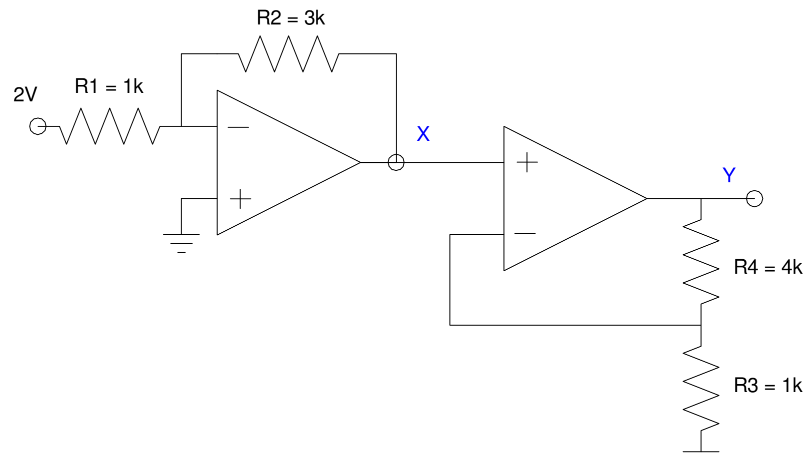
ECE 341 - Homework #9

Weibull Distribution, Central Limit Theorem. Summer 2023

Weibull Distribution

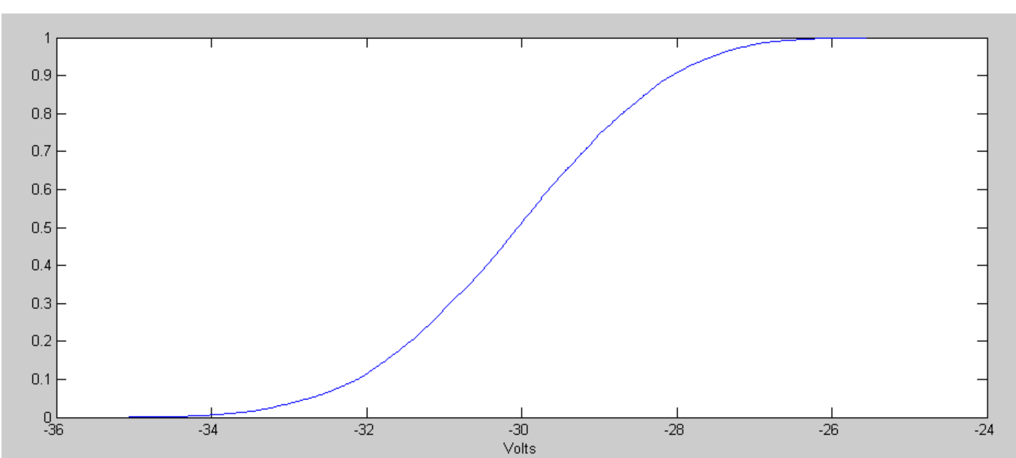
1) Determine and plot the cdf for the voltage, Y, in homework set #7 problem #3



Homework #7, problem #3. Find the pdf for the voltage at Y. All resistors are 5% tolerance

Matlab Code

```
result = [];

for i=1:10000
   R1 = (1 + 0.05*(2*rand-1)) * 1000;
   R2 = (1 + 0.05*(2*rand-1)) * 3000;
   R3 = (1 + 0.05*(2*rand-1)) * 1000;
   R4 = (1 + 0.05*(2*rand-1)) * 4000;
   Y = -(R2/R1) * (1 + R4/R3) * 2;
   result = [result ; Y];
end

result = sort(result);
p = [1:length(result)]';
p = p / length(result);

plot(result,p);
```



2) Determine and plot the pdf for this voltage using a Weibull approximation for the cdf

