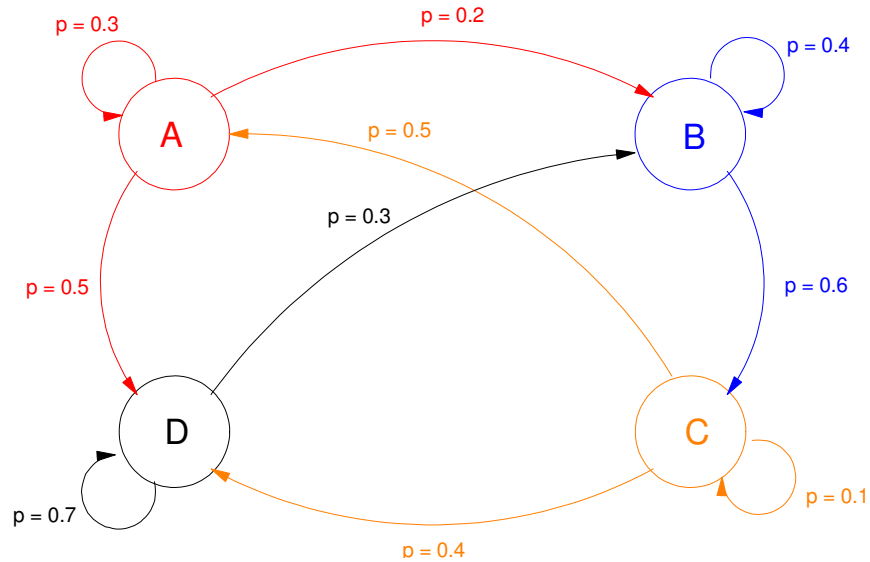


# ECE 341 - Homework #11

Markov Chains. Summer 2024

## Markov Chains

Four people are playing a game of hot potato. Each second, a player can keep the potato or pass it to another player. The probability of each decision are as follows:



- 1) Assume player A starts with the potato. Determine the probability that each player has the potato after 10 tosses using matrix multiplication.
- 2) Assume player A starts with the potato. Determine the probability that player A has the potato after  $k$  tosses using z-transforms.
  - What is the probability that player A has the potato after infinite tosses?
- 3) Assume player A starts with the potato. Determine the probability that player A has the potato after  $k$  tosses using eigenvalues and eigenvectors.

## Markov Chains with Absorbing States

Problem 4 & 5: Two teams, A and B, are playing a match made up of  $N$  games. For each game

- Team A has a 40% chance of winning
- There is a 25% chance of a tie, and
- Team B has a 35% chance of winning

In order to win the match, a team must be up by 2 games.

- 4) Determine the probability that team A wins the match after  $k$  games for  $k = \{0 \dots 10\}$  using matrix multiplication.
- 5) Determine the z-transform for the probability that A wins the match after  $k$  games
  - From the z transforms, determine the explicit function for  $p(A)$  wins after game  $k$ .

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Problem 6: Two players are playing a game of tennis. To win a game, a player must win 4 points *and* be up by 2 points.

- If player A reaches 4 points and player B has less than 3 points, the game is over and player A wins.
- If player A reaches 3 points and player B has 3 points, then the game reverts to 'win by 2' rules. Both players keep playing until one of them is up by 2 games.

Suppose:

- Player A has a 65% chance of winning any given point
- Player B has a 35% chance of winning any given point.

What is the probability that player A wins the game (first to 4 games, win by 2)?

- Note: This is a combination of a binomial distribution (A has 4 points while B has 0, 1, or 2 points) along with a Markov chain (A and B both have 3 points, at which point it becomes a win-by-2 series)