Student t Distribution with >2 Populations

ECE 341: Random Processes

Lecture #24b

note: All lecture notes, homework sets, and solutions are posted on www.BisonAcademy.com

Student-t Test with One Population

The Student-t Test is designed for a single population

Population	mean	st dev	sample size
A	90.00	10.00	5

What is the chance A scores more than 100 points?

Find the t-score

$$t = \left(\frac{100 - 90}{10}\right) = 1.00$$

Use a t-table to convert to a probability

t-Test with Two Populations

Compare two populations: A and B

- What is the chance A wins the next game?
- What is the chance A is the better team?

Solution:

- Create a new variable: W = A - B
- You now have a t-test with one population

Population	mean	st dev	df
A	90.00	10.00	5
B	85.00	11.00	6
W A - B	5.00	14.87 individual	5 approx

t-Test with >2 Populations

Four people are playing Hungry Hungry Hippo

- What is the chance that A will win the next game?

Population	mean	st dev	df
A	90.00	10.00	5
B	85.00	11.00	6
C	84.00	12.00	3
D	83.00	13.00	7

Option #1: Create three variables

- W1 = A - B
- W2 = A - C
- W3 = A - D

Population	mean	st dev	df
A	90.00	10.00	5
B	85.00	11.00	6
C	84.00	12.00	3
D	83.00	13.00	7
W1 A - B	5.00	14.866	5
W2 A - C	6.00	15.620	3
W3 A - D	7.00	16.401	5

Find the probability A wins each case

Population	mean	st dev	df	t-Score	p(A Wins)
W1 A - B	5.00	14.866	5	0.3363	0.62485
W2 A - C	6.00	15.620	3	0.3841	0.63641
W3 A - D	7.00	16.401	5	0.4286	0.65697

Multiply all three probabilities together

$$p = p1 * p2 * p3$$

$$p = 0.2613$$

Note: This probabilty is low

- This is actually the odds that A defeats each other play one at a time
  - A runs the gauntlet of player B then C then D
- The odds that A wins a single game against three oponents is higher.

Option #2: Combine B, C, & D

- A's score is more than the max(B, C, D)
- Create a new variable, F = max(B, C, D)

You now have two variables (A & F)

- Problem has been previously solved

Game	Player A	max(B, C, D)	Player B	Player C	Player D
1	95	95	89	95	89
2	95	98	98	80	76
3	73	103	93	80	103
4	89	82	76	82	64
5	86	86	86	66	84
6	101	100	68	100	82
mean	89.8333	94.00			
st dev	9.7656	8.2704			

The probability of A winning any given game is then

$$t = \left(\frac{x_a - x_f}{\sqrt{s_a^2 + s_f^2}} \right) = -0.3256$$

6 games means 5 degrees of freedom

p = 0.37896

Player A has a 37.896% chance of winning any given game

- vs. 26.13% if A had to run the gauntlet

Option #3

Run a Monte-Carlo simulation to find the pdf for max(B, C, D)

```
>> B = 11*randn(1000,1) + 85;
>> C = 12*randn(1000,1) + 84;
>> D = 13*randn(1000,1) + 83;
>> F = max([B,C,D]')';

>> Xf = mean(F)

Xf =   94.2967

>> Sf = std(F)

Sf =    8.8662
```

Option #4: Run a Monte-Carlo Simulation

```
Wins = 0;

for n=1:1e5
    A = 10*randn + 90;
    B = 11*randn + 85;
    C = 12*randn + 84;
    D = 13*randn + 83;
    if(A > max([B,C,D])) Wins = Wins + 1; end
end

Wins / 1e5

>> ans =    0.3810
```

A has a 38.10% chance of winning any given game

Option #5: ANOVA

Student t-Tests are just one type of statistical test

- Assumes a single population
- You can play with the data to make it work with 2 populations

There are statistical tests design for more than 2 populations

- Analysis of Variance is one such test
- Coming soon...

Summary

With a t-test, you can compare two populations

- Create a new variable, $W = A - B$
- Determine the probability that $W > 0$

Only really works with two populations

- If you have more than two populations, you need a different tool
- ANOVA is one such tool (upcoming....)