Save the data in a file

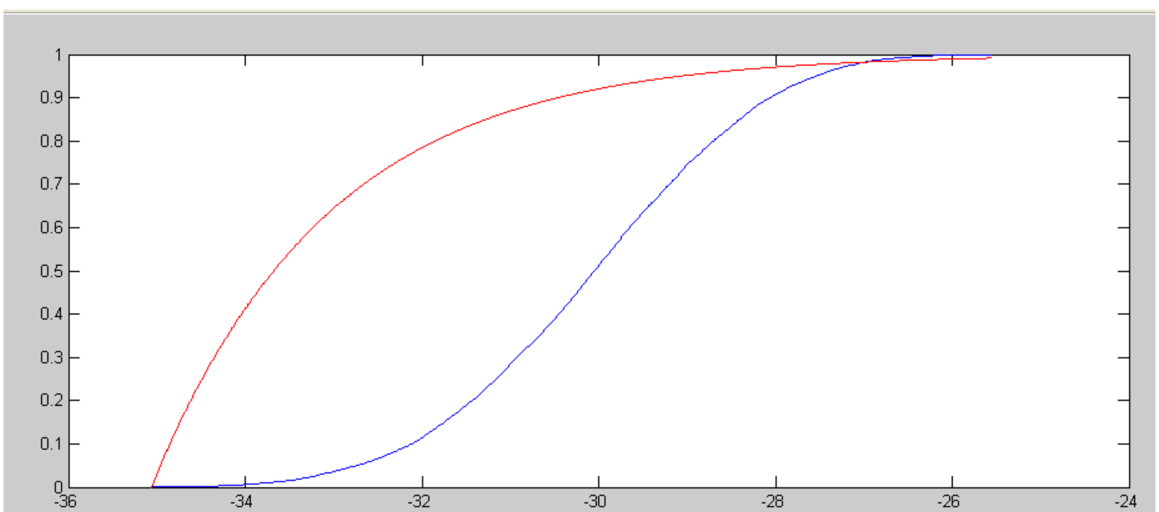
```
>> Volts = result;
>> save HW7 Volts p
```

Write an m-file to compare the data to a Weibull distribution

```
function y = Weibull(z)
% Weibull distribution curve fit
 k = z(1);
 L = z(2);

 load HW7

 x = Volts;
 X0 = min(x);

% p(Vce) = target
 W = 1 - exp( -( ((x-X0)/L) .^ k ) );

 e = p - W;

 plot(Volts,p,'b',Volts,W,'r');
 pause(0.01);
 y = sum(e.^2);

end
```

Try a couple of guesses for k and lambda:

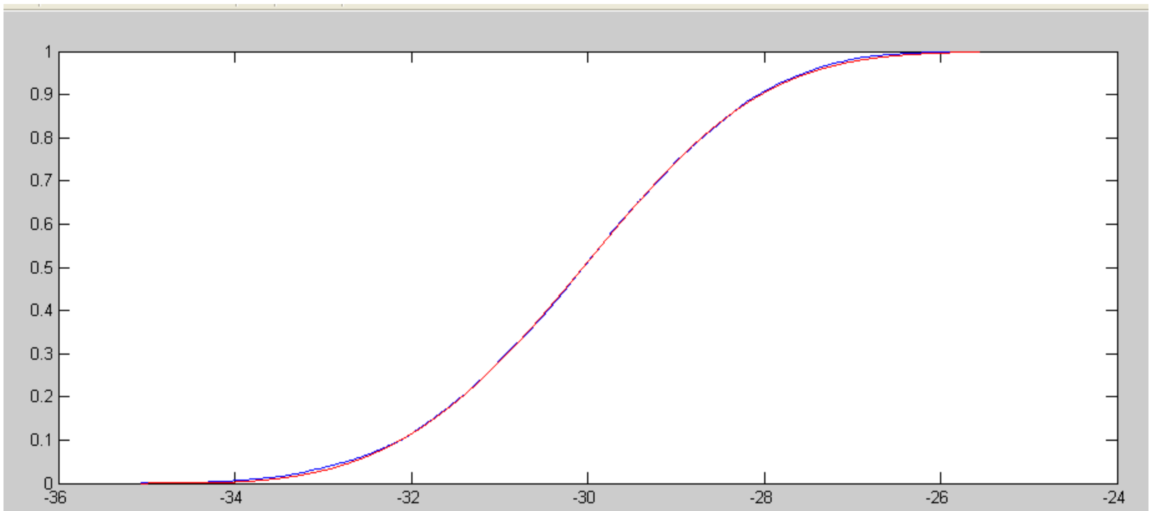
```
>> Weibull([1,2])

ans =  1.9566e+003
```



Use fminsearch to do better

```
>> [Z,e] = fminsearch('Weibull', [1,2])
```

```
            k        lambda
Z =      3.5479     5.5610
e =      0.0652
```



cdf for y (blue) and Weibull approximation (red)

So, the cdf is approximately

$$F(x) \approx \left(1 - \exp\left(-\left(\frac{x}{5.5610}\right)^{3.5479}\right)\right)$$

