Circuit Analysis with Phasors

With phasors, you can also analyze electrical circuits. For a resistor

$$V = IR$$

For an inductor

$$V = L\frac{dI}{dt}$$

Using phasors, differentiation becomes multiplication by jw

$$V = (j\omega L)I$$

so inductors look like resistors with an impedance of jwL. For a capacitor

$$I = C\frac{dV}{dt}$$

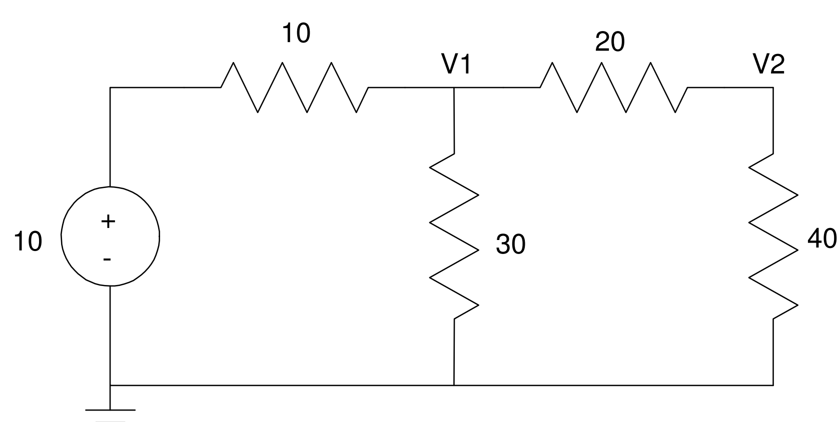
$$I = Cj\omega V$$

$$V = \left(\frac{1}{j\omega C}\right)I$$

Capacitors look like resistors with an impedance of $\left(\frac{1}{j\omega C}\right)$. Using this we can analyze AC circuits just like we analyze DC circuits, only with complex numbers. In summary

Component	Impedance	Impedance
Input	e^{st}	e^{jwt}
R	R	R
L	Ls	jwL
C	1 / Cs	1 / jwC

For example, find the votlages for the following circuit (EE 206 example: DC circuit)



Using voltage nodes, the nodal equations would be

$$\left(\frac{V_1-10}{10}\right)+\left(\frac{V_1}{30}\right)+\left(\frac{V_1-V_2}{20}\right)=0$$

$$\left(\frac{V_2-V_1}{20}\right)+\left(\frac{V_2}{40}\right)=0$$

Grouping terms

$$\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{20}\right)V_1-\left(\frac{1}{20}\right)V_2=\left(\frac{1}{10}\right)10$$

