ECE 341 - Test #1

Combinations, Permitations, and Discrete Probability - Summer 2023

Open-Book, Open Notes. Calculators & Tarot cards allowed. Chegg or other people *not* allowed.

1) Permutations & Combinations in Bison Poker

Assume a 50-card deck of playing cards

- 10 card values (ace .. ten)
- Five suits (clubs, diamonds, hearts, spades, bison)

Each player is dealt six cards. The best 5-card hand makes up your band in Bison poker.

Calculate the odds of being dealt a full-house:

- best five cards include a 3-of-a-kind and a pair
- hand = {xxxyyz or xxxyyy},
- {x, y, z} all have different values, suit doesn't matter.

Number of hands possible:

$$N = \binom{50}{6} = 15,890,700$$

Hands = xxx yy z

(10c1 value for x)(5c3 x's)(9c1 value for y)(5c2 y's)(8c1 value for z)(5c1 z)

$$M_1 = \binom{10}{1}\binom{5}{3}\binom{9}{1}\binom{5}{2}\binom{8}{1}\binom{5}{1} = 360,000$$

Hand = xxx yyy

(10c2 values for x and y)(5c3 five x's choose three)(5c3 five y's choose three)

$$M_2 = \binom{10}{2}\binom{5}{3}\binom{5}{3} = 4,500$$

The total number of hands that result in a full-house are

$$M = M_1 + M_2 = 364,500$$

The odds are then

$$p = \left(\frac{M_1+M_2}{N}\right) = 2.2938\%$$

2) Conditional Probability

Assume you play the following game:

- Start by rolling a four-sided die. Then,
- Whatever number you rolled (1-4), roll that many six-sided dice:
- Your score is the sum of all dice rolled

(For example, if you roll a 3 on a 4-sided die, you then roll three six-sided dice)

Determine the probability that the sum of all dice rolled is seven.

Case 1: d4 = 1

roll a six on the d6 for the total to be seven

$$p_1 = p(7 | d4 = 1) p(d4 = 1)$$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{4}\right) = \frac{1}{24}$$

case 2: d4 = 2

roll a five with two 6-sided dice

(1,4), (2,3), (3,2), 4,1) by enumeration

$$p(2d6 = 5) = \frac{4}{36}$$

$$p_2 = p(7 | d4 = 2) p(d4 = 2)$$

$$= \left(\frac{4}{36}\right)\left(\frac{1}{4}\right) = \left(\frac{1}{36}\right)$$

case 3: d4 = 3

roll a four with three 6-sided dice

(1,1,2), (1,2,1), (2,1,1) by enumeration

$$p(3d6 = 4) = \frac{3}{216}$$

$$p_3 = p(7 | d4 = 3) = \left(\frac{3}{216}\right)\left(\frac{1}{4}\right) = \left(\frac{3}{864}\right)$$

case 4: d4 = 4

roll a three with four 6-sided dice

p = 0

Total Odds:

$$p = p_1 + p_2 + p_3 + p_4$$

$$p = \left(\frac{1}{24}\right) + \left(\frac{1}{36}\right) + \left(\frac{3}{864}\right) + 0$$

$$p = 7.5442\%$$

3. Binomial Distribution

Let X be the number of 1's and 2's you get when rolling fifteen 6-sided dice.

- die roll = {1, 2}   1 point
- die roll = {3, 4, 5, 6}   0 points

Determine the probability that X = m+1 where m is your birth month (1..12)

m+1 birth month plus one (2..13)	probability X = m+1 with 15 die rolls
6	**p = 17.8589%**

$$p = \binom{15}{6}\left(\frac{1}{3}\right)^{6}\left(\frac{2}{3}\right)^{9}$$

$$p = 0.178589$$

4. Convolution

Use convolution to determine the product of two polynomials:

$$y(x) = (3x+4)(2x^2+6x+9)$$

Note: Show your work to get full credit

a) x^0 term (determine using convolution) = 36

	-2	-1	0	1	2	3
$9+6x+2x^2$	0	0	9	6	2	0
$3x+4$	0	3	4	0	0	0
product	0	0	36	0	0	0

b) x^1 term (determine using convolution) = 51

	-2	-1	0	1	2	3
$9+6x+2x^2$	0	0	9	6	2	0
$3x+4$	0	0	3	4	0	0
product	0	0	27	24	0	0

c) x^2 term (determine using convolution) = 26

	-2	-1	0	1	2	3
$9+6x+2x^2$	0	0	9	6	2	0
$3x+4$	0	0	0	3	4	0
product	0	0	0	18	8	0

d) x^3 term (determine using convolution) = 6

	-2	-1	0	1	2	3
$9+6x+2x^2$	0	0	9	6	2	0
$3x+4$	0	0	0	0	3	4
product	0	0	0	0	6	0

5. Geometric & z-Transforms

Let

- X be the number of rolls of an 10-sided die until you get a one with the following moment-generating function:

  $$X = \left(\frac{0.1}{z-0.9}\right)$$

- Y be the number of rolls of a 20-sided die until you get a one with the following moment-generating function:

  $$Y = \left(\frac{0.05}{z-0.95}\right)$$

Determine the pdf for W = X + Y using z-transforms

(the number of times you have to roll a 10-sided die until you get a 1, then roll a 20-sided die until you get a 1)

$$W = \left(\frac{0.1}{z-0.9}\right)\left(\frac{0.05}{z-0.95}\right)$$

doing partial fractions

$$W = \left(\frac{-0.1}{z-0.9}\right) + \left(\frac{0.1}{z-0.95}\right)$$

multiply both sides by z

$$zW = \left(\frac{0.1z}{z-0.95}\right) - \left(\frac{0.1z}{z-0.9}\right)$$

Take the inverse z-transform

$$zw(k) = \left(0.1(0.95)^k - 0.1(0.9)^k\right)u(k)$$

Divide by z

$$w(k) = \left(0.1(0.95)^{k-1} - 0.1(0.9)^{k-1}\right)u(k-1)$$

You can also write this as

$$w(k) = \left(0.105263(0.95)^k - 0.111111(0.9)^k\right)u(k-1)$$