The pdf is

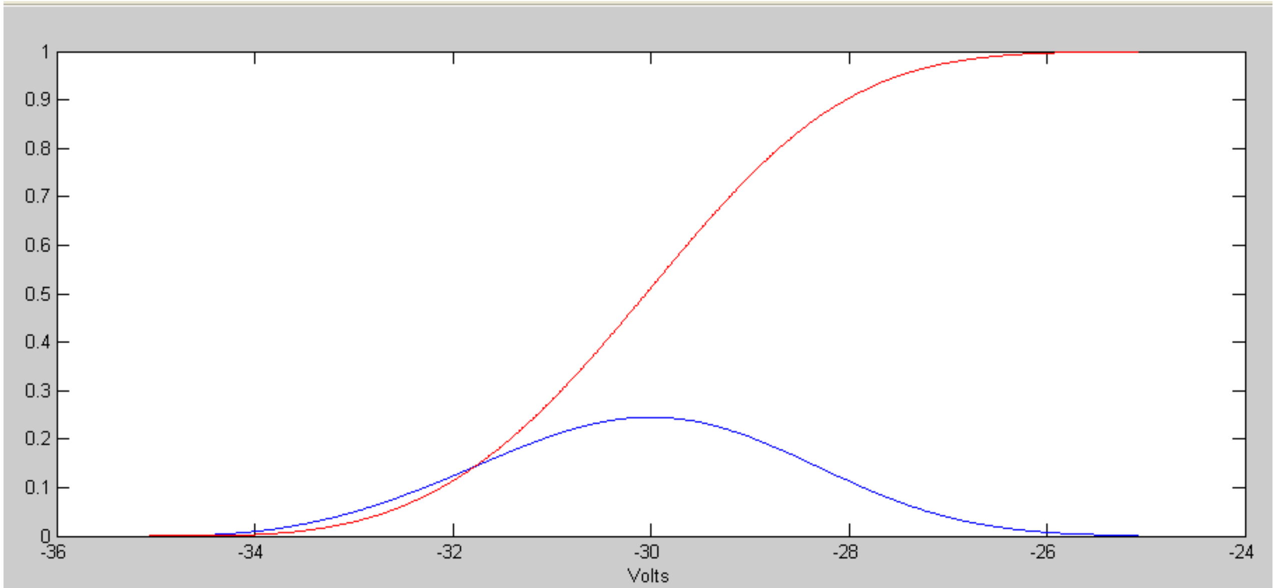
$$f(x) \approx \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right) u(k)$$

Shifting by x0

$$f(x - x_0) \approx \left(\frac{3.5479}{5.5610}\right)\left(\frac{x}{5.5610}\right)^{2.5479} \exp\left(-\left(\frac{x}{5.5610}\right)^{3.5479}\right) u(x - x_0)$$

Plotting this in Matlab

```
>> k = Z(1);
>> L = Z(2);
>> x0 = min(Volts)

x0 =  -35.0635

>> x = [0:0.01:10]';
>> pdf = (k/L) * (x/L).^(k-1) .* exp(-(x/L).^k);
>> cdf = 1 - exp(-(x/L).^k);
>> plot(x+x0,pdf,'b',x+x0,cdf,'r')
>> xlabel('Volts');
```



pdf (blue) and cdf (red) for the votlage at Y

Central Limit Theorem

The mean and standard deviation for a 4, 6, and 8-sided die are

Die	d4	d6	d8
mean	2.5	3.5	4.5
standard deviation	1.1180	1.7078	2.2191
variance	1.2500	2.9166	5.2487

5) Let Y be the sum of rolling six 6-sided dice (homework #4 problem 4):

