ECE 341 - Homework #15

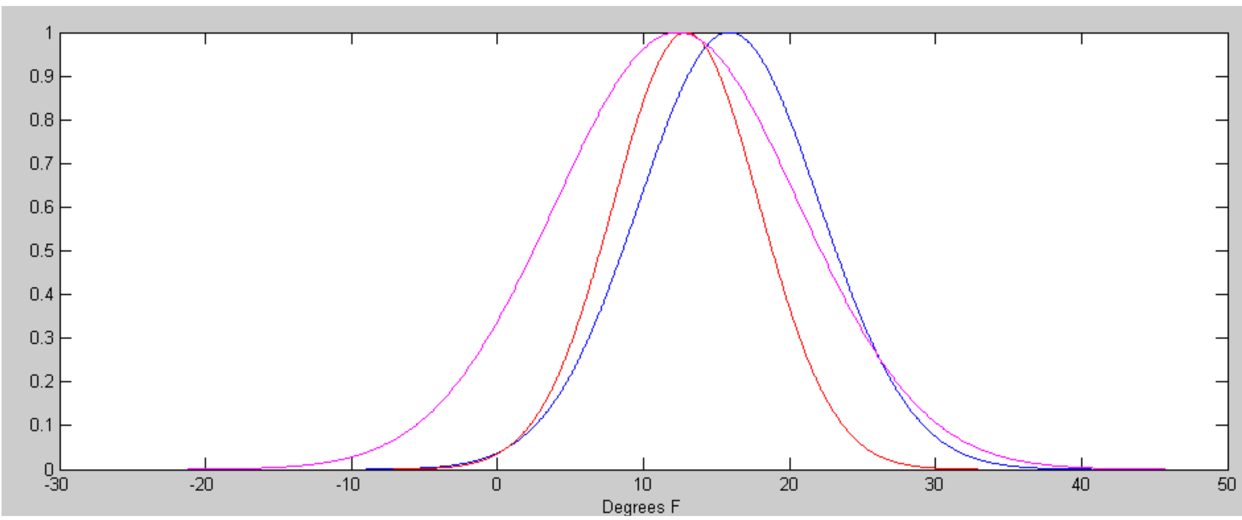
F-Test and ANOVA. Summer 2023

Test of a 3+ Populations

1) The average temperature in Fargo for the past 10 years is: (units: degrees F)

- Source: Hector Airport

		mean	std	n
A	Dec	15.8800	6.2097	10
B	Jan	12.9380	5.0091	10
C	Feb	12.3210	8.2684	10



pdf for A (blue), B (red), and C (magenta).

Determine if the means are the same using an ANOVA test.

Determine the global mean

$$\overline{G} = \left(\frac{1}{N}\right)\left(n_a\overline{A} + n_b\overline{B} + n_c\overline{C}\right)$$

Determine MSSb and MSSw

$$MSS_b = \left(\frac{1}{k-1}\right)\left(n_a\left(\overline{A}-\overline{G}\right)^2 + n_b\left(\overline{B}-\overline{G}\right)^2 + n_c\left(\overline{C}-\overline{G}\right)^2\right)$$

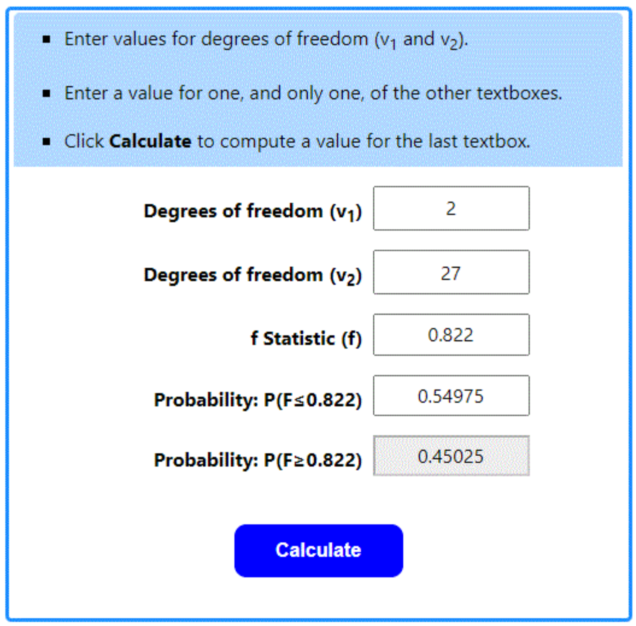
$$MSS_w = \left(\frac{1}{N-k}\right)\left((n_a-1)s_a^2 + (n_b-1)s_b^2 + (n_c-1)s_c^2\right)$$