$$\left(\frac{1}{20}+\frac{1}{40}\right)V_2-\left(\frac{1}{20}\right)V_1=0$$

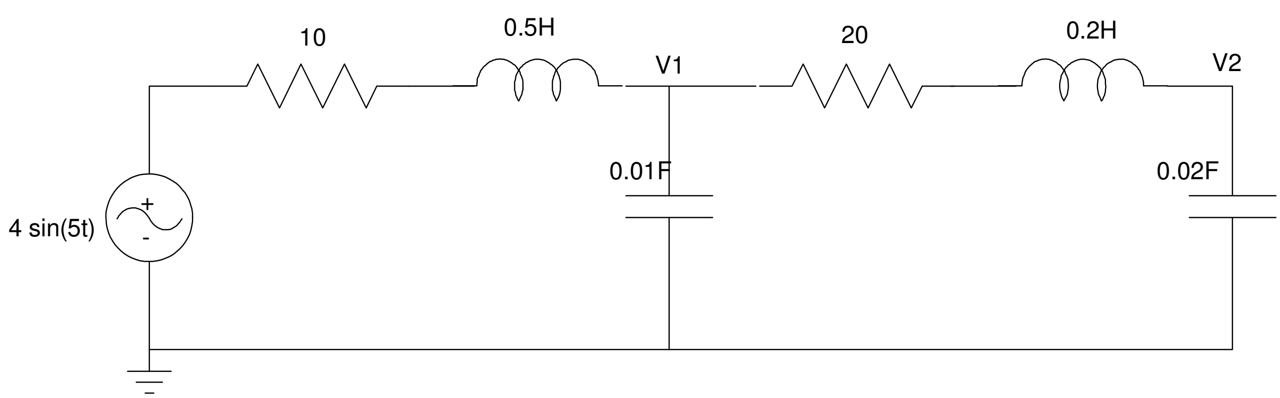
Placing in matrix form

$$\begin{bmatrix} \left(\frac{1}{10}+\frac{1}{30}+\frac{1}{20}\right) & \left(\frac{-1}{20}\right) \\ \left(\frac{1}{20}\right) & \left(\frac{1}{20}+\frac{1}{40}\right) \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} \left(\frac{10}{10}\right) \\ 0 \end{bmatrix}$$

Solving 2 equations for 2 unknowns in Matlab:

```
-->A = [1/10+1/30+1/20,-1/20 ; -1/20,1/20+1/40]

    0.1833333  - 0.05
  - 0.05         0.075

-->B = [1;0]

    1.
    0.

-->inv(A)*B

V1    6.6666667
V2    4.4444444
```

Now find the voltages for the following circuit



Step 1: Convert to phasor notation. The input becomes

$$4\sin(5t) \rightarrow 0 - j4$$

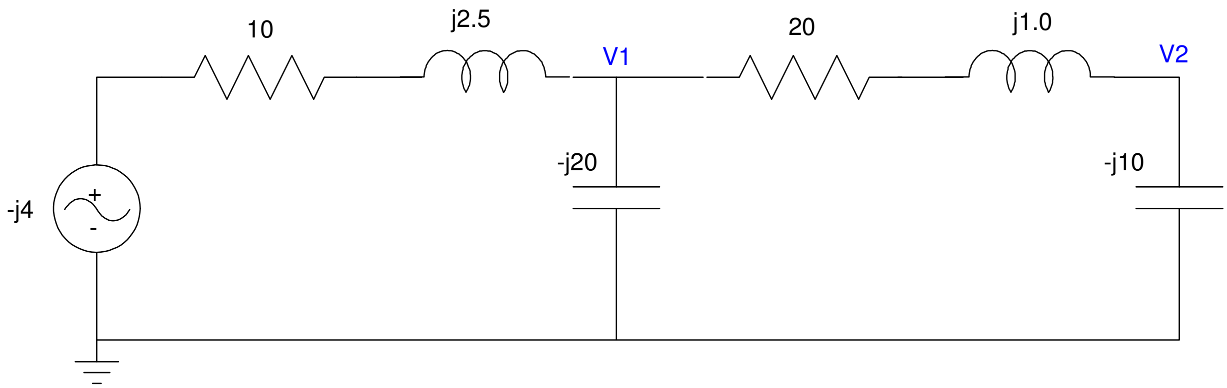
The resistors remain uncanged. The inductors become

$$L \rightarrow j\omega L$$

where ω is 5 rad/sec. The capacitors become

$$C \rightarrow \frac{1}{j\omega C}$$

This results in



Now write the voltage node equations just like before

$$\left(\frac{V_1-(-j4)}{10+j2.5}\right) + \left(\frac{V_1}{-j20}\right) + \left(\frac{V_1-V_2}{20+j1}\right) = 0$$

$$\left(\frac{V_2-V_1}{20+j1.0}\right) + \left(\frac{V_2}{-j10}\right) = 0$$

Group terms

$$\left(\frac{1}{10+j2.5} + \frac{1}{-j20} + \frac{1}{20+j1}\right)V_1 - \left(\frac{1}{20+j1}\right)V_2 = \left(\frac{-j4}{10+j2.5}\right)$$

$$\left(\frac{-1}{20+j}\right)V_1 + \left(\frac{1}{20+j} + \frac{1}{-j10}\right)V_2 = 0$$

Place in matrix form

$$\begin{bmatrix} \left(\frac{1}{10+j2.5} + \frac{1}{-j20} + \frac{1}{20+j1}\right) & \left(\frac{-1}{20+j1}\right) \\ \left(\frac{-1}{20+j1}\right) & \left(\frac{1}{20+j} + \frac{1}{-j10}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \left(\frac{-j4}{10+j2.5}\right) \\ 0 \end{bmatrix}$$