Y = 6d6

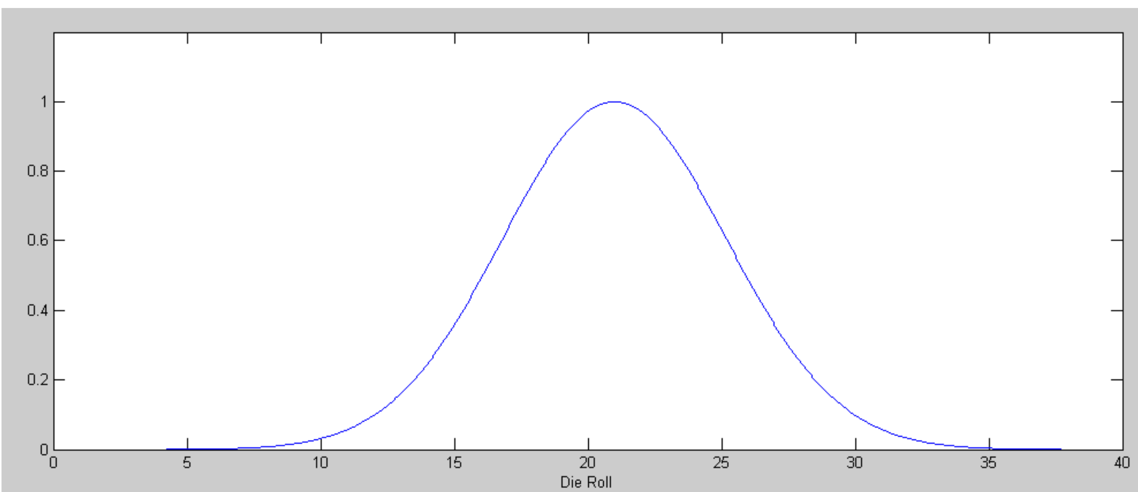
a) What is the mean and standard deviation of Y?

The mean and variance add

$$\mu_y = 6 \cdot 3.5 = 21$$

$$\sigma_y^2 = 6 \cdot 2.9166 = 17.4996$$

$$\sigma_y = \sqrt{17.4996} = 4.1833$$



Normal Approximation for the sum of 6d6

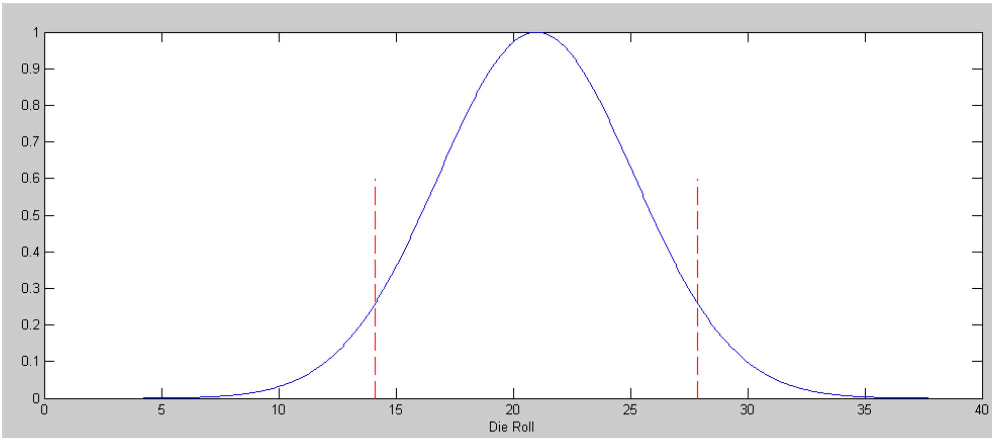
b) Using a normal approximation, what is the 90% confidence interval for Y?

The z-score for 5% tails is 1.64485

The 90% confidence interval is

$$\mu - 1.64485\sigma < roll < \mu + 1.64485\sigma$$

$$14.1192 < roll < 27.8808$$



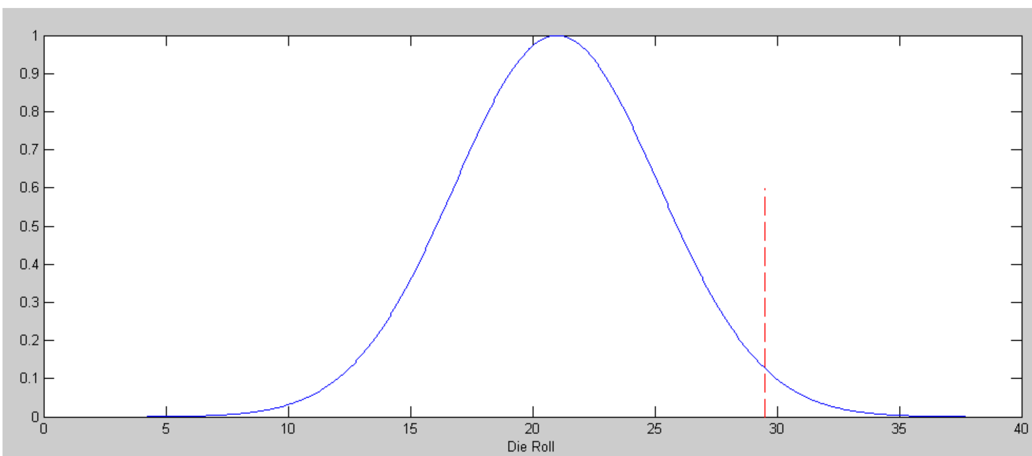
90% confidence interval for the sum of 6d6 (Normal approximation)

c) Using a normal approximation, what is the probability that the sum the dice will be more than 29.5?