Matlab Code:

```
Xa = 15.88;
Sa = 6.2097;
Xb = 12.938;
Sb = 5.0091;
Xc = 12.321;
Sc = 8.2684;
Na = 10;
Nb = 10;
Nc = 10;
k = 3;
N = Na + Nb + Nc
G = (Na*Xa + Nb*Xb + Nc*Xc) / N
MSSb = (Na*(Xa-G)^2 + Nb*(Xb-G)^2 + Nc*(Xc-G)^2) / (k-1)
MSSw = ((Na-1)*Sa^2 + (Nb-1)*Sb^2 + (Nc-1)*Sc^2) / (N-k)
F = MSSb / MSSw
```

Result:

```
N =       30
G =       13.7130
MSSb =    36.1709
MSSw =    44.0060
F =        0.8220
```



Now calculate the probability

- The numerator has (k-1) degrees of freedom
- The denominator has (n-k) degrees of freedom

This corresponds to a probability of 0.54975

Translation:

a) I can reject the null hypothesis (the means are the same) with 54% certainty.

b) The populations may be different. Combining these into an overall population (winter) may not be valid.

You can also get the same answer with an ANOVA table

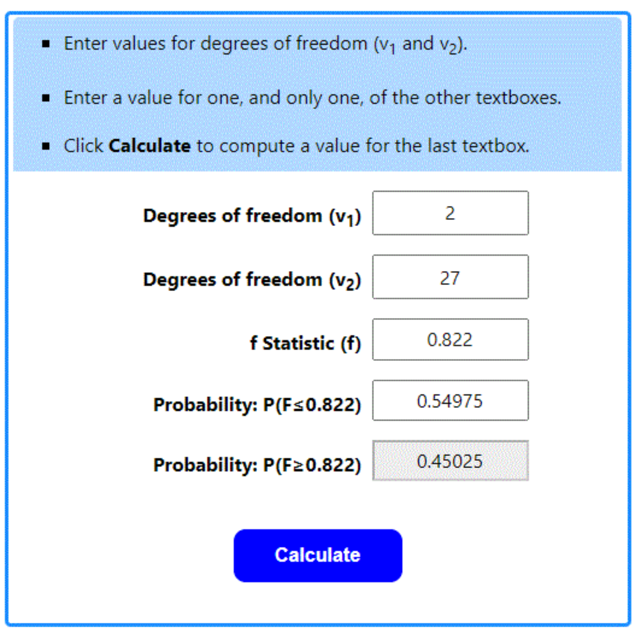
A	B	C	A	B	C
			6.2097 std(A)	5.0091 std(B)	8.2684 std(C)
Na = 10	Nb = 10	Nc = 10	347.04 sum of squares	225.81 sum of squares	615.29 sum of squres
N = 30			1320.2 sum of squares		
15.88 mean(A)	12.938 mean(B)	12.321 mean(C)	MSSw = 44.0060		
13.713 (G: Global Mean)					
46.9589 Na $(A - G)^2$	6.0062 Nb $(B - G)^2$	19.3766 Nc $(C - G)^2$			
72.3418 sum of squres					
MSSb = 36.1709					

```
F = MSSb / MSSw
F =       0.8220
```

Now use an F table with

- numerator = 2 degrees of freedom (k-1)
- denominator = 27 degrees of freedom (N-k)

