ECE 341 - Solution to Homework #2

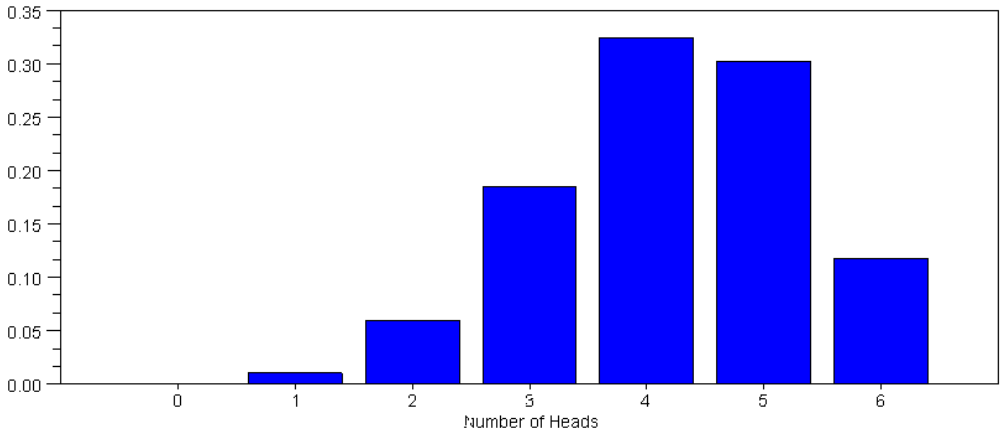
Binomial Distributions - Monte Carlo Simulations in MATLAB - Due Tuesday, September 5

1a) Determine the probability density function for flipping six coins with a probabilty of heads being 0.7.

$$f(x|n=6,p=0.7)=\binom{6}{x}(0.7)^{x}(0.3)^{6-x}$$

1b) Plot the pdf (bar graph preferred)

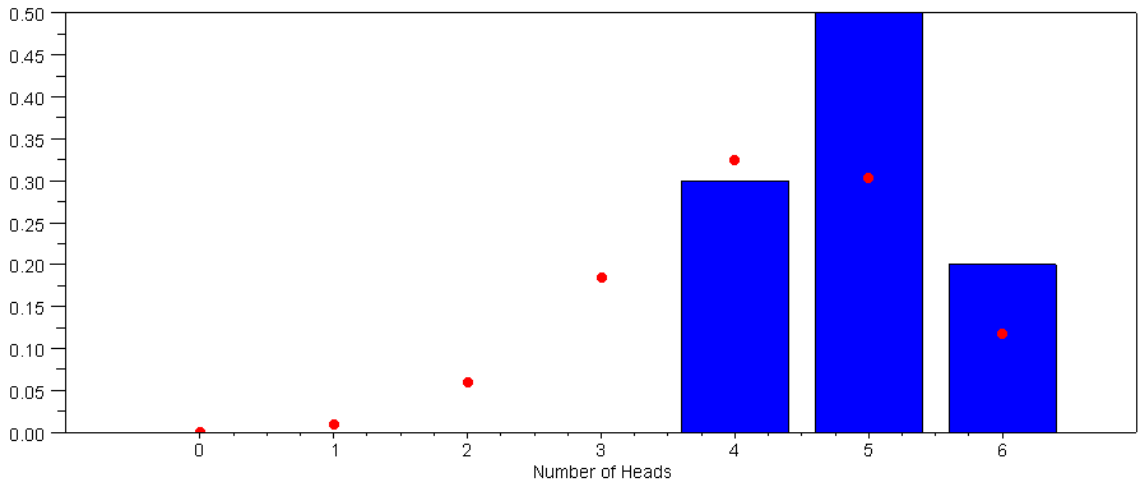
```
-->p1 = zeros(7,1);

-->for i=1:7
-->   n = i-1;
-->   p1(i) = factorial(6)/(factorial(n)*factorial(6-n))*(0.7^n)*(0.3^(6-n));
-->   end

-->N = [0:6]';

-->bar(N,p1)

-->xlabel('Number of Heads');
```



2) Using MATLAB (or like program), plot the resulting distribution for flipping six coins with a probabilty of heads being 0.7

