ECE 341 - Homework #13

t-Tests with Two Populations. Summer 2023

Let

- X = 5d10 (the sum of five 10-sided dice) plus 0.5 (X wins on ties)
- Y = 2d4 + 3d6 + 4d8

Monte-Carlo Simulation

1) Run a Monte-Carlo simulation with 100,000 rolls for X and Y. From this, determine the probability that X will win any given game.

In Matlab

```
WIN = 0;
for n=1:1e5
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = sum(d10) + 0.5;
   Y = sum(d4) + sum(d6) + sum(d8);
   if(X > Y)
       WIN = WIN + 1;
   end
end
disp(WIN / 1e5)
  0.2630
```

X has a 26.30% chance of winning any given game

t-Test: Sample Size = 4

2) Take four measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test.

In Matlab

```
X = [];
Y = [];
for n=1:4
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end
disp([X,Y])
     X      Y
   37.5000   36.0000
   37.5000   41.0000
   24.5000   27.0000
   23.5000   38.0000
```

Find the mean and standard deviation of X and Y

```
>> Mx = mean(X)

Mx =   30.7500

>> Sx = std(X)

Sx =    7.8049

>> My = mean(Y)

My =   35.5000

>> Sy = std(Y)

Sy =    6.0277
```

Find the probability that X will win any given game. Create a new variable, W

$$W = X - Y$$

The mean and standard deviation of W are then

```
>> Mw = Mx - My

Mw =   -4.7500

>> Sw = sqrt(Sx^2 + Sy^2)

Sw =    9.8615
```

```
>> t = Mw/Sw

t =   -0.4817
```

Convert this to a probability using a t-table with 3 degrees of freedom (sample size = 4)

p = 0.33148

The probability of X winning any given game is

26.30%	**Monte-Carlo with sample size of 100,000**
33.148%	**t-test with a sample size of 4**

t-Test: Sample Size = 20

3) Take twenty measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test

```
X = [];
Y = [];
for n=1:20
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end
disp([X,Y])

   23.5000   39.0000
   30.5000   32.0000
   31.5000   31.0000
   28.5000   26.0000
   37.5000   29.0000
   25.5000   28.0000
   29.5000   23.0000
   28.5000   30.0000
   29.5000   24.0000
   27.5000   36.0000
   38.5000   39.0000
   26.5000   22.0000
   20.5000   31.0000
   30.5000   32.0000
   31.5000   32.0000
   32.5000   35.0000
   21.5000   41.0000
   26.5000   30.0000
   18.5000   39.0000
   20.5000   35.0000

>> Mw = mean(X) - mean(Y)

Mw =   -3.7500

>> Sw = sqrt(var(X) + var(Y))

Sw =    7.6611

>> t = Mw/Sw

t =   -0.4895
```

From StatTrek, this corresponds to a probability of 31.503%

t-Test: Sample Size = 100

4) Take 100 measurements of X and Y. From this data, determine

- The mean and standard devation of X
- The mean and standard devation of Y
- The probability that X will win any given game using a student-t test

```
X = [];
Y = [];
for n=1:100
   d4 = ceil(4*rand(1,2));
   d6 = ceil(6*rand(1,3));
   d8 = ceil(8*rand(1,4));
   d10 = ceil(10*rand(1,5));

   X = [X ; sum(d10) + 0.5];
   Y = [Y; sum(d4) + sum(d6) + sum(d8)];
   end


>> Mw = mean(X) - mean(Y)

Mw =   -5.1300

>> Sw = sqrt(var(X) + var(Y))

Sw =    9.2028

>> t = Mw/Sw

t =   -0.5574
```

From StatTrek, this corresponds to a probability of 28.926%

Case	# rolls	p(X wins)
Mote-Carlo	100,000	26.3%
t-test	4	33.148%
t-test	20	21.50%
t-test	100	28.92%

>>

Reaction Time

5) Go to the Human Benchmark Dashboard

`https://humanbenchmark.com/tests/reactiontime`

(population A): Record your reaction time with both eyes open

Time (ms) = {310, 257, 278, 351, 292}

(population B): Record a different reaction time (20 minutes later. Does practice improve my scrores?)

Time(ms) = {285, 272, 284, 280, 264}

6) From your results, determine the probability that

- A's time will be less than B's time next time you run the experiment
- A's average time is less than B's average time

```
>> A = [310, 257, 278, 351, 292];
>> B = [285, 272, 284, 280, 264];
>> Xa = mean(A)

Xa =  297.6000

>> Sa = std(A)

Sa =   35.5992

>> Xb = mean(B)

Xb =   277

>> Sb = std(B)

Sb =    8.8882
```

Create a new variable: W = A - B

```
>> Xw = Xa - Xb

Xw =   20.6000

>> Sw = sqrt(Sa^2 + Sb^2)

Sw =   36.6920
```

Find the t-score for a single game

```
>> t = Xw / Sw

t =    0.5614
```

This corresponds to a probability of p = 0.30224

Player A has a 30.224% chance of winning the next game (lower reaction time)

There is a 30.224% chance my reaction time after practice will be worse for one game

Find the t-score for the populations (A's average is less than B's average, or A wins an infinite series)

```
>> Sw = sqrt((Sa^2)/5 + (Sb^2)/5)
Sw =   16.4091

>> t = Xw / Sw
t =    1.2554

>>
```

This corresponds to a probability of 0.13883

Player A has a 13.833% chance of winning an infinite series

There is a 13.833% chance my overall reaction time gets worse with practice

Conclusion: There is learning going on.

- I'm 86.2% certain my reaction time gets better with practice

Aim Trainer

7) Go to the Human Benchmark Dashboard

`https://humanbenchmark.com/tests/aim`

(population A): Record your time to hit 30 targets with both eyes open

- repeat to get at least two measurements

(population B): Record your time to hit 30 targets with a different condition (different person, one eye closed, opposite hand, your pick)

- repeat to get at least two measurements

A (dominant hand)

Time (ms) = {752, 661, 758}

B (nondominant hand)

Time(ms) = {1049, 1025, 908}

8) From your results, determine the probability that

- A's time will be less than B's time next time you run the experiment
- A's average time is less than B's average time

```
>> A = [752, 661, 758];
>> B = [1049, 1025, 908];
>> Xa = mean(A)

Xa =  723.6667

>> Sa = std(A)

Sa =   54.3538

>> Xb = mean(B)

Xb =   994

>> Sb = std(B)

Sb =   75.4387
```

Form W = A - B:

```
>> Xw = Xa - Xb

Xw = -270.3333

>> Sw = sqrt(Sa^2 + Sb^2)

Sw =   92.9803

>> t = Xw / Sw

t =   -2.9074
```

From StatTrek, this corresponds to a probability of 0.05307

There is a 5.307% chance my non-dominant hand will win the next game (lower reaction time)

Population:

```
>> Sw = sqrt((Sa^2)/3 + (Sb^2)/3)

Sw =   53.6822

>> t = Xw / Sw

t =   -5.0358
```

From StatTrek, this corresponds to a probabiliyty of 0.01862

There is a 1.862% chance my non-dominant hand has faster reaction times than my dominant hand