 3d6 + 4d8');
>> ylabel('p(x)')
>> xlim([0,58])
>>
```



Mean & variance

```
>> x = sum(d4d6d8 .* N)
x =   33.5000                    mean
>> s2 = sum(d4d6d8 .* (N-x).^2)
s2 =   32.2500                   variance
>> s = sqrt(s2)
s =    5.6789                    standard deviation
```

Probability of 40+ damage

```
>> d4d6d8(41)
ans =    0.0373                  probability of 40 damage
>> sum(d4d6d8(41:58))
ans =    0.1484                  probability of 40+ damage
>> sum(d12)
ans =    1.0000                  all probabilities add to one
```