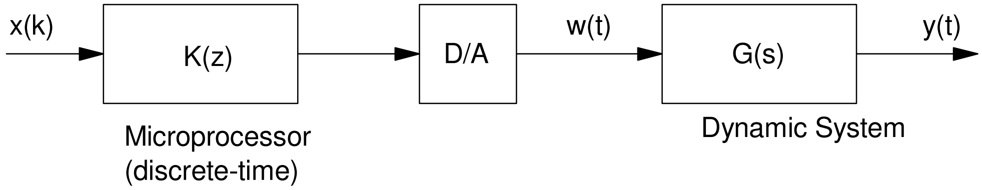
Evaluating Hybrid Systems

Now, find the output of a hybrid system: part digital, part analog.



Hybrid System: Part Discrete-Time, Part Continuous Time

Example 1: Find the step response where

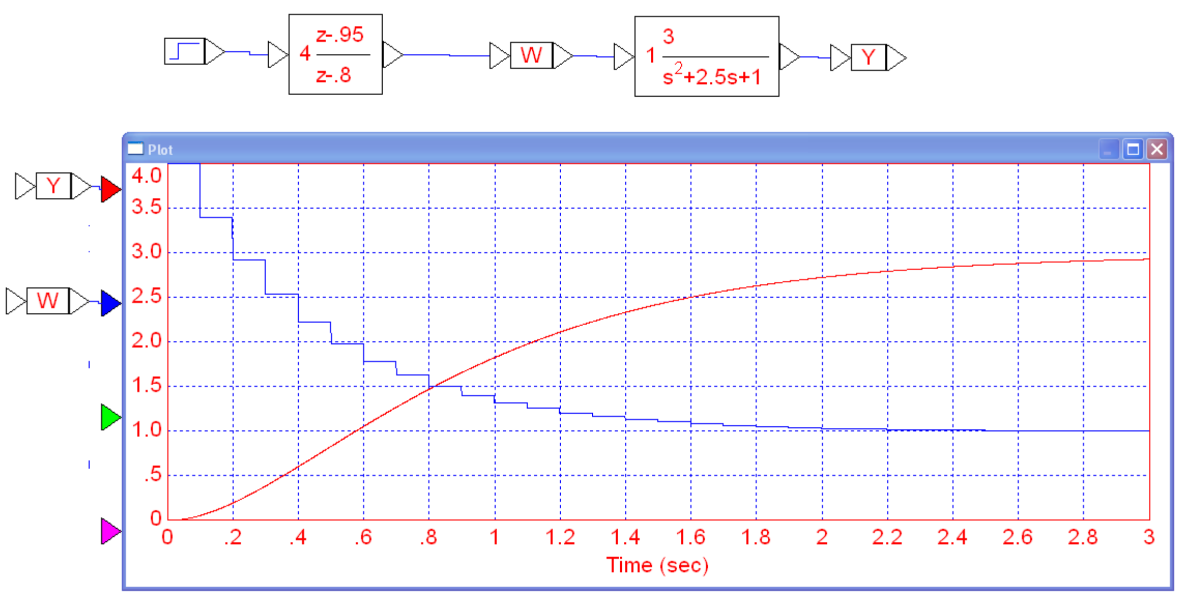
$$K(z) = 4\left(\frac{z-0.95}{z-0.8}\right) \qquad T = 100\text{ms}$$

$$G(s) = \left(\frac{3}{s^2+2.5s+1}\right)$$

and

$$x(k) = u(k)$$

What the output should look like is something like this:



VisSim simulation of a hybrid system: a discrete-time system with a sampling rate of 100ms driving an analog system
Note that w(t) changes at discrete times (the sampling rate of K(z) whereas y(t) is analog

Solution #1: Convert everything to the z-plane with a sampling rate of T = 100ms

$$G(s) = \left(\frac{3}{s^2+2.5s+1}\right) = \left(\frac{3}{(s+0.5)(s+2)}\right)$$

The poles map as

$$z = e^{sT}$$

s = -0.5

$$z = e^{-0.5\cdot 0.1} = 0.9512$$

s = -2

$$z = e^{-2\cdot 0.1} = 0.8187$$

so

$$G(z) \approx \left(\frac{k}{(z-0.9512)(z-0.8187)}\right)$$

Pick 'k' to make the DC gain match up

$$\left(\frac{3}{(s+0.5)(s+2)}\right)_{s=0} = 3$$

$$\left(\frac{k}{(z-0.9512)(z-0.8187)}\right)_{z=1} = 3$$

$$k = 0.0265$$

$$G(z) \approx \left(\frac{0.0265}{(z-0.9512)(z-0.8187)}\right)$$

Add zeros at z=0 to match the phase at some frequency, such as 0.1 rad/sec

$$\left(\frac{3}{(s+0.5)(s+2)}\right)_{s=j0.1} = 2.9381\angle -14.1723^0$$

$$\left(\frac{0.0265}{(z-0.9512)(z-0.8187)}\right)_{z=e^{j0.1T}} = 2.9335\angle -14.7499^0$$

Each zero at z=0 adds 0.57 degrees.