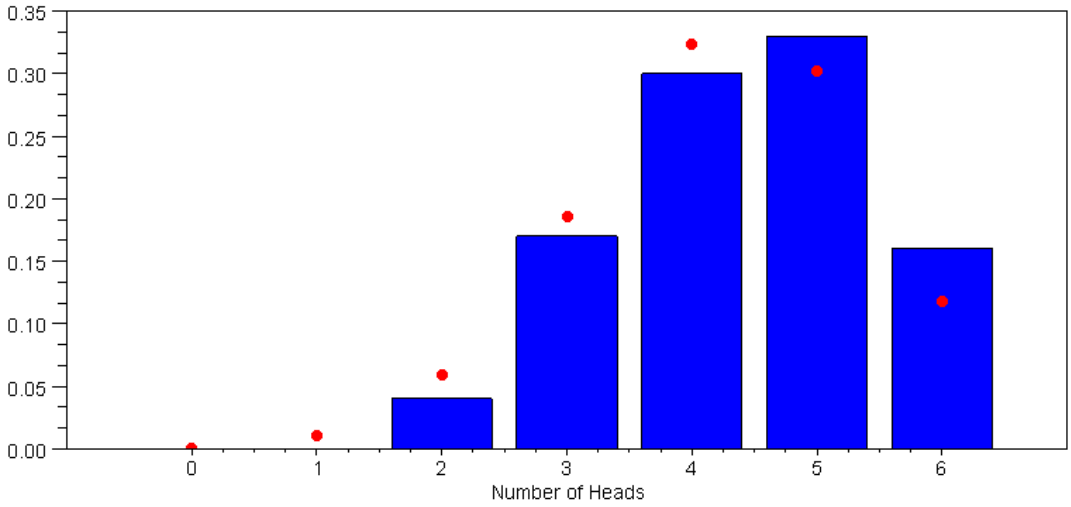
2a) Using a Monte-Carlo simulation with 10 trials (toss six coins 10 times and take the average)

```
-->p2a = zeros(7,1);

-->for i=1:10
-->   X(i) = sum(1*(rand(6,1)<0.7));
-->   end

-->for i=1:7
-->   n = i-1;
-->   p2a(i) = sum(X == n);
-->   end

-->p2a = p2a / 10;

-->bar(N,p2a)
-->plot(N,p1,'.r');
-->xlabel('Number of Heads');
```



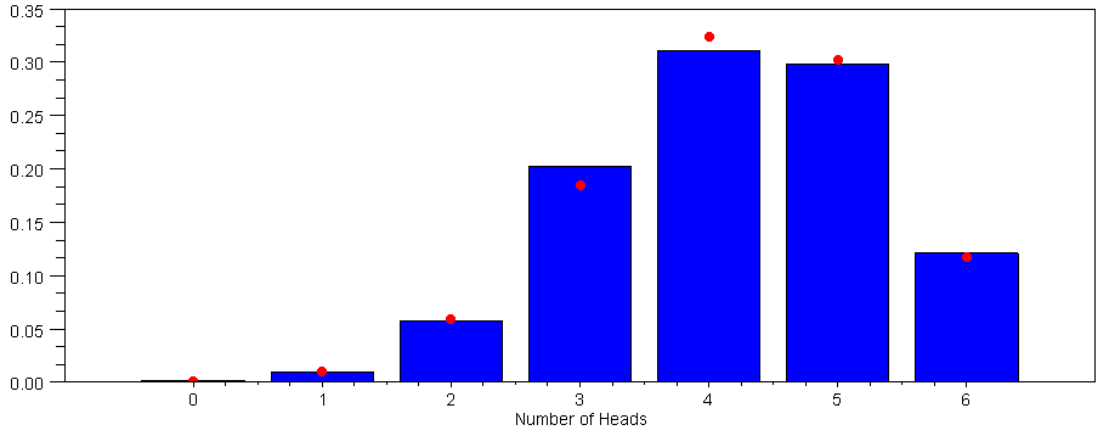
pdf for flipping six coins: Theoretical (dots) and experimental (bars)

2b) Using a Monte-Carlo simulation with 100 trials (toss six coins 100 times and take the average)



pdf for flipping six coins: Theoretical (dots) and experimental (bars)

2c) Using a Monte-Carlo simulation with 1000 trials (toss six coins 1000 times and take the average)