Solve in Matlab

```
-->a11 = 1/(10+j*2.5)-1/(j*20)+1/(20+j);
-->a12 = -1/(20+j);
-->a21 = -1/(20+j);
-->a22 = 1/(20+j) + 1/(-j*10);
```

```
-->A = [a11, a12 ; a21, a22]

    0.1439930 + 0.0239768i  - 0.0498753 + 0.0024938i
  - 0.0498753 + 0.0024938i    0.0498753 + 0.0975062i

-->B = [-j*4/(10+j*2.5) ; 0]

  - 0.0941176 - 0.3764706i
    0

-->V = inv(A)*B

V1  - 1.4559151 - 2.2895748i
V2  - 1.2244227 + 0.1769673i
```

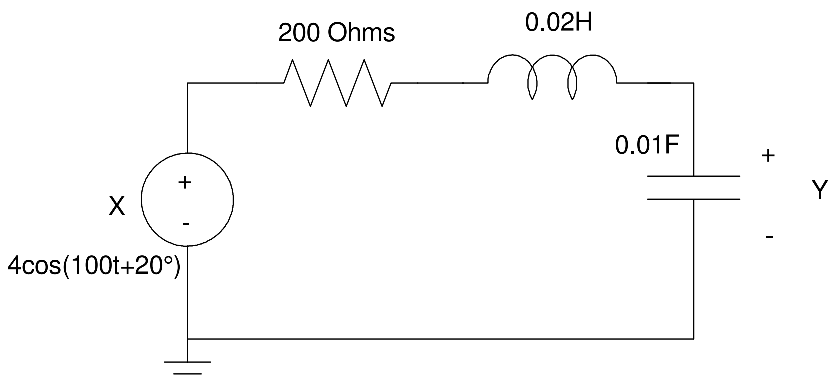
meaning

$$V_1(t) = -1.4559\cos(5t) + 2.289\sin(5t)$$

$$V_2(t) = -1.224\cos(5t) - 0.176\sin(5t)$$

Example 3) Voltage Division

Everything we did in EE 206 for DC analysis also works for AC circuits - only with complex numbers. For example, use voltage division to find y(t):



First, convert to phaser notation. For this forcing function, x(t),

$$s = j100$$

Change the input to phaser notation:

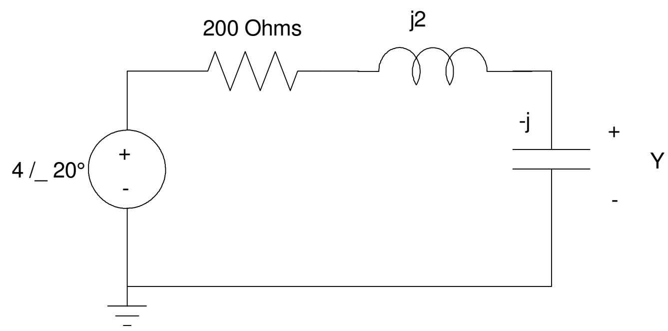
$$X = 4\angle 20^0$$

Change RLC to phaser impedances:

$$R \to R = 200\Omega$$

$$L \to Ls = j2\Omega$$

$C \rightarrow \frac{1}{Cs} = -j\Omega$



Now, solve using voltage division:

