

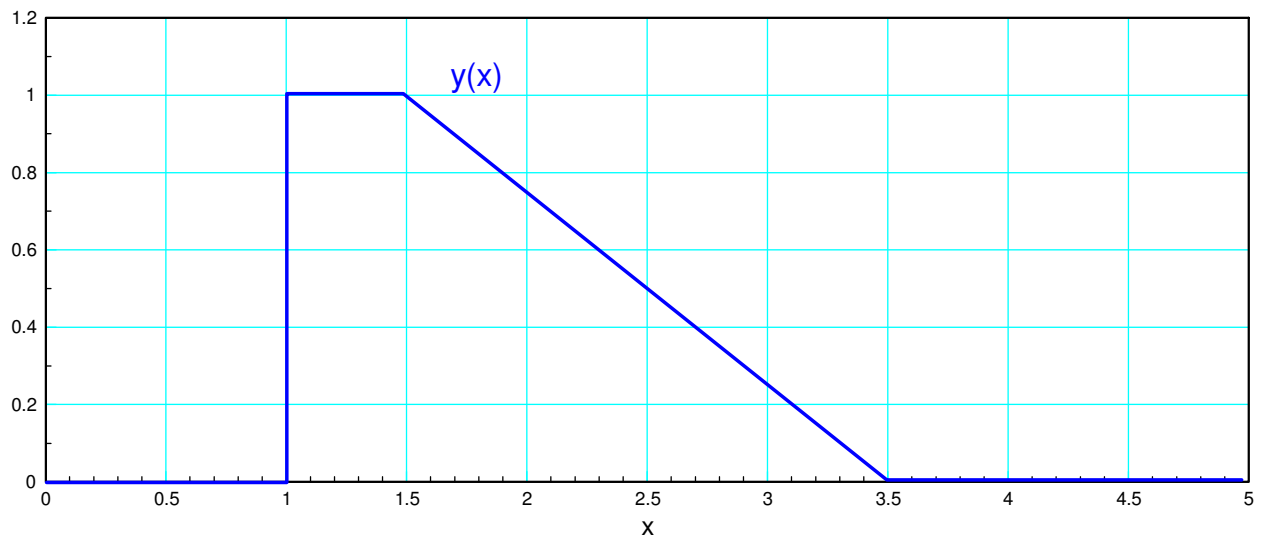
ECE 341 - Test #2

Continuous Probability - Summer 2023

1) Continuous PDF

Let

$$y = \begin{cases} \alpha \cdot 1 & 1 > x > 1.5 \\ \alpha \cdot (1.75 - 0.5x) & 1.5 < x < 3.5 \\ 0 & \text{otherwise} \end{cases}$$



a) Determine the scalar, α , so that this is a valid pdf (i.e. the total area = 1.0000)

b) Determine the moment generating function (i.e. LaPlace transform)

2) Uniform Distributions

Let A, B, and C be continuous uniform distributions

- A = uniform over the interval of (0, 2)
- B = uniform over the interval of (1, 4),
- $X = A + B$

Use moment generating functions to determine the pdf for X (i.e. LaPlace Transforms)

3) Gamma CDF

Let A, B be continuous exponential distributions:

- A has a mean of 3 seconds $a(t) = \frac{1}{3}e^{-t/3}u(t)$ $A(s) = \left(\frac{1/3}{s+1/3}\right)$
- B has a mean of 4 seconds $b(t) = \frac{1}{4}e^{-t/4}u(t)$ $B(s) = \left(\frac{1/4}{s+1/4}\right)$

Determine the equation for the cdf (i.e. the integral of the pdf) for $Y = A + B$ using moment generating functions (i.e. Laplace transforms)

4) Central Limit Theorem

The Dungeons and Dragons spell *Insect Plague* does 4-40 damage (the sum of four 10-sided dice). Use a normal approximation to determine the probability that the total damage is less than 9.5.

Note: for a single 10-sided die

- mean = 5.5
- variance = 8.25

mean of 4d10	standard deviatio of 4d10	z-score for sum = 9.5	p(sum < 9.5)

5) Testing with Normal pdf

Let

- x have a uniform distribution over the range of (1, 10)
- A be the sum of three x 's (range = 3..30)
- B be the sum of four x 's (range = 4..40)

Use a normal approximation to determine the probability that $A > B$

Note: The mean and variance for x (a uniform distribution over the range of (1,10)) is

- $\text{mean}(x) = 5.5$
- $\text{variance}(x) = 6.75$