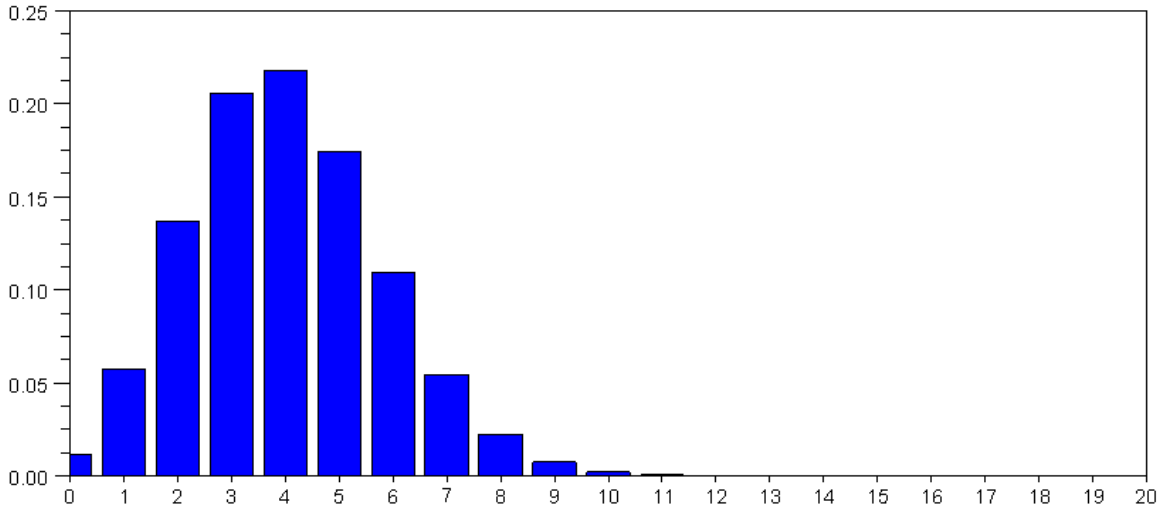
pdf for flipping six coins: Theoretical (dots) and experimental (bars)

3a) Determine the probability density function for flipping 20 coins with a probability of heads being 0.2.

$$f(x|n=20,p=0.2)=\binom{20}{x}(0.2)^{x}(0.8)^{20-x}$$

3b) Plot the pdf (bar graph preferred)

```
-->p3 = zeros(21,1);

-->for i=1:21
-->   n = i-1;
-->   p3(i) = factorial(20)/(factorial(n)*factorial(20-n))*(0.2^n)*(0.8^(20-n));
-->   end

-->N = [0:20]';

-->bar(N,p3)
```

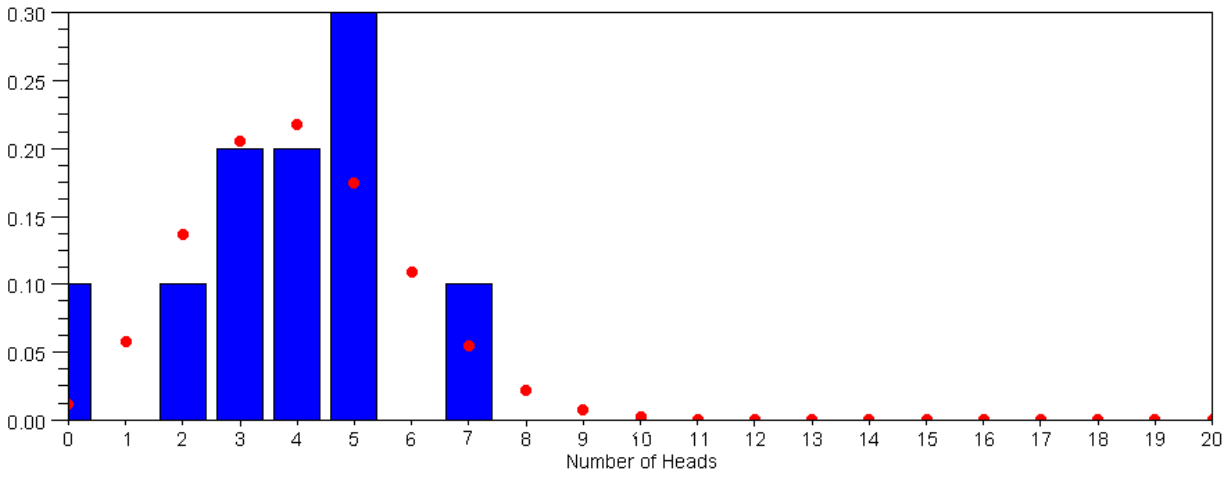


pdf for flipping 20 coins

4) Using MATLAB (or like program), plot the resulting distribution for flipping 20 coins with a probabilty of heads being 0.2