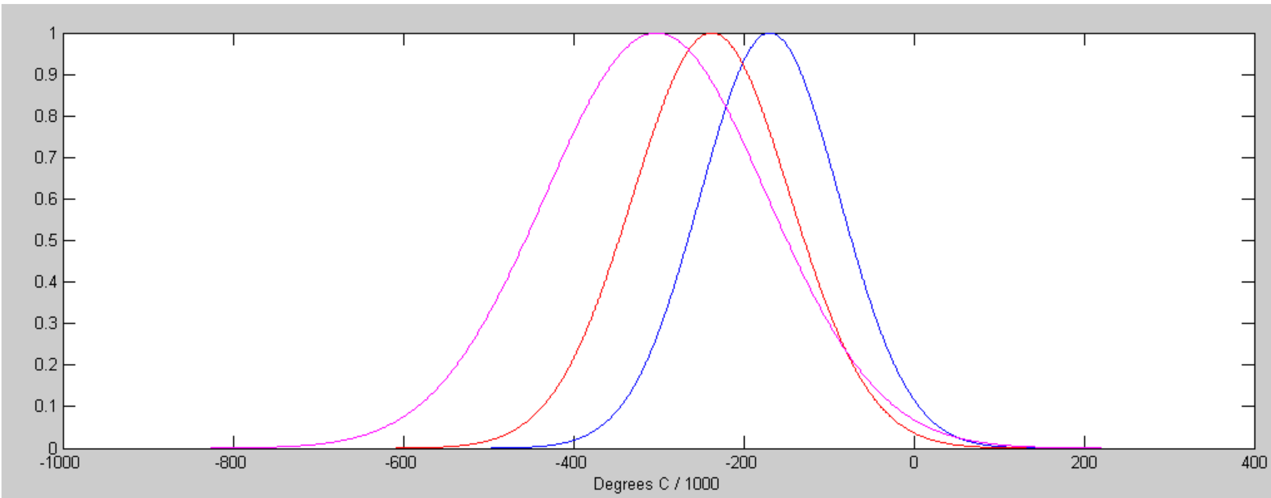
This corresponds to a probability of 0.54975



2) The global average temperature for three decades are (units: degrees C x 1000)

- source: NASA Goddard

		mean	std	n
A	1880-1889	-169.2	81.8	10
B	1890 - 1899	-237.5	92.68	10
C	1900 - 1909	-302.5	130.8	10



pdf for 1880's (blue), 1890's (red), 1900's (magenta)
Note: temperatures were dropping at the start of the 1900's (with worries of another ice age coming on)

Determine if the means are the same using an ANOVA test.

```
Xa = -169.2;
Sa =  81.8;
Xb =  -237.5;
Sb =  92.68;
Xc =  -302.5;
Sc =  130.8;
Na =  10;
Nb =  10;
Nc =  10;
k = 3;
N = Na + Nb + Nc
G = (Na*Xa + Nb*Xb + Nc*Xc) / N
MSSb = (Na*(Xa-G)^2 + Nb*(Xb-G)^2 + Nc*(Xc-G)^2) / (k-1)
MSSw = ((Na-1)*Sa^2 + (Nb-1)*Sb^2 + (Nc-1)*Sc^2) / (N-k)
F = MSSb / MSSw
```

Result:

```
N    =    30
G    = -236.4000
MSSb =  44,431
MSSw =  10,796
F    =    4.1153
```

Now use an F table with

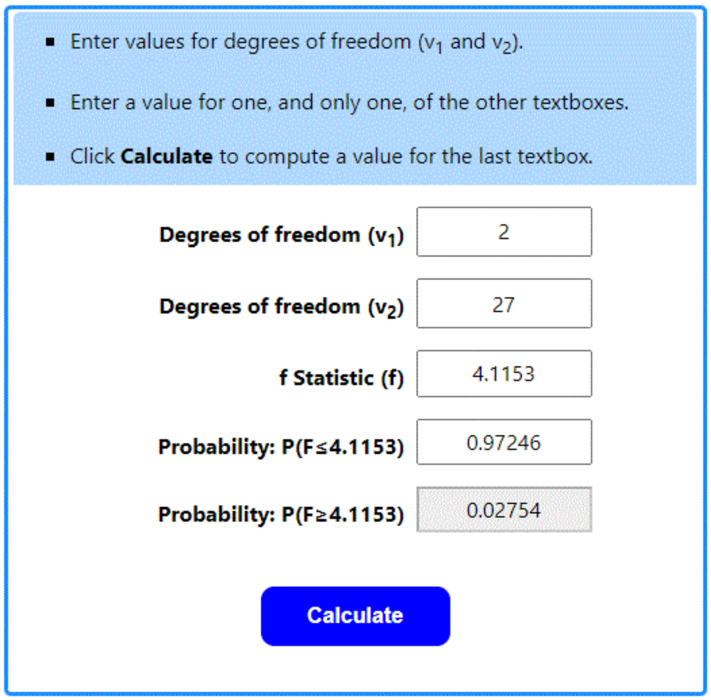
- numerator = 2 degrees of freedom (k-1)
- denominator = 27 degrees of freedom (N-k)