$$Y = \left(\frac{-j}{200+2j-j}\right) \cdot 4\angle 20^0$$

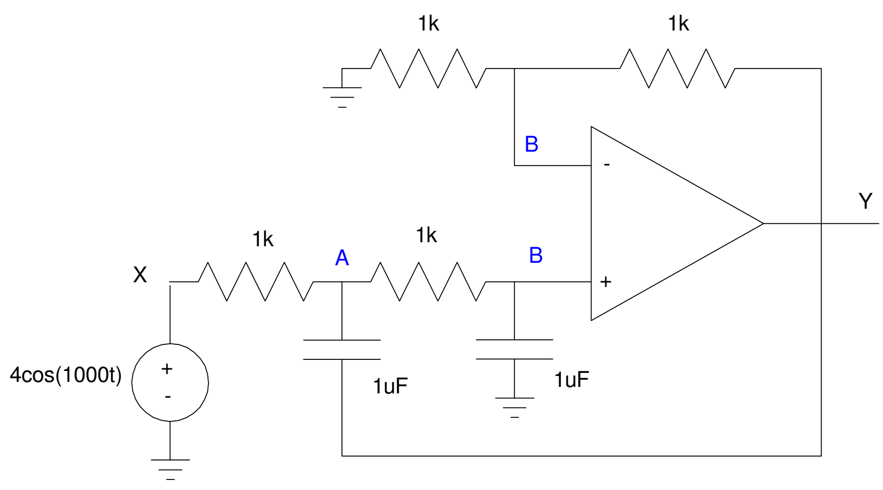
$$Y = 0.02\angle -70.3^0$$

$$y(t) = 0.02\cos\left(100t - 70.3^0\right)$$

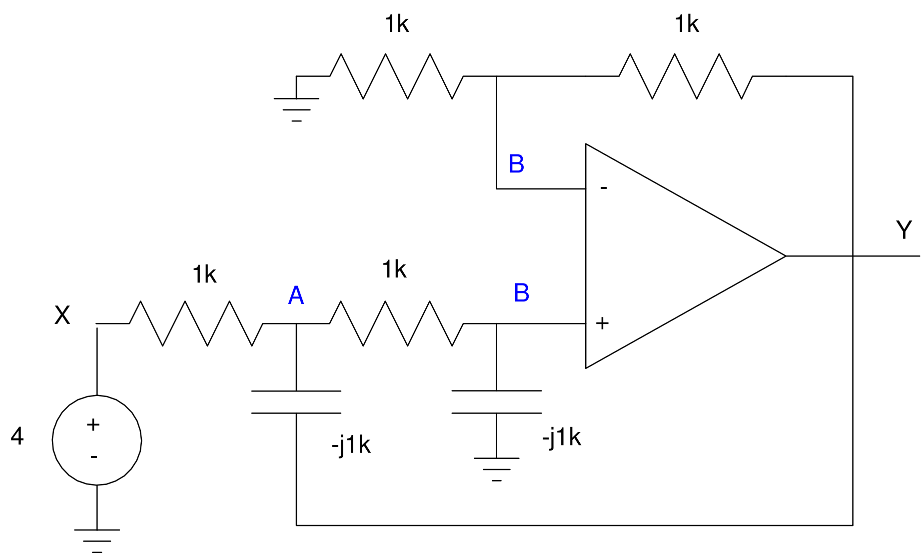
Note that these answers were *much* easier to obtain than before.

Example 4: Op-Amp Circuits

Find y(t):



First. convert to phaser notation. In this case, $s = j1000$



Next, write the voltage node equations:

A $\left(\frac{A-4}{1000}\right)+\left(\frac{A-Y}{-j1000}\right)+\left(\frac{A-B}{1000}\right)=0$

V- $\left(\frac{B-A}{1000}\right)+\left(\frac{B-0}{-j1000}\right)=0$

V+ $\left(\frac{B-0}{1000}\right)+\left(\frac{B-Y}{1000}\right)=0$

Note that you can't write a voltage node equation at Y. The op-amp puts out whatever current it takes to force V- = V+. Since you don't know this current, you can't sum the currents to zero. The fourth equation already incorporated in the above three is

$$V^{+}=V^{-}$$

To make the numbers nicer, lets scale all three equations by 1000 and group terms:

A $(2+j)A+(-j)Y+(-1)B=4$

V- $(-1)A+(0)Y+(1+j)B=0$

V+ $(0)A+(-1)Y+(2)B=0$

Solving three equations for three unknowns (again using MATLAB):

```
-->W = [2+j,-j,-1;-1,0,1+j;0,-1,2]
 W  =
```

```
     2. + i    - i     - 1.
   - 1.          0       1. + i
     0         - 1.      2.


-->inv(W)*[4;0;0]
 ans  =

Va     4. - 4.i
Vy   - 8.i
Vb   - 4.i
```

The way I set up the problem, Y was the second column in the 3x3 matrix. Likewise, Y will be the second entry in the answer:

$$Y = -j8$$

and

$$y(t) = 8\sin(1000t)$$