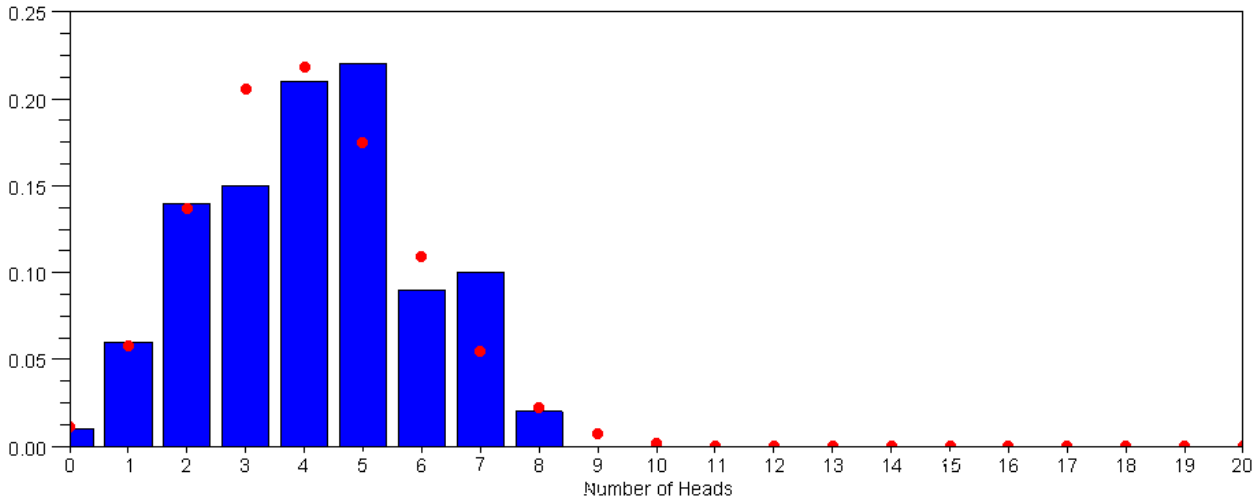
4a) Using a Monte-Carlo simulation with 10 trials (toss 20 coins 10 times and take the average)

```
X = zeros(10,1);

for i=1:10
   X(i) = sum(1*(rand(20,1)<0.2));
   end

for i=1:21
   n = i-1;
   p4(i) = sum(X == n);
   end

p4 = p4 / 10;

bar(N,p4)
plot(N,p3,'.r');

xlabel('Number of Heads');
```



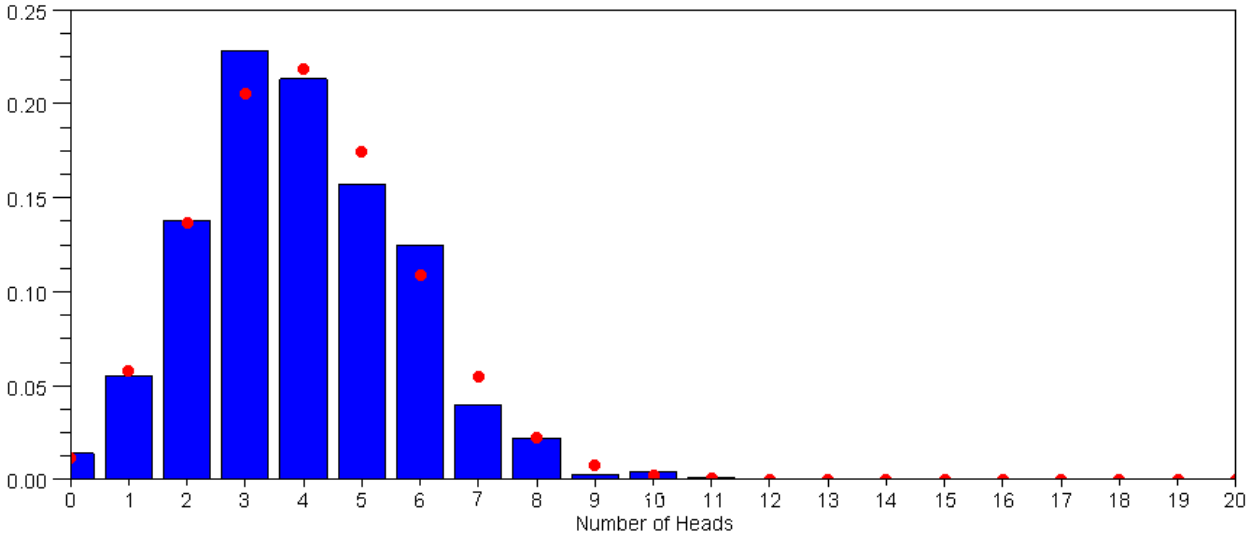
pdf for flipping 20 coins: theoretical (dots) and experimental (bars)