This corresponds to a probability of 0.97246

Translation

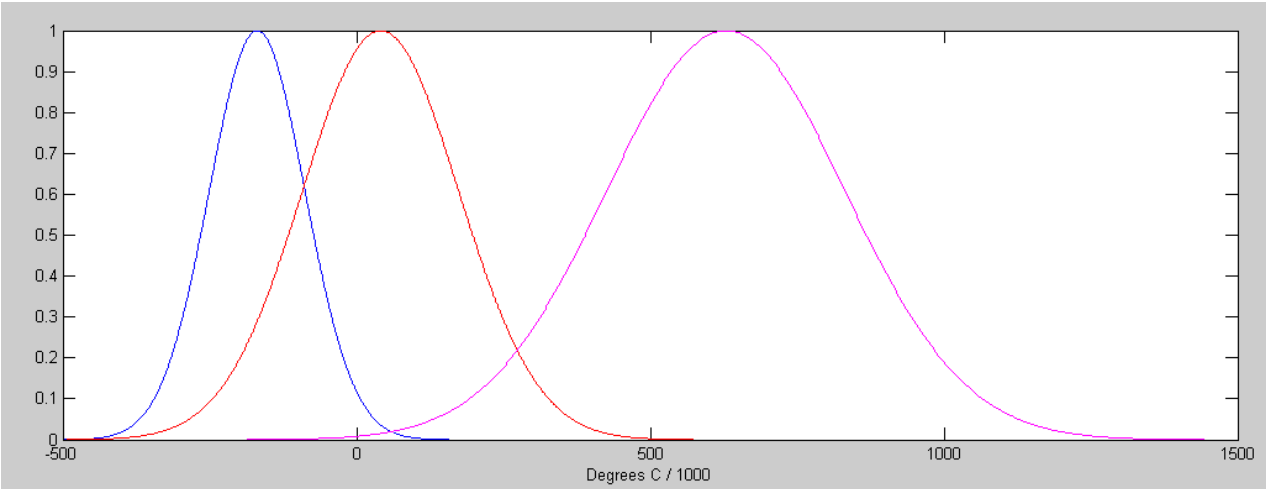
I'm 97% certain that these three populations have different means.

You shouldn't combine these into a single population - the means are changing.



3) The global average temperature for three time-spans are (units: degrees C x 1000)

		mean	std	n
A	1880-1889	-169.2	81.8	10
B	1940 - 1959	41.05	133.4	20
C	1992 - 2021	628.94	203.63	30



pdf for 1800 (blue), 1940 (red), 1992 (magenta)

Determine if the means are the same using an ANOVA test.

```
Xa = -169.2;
Sa =  81.8;
Xb =  41.05;
Sb =  133.4;
Xc =  628.94;
Sc =  203.63;
Na =  10;
Nb =  20;
Nc =  30;
k = 3;
N = Na + Nb + Nc
G = (Na*Xa + Nb*Xb + Nc*Xc) / N
MSSb = (Na*(Xa-G)^2 + Nb*(Xb-G)^2 + Nc*(Xc-G)^2) / (k-1)
MSSw = ((Na-1)*Sa^2 + (Nb-1)*Sb^2 + (Nc-1)*Sc^2) / (N-k)
F = MSSb / MSSw
```

Results

```
N    =    60
G    =  299.9533
MSSb =  3.3943e+006
MSSw =  2.8085e+004
F    =  120.8601
```

Now use an F table with

- numerator = 2 degrees of freedom (k-1)
- denominator = 57 degrees of freedom (N-k)

This corresponds to a probability of 1.0000

Translation: I'm over 99.995% certain that these populations have different means.