ECE 376 - Homework #11 (revised)

z-Transforms and Digital Filters.   Due Friday, April 21st
Please email to jacob.glower@ndsu.edu, or submit as a hard copy, or submit on BlackBoard

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{5(s+2)}{(s^2+4s+30)}\right)X$$

a) What is the differential equation relating X and Y?

b) Find y(t) assuming

$$x(t) = 6 + 5\sin(4t)$$

2) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{0.01(z+1)}{(z-0.96)(z-0.9)}\right)X$$

a) What is the difference equation relating X and Y?

b) Find y(t) assuming a sampling rate of T = 0.01 second

$$x(t) = 6 + 5\sin(4t)$$

3) Assume G(s) is a low-pass filter with real poles:

$$G(s) = \left(\frac{100}{(s+4)(s+5)(s+6)}\right)$$

3) Design a digital filter, G(z), which has approximately the same gain vs. frequency as G(s). Assume a sampling rate of T = 0.01 second.

Plot the gain vs. frequency for both filters from 0 to 50 rad/sec.

4) Assume G(s) is the following band-pass filter:

$$G(s) = \left(\frac{30s}{(s+2+j15)(s+2-j15)}\right)$$

Design a digital filter, G(z), which has approximately the same gain vs. frequency as G(s). Assume a sampling rate of T = 0.01 second.

Plot the gain vs. frequency for both filters from 0 to 50 rad/sec.

5) Write a C program to implement the digital filter, G(z)

Filters & Range Measurement

6) In Matlab, create data (x) to represent ultrasonic range sensor readings at a distance of 100mm. For the raw data (x), determine

- The mean of x
- The standard deviation of x
- The 90% confidence interval for the next value of x.

Also plot the raw data, x(k).

```
>> x = 100 + 3*randn(1000,1);
>> k = [1:1000]';
>> plot(k,x)
```

7) Filter the data with a FIR filter (the average of the last five data points)

$$Y = \left(\frac{1}{5}\right)\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \frac{1}{z^4}\right)X$$

For the filtered data (y), determine

- The mean of y
- The standard deviation of y
- The 90% confidence interval for the next value of y

Also plot the filtered data, y(k)

```
>> x = 100 + 3*randn(1000,1);
>> y = 0*x;
>> y(1:4) = 100;
>> for k=5:1000
     y(k) = mean(x(k-4:k);
     end
>> k = [1:1000]';
>> plot(k,y)
```

8) Filter the data with the following low-pass filter:

$$Y = \left(\frac{8}{s^2+4s+8}\right)X$$ s-plane, poles at s = -2 +/- j2

$$Y = \left(\frac{0.0008}{z^2-1.9600z+0.9608}\right)X$$ same filter in the z-plane with T = 10ms

For the filtered data (y), determine

- The mean of y
- The standard deviation of y
- The 90% confidence interval for the next value of y

Also plot the filtered data, y(k)

```
>> x = 100 + 3*randn(1000,1);
>> y = 0*x;
>> y(1:2) = 100;
>> for k=3:1000
     y(k) = 1.9600*y(k-1)-0.9608*y(k-2)+0.0008*x(k-2);
     end
>> k = [1:1000]';
>> plot(k,y)
```

note: You can do a lot better than just taking the average of the last five data points.