4b) Using a Monte-Carlo simulation with 100 trials (toss 20 coins 100 times and take the average)



pdf for flipping 20 coins: theoretical (dots) and experimental (bars)

4c) Using a Monte-Carlo simulation with 1000 trials (toss 20 coins 1000 times and take the average)



pdf for flipping 20 coins: theoretical (dots) and experimental (bars)

5) Compute the probabilty density function for the following:

- Roll 20 dice, Count the number of times a 1 appears.
- Flip that many coins with a probability of heads being 0.5. Count the number of heads.
- Each heads is a success.

(hint: think about this one - it's a Binomial distribution)

This is a binomial distribution with a probability of success being 1/12

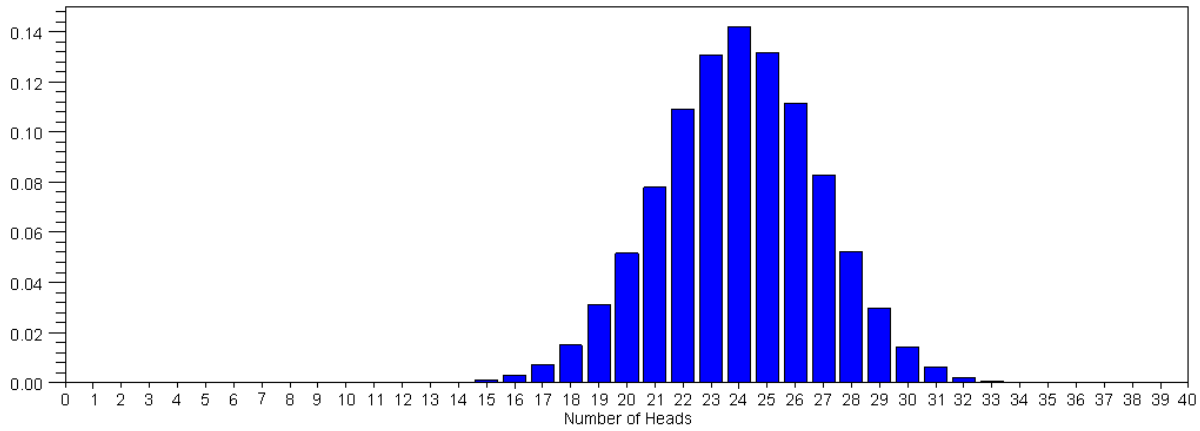
$$f(x|n=20, p=\tfrac{1}{12}) = \begin{pmatrix} 20 \\ x \end{pmatrix} \left(\tfrac{1}{12}\right)^{x} \left(\tfrac{11}{12}\right)^{20-x}$$

6) Use a Monte-Carlo simulation to determine the (approximate) probability density function for the following:

- Toss 20 coins with a probability of heads being 0.8
- Toss another 20 coins with a probability of heads being 0.4
- Count the total number of heads

Just to make the Monte Carlo run somewhat close, let's run the experiment 100,000 times:

```
npt = 100000;

p6 = zeros(41,1);
N = [0:40]';

X = zeros(npt,1);

for i=1:npt
   X(i) = sum(1*(rand(20,1)<0.8)) + sum(1*(rand(20,1)<0.4));
   end

for i=1:41
   n = i-1;
   p6(i) = sum(X == n);
   end

p6 = p6 / npt;

bar(N,p6)

xlabel('Number of Heads');
```



pdf for the sum of 40 coin tosses