$$z = e^{j0.1T} = 1\angle 0.5730^0$$

To make the phase add up exactly, you need to add 1.0081 zeros at z = 0

$$n = \left(\frac{14.7499^0 - 14.1723^0}{0.5730^0}\right) = 1.0081$$

Round to 1.00. This gives

$$G(z) \approx \left(\frac{0.0265z}{(z-0.9512)(z-0.8187)}\right)$$

Now we can analyze the system

$$Y(z) = G(z) \cdot K(z) \cdot X(z)$$

$$Y = \left(\frac{0.0265z}{(z-0.9512)(z-0.8187)}\right) \cdot 4\left(\frac{z-0.95}{z-0.8}\right) \cdot \left(\frac{z}{z-1}\right)$$

$$Y = \left(\frac{0.1060z^2(z-0.95)}{(z-1)(z-0.9512)(z-0.8187)(z-0.8)}\right)$$

$$Y = z\left(\frac{2.9952}{z-1} + \frac{-0.1238}{z-0.9512} + \frac{-25.3653}{z-0.8187} + \frac{22.4938}{z-0.8}\right)$$

which gives

$$y(k) \approx (2.9952 - 0.1238 \cdot 0.9512^k - 25.3653 \cdot 0.8187^k + 22.4838 \cdot 0.8^k)u(k)$$

Solution #2: Convert everything to the s-plane

$$K(z) = 4\left(\frac{z-0.95}{z-0.8}\right)$$

z = 0.95

$$z = e^{sT}$$

$$s = \frac{1}{T}\ln(z) = -0.5129$$

z = 0.8

$$s = \frac{1}{T}\ln(z) = -2.2314$$

so

$$K(s) \approx k\left(\frac{s+0.5129}{s+2.2314}\right)$$

Matching the DC gain

$$4\left(\frac{z-0.95}{z-0.8}\right)_{z=1} = 1.00$$

$$k\left(\frac{s+0.5129}{s+2.2314}\right)_{s=0} = 1.00$$

$$k = 4.3506$$

so

$$K(s) \approx 4.3506\left(\frac{s+0.5129}{s+2.2314}\right)$$

This results Y(s) being

$$Y(s) = G(s) \cdot K(s) \cdot X(s)$$

$$Y(s) = \left(\frac{3}{(s+0.5)(s+2)}\right) \cdot 4.3506\left(\frac{s+0.5129}{s+2.2314}\right) \cdot \frac{1}{s}$$

$$Y(s) = \left(\frac{13.0517(s+0.5129)}{s(s+0.5)(s+2)(s+2.2314)}\right)$$

$$Y(s) = \left(\frac{3}{s} + \frac{-0.1297}{s+0.5} + \frac{-27.9591}{s+2} + \frac{25.0887}{s+2.2314}\right)$$

Taking the invers LaPlace transform

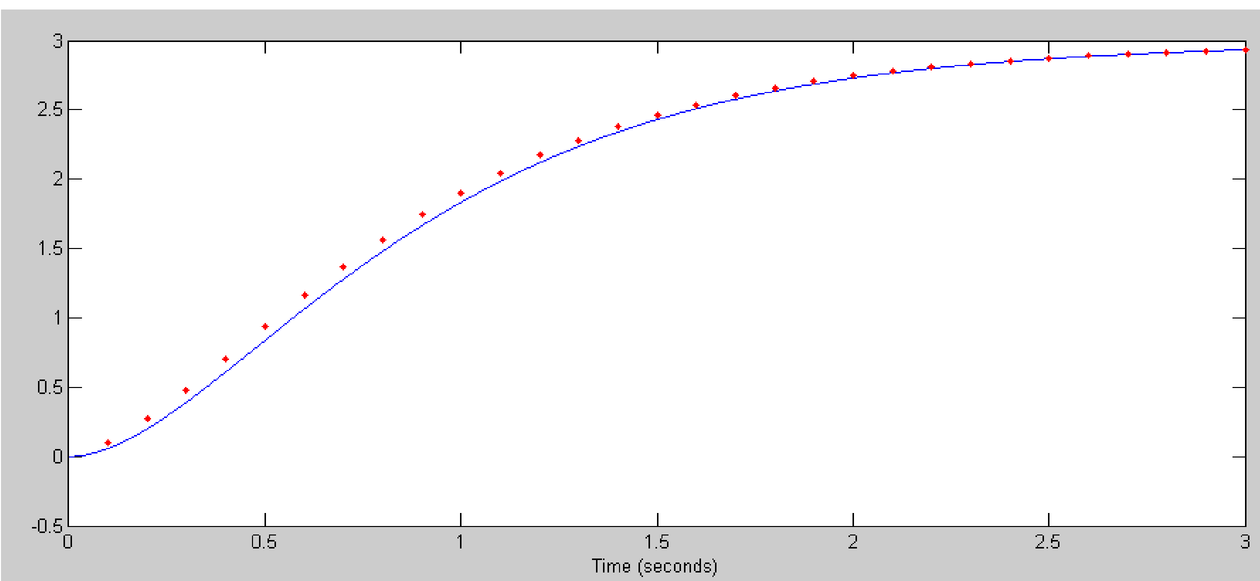
$$y(t) \approx (3 - 0.1297e^{-0.5t} - 27.9591e^{-2t} + 25.0887e^{-2.2314t}) \cdot u(t)$$

Comparing the two solutions in Matlab:

```
Ys = zpk(-0.5129,[0,-0.5,-2,-2.2314],13.0517);
t = [0:0.001:3]';
y = impulse(Ys,t);

k = [0:30]';
yk = 2.9952 - 0.1238*(0.9512.^k) - 25.3653*(0.8187.^k) + 22.4838 * (0.8 .^ k);
T = 0.1;

plot(t, y, '-', k*T,yk,'r.');
xlabel('Time (seconds)')
```



Solution for y: Converting to the s-plane (blue) and converting to the z-plane (red dots).