Find the z-score

$$z = \left(\frac{29.5-\mu}{\sigma}\right) = \left(\frac{29.5-21}{4.1833}\right) = 2.0319$$

From StatTrek, this corresponds to a probability of 2.108%



d) Compare these results to the actual odds (from homework #4)

From homework #4, the actual odds are 1.94%

- found using convolution

6) Let Y be the sum of rolling twelve 6-sided dice (homework #4 problem 5):

Y = 12d6

a) What is the mean and standard deviation of Y?

$$\mu = 12 \cdot 3.5 = 42$$

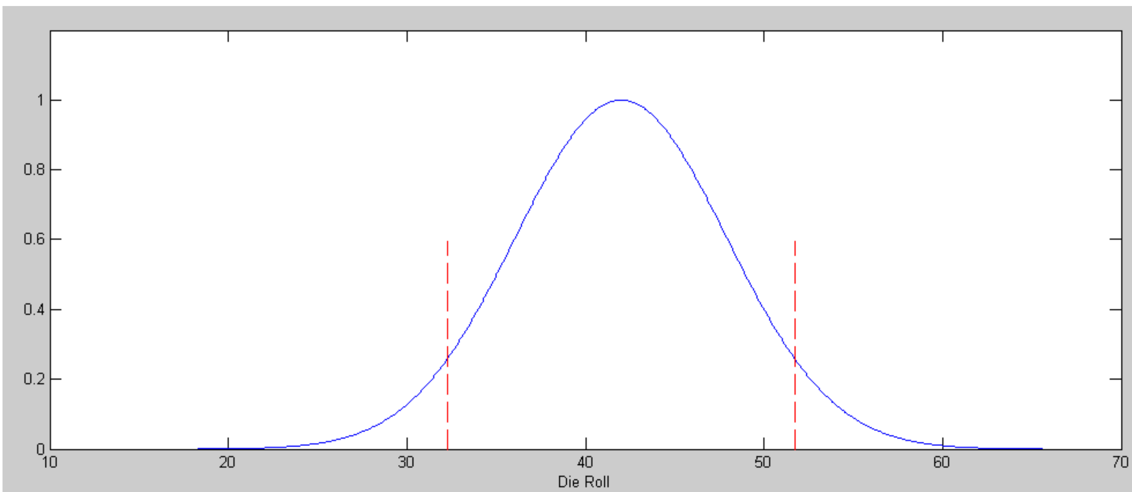
$$\sigma^2 = 12 \cdot 2.9166 = 34.9992$$

$$\sigma = 5.9160$$

b) Using a normal approximation, what is the 90% confidence interval for Y?

$$\mu - 1.64485\sigma < roll < \mu + 1.64485\sigma$$

$$32.2690 < roll < 51.7310$$



Normal Approximation for the sum of 12d6 & 90% Confidence Interval

c) Using a normal approximation, what is the probability that the sum the dice will be more than 49.5?

Find the z-score

$$z = \left(\frac{49.5 - \mu}{\sigma}\right) = \left(\frac{49.5 - 42}{5.9160}\right) = 1.2170$$

From StatTrek, this corresponds to a tail area of **11.18%**

d) Compare these results to the actual odds (from homework #4)

Area = 10.36%

found using convolution

7) Let Y be the sum of rolling 2d4 + 3d6 + 4d8 (homework #4 problem 6)

Y = 2d4 + 3d6 + 4d8

a) What is the mean and standard deviation of Y?

The means and variances add

$$\mu = 2 \cdot 2.5 + 3 \cdot 3.5 + 4 \cdot 4.5$$

$$\mu = 33.50 \qquad \textit{mean}$$

$$\sigma^2 = 2 \cdot 1.25 + 3 \cdot 2.9166 + 4 \cdot 5.2487$$

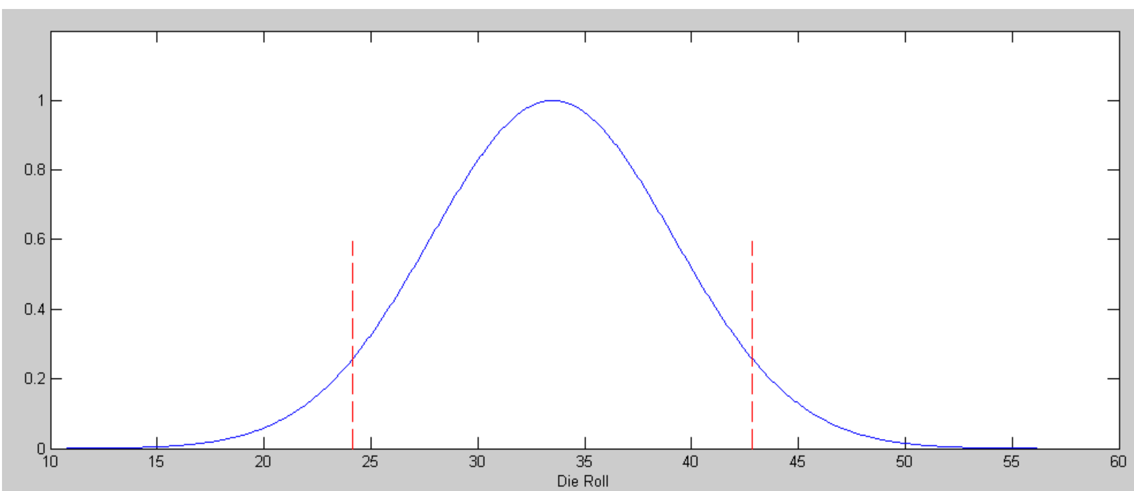
$$\sigma^2 = 32.2446$$

$$\sigma = 5.6784 \qquad \textit{standard deviation}$$

b) Using a normal approximation, what is the 90% confidence interval for Y?

$$\mu - 1.64485\sigma < roll < \mu + 1.64485\sigma$$

$$24.1598 < roll < 42.8402$$



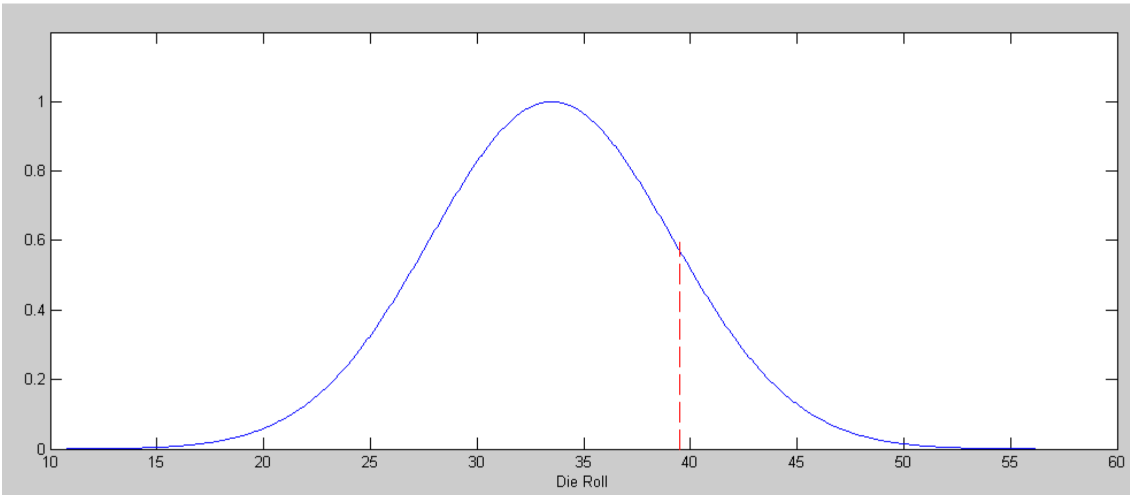
Normal approximation for 2d4 + 3d6 + 4d8 & 90% confidence interval

c) Using a normal approximation, what is the probability that the sum the dice will be more than 39.5?

Compute the z-score

$$z = \left(\frac{39.5-\mu}{\sigma}\right) = \left(\frac{39.5-33.5}{5.6784}\right) = 1.05663$$

From a standard normal table, this corresponds to a probability of 14.837%



Normal approximation for Y. Area to the right of 3.9 = p(rolling 40 or more) (approx)

d) Compare these results to the actual odds (from homework #4)

Actual probability = 14.84% (from convolution)