Instrumentation Amplifiers & Calibration

Calibration and Curve Fitting

Objective

- Design a circuit which outputs 0 .. 5V for a sensor (instrumentation amplifier)
- Determine a calibration function so that a PIC can determine temperature from the A/D reading

Sensors

Sensors convert something you want to measure to something you can measure. For example, some sensors available in lab are:

- An ultrasonic range sensor which converts distance to a voltage where the pulse-width is proportional to distance.
- A light sensor which converts light level to resistance as

$$R \approx \frac{100,000}{Lux}\ \Omega$$

- A temperature sensor which converts temperature to resistance as

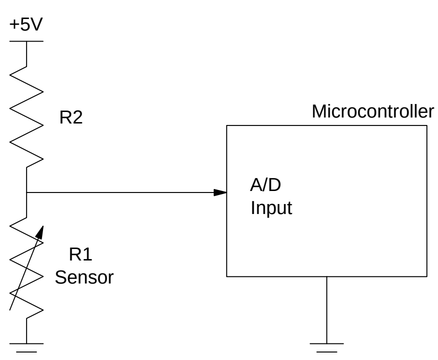
$$R \approx 1000 \cdot \exp\left(\frac{3533}{T} - 11.85\right)\ \Omega$$

Usually, but not always, the output of a sensor is a resistance. This is because

- It takes no energy to produce a resistance. (such sensors do not need to be powered)
- If you find a material which changes resistance with changing light / temperature / humidity / etc, you have a sensor.

What this means is that if you can measure a resistance, you can measure whatever the sensor detects.

One way to convert a resistance (which your microcntroller cannot measure) into a voltage (which your microcontoller can measure) is to use a voltage divider:

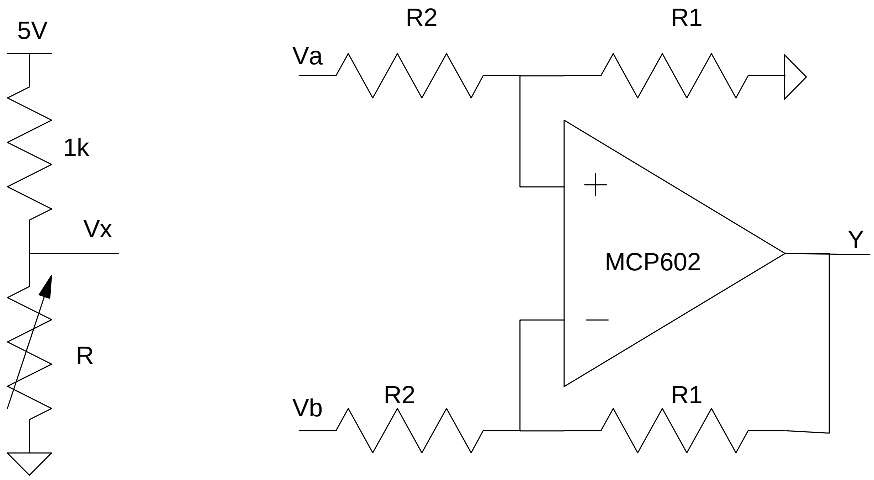


To convert a resistance to a voltage, a voltage divider can be used.
Note that the sensor and microcontroller need a common ground

If the resulting variation in voltage (and A/D reading) is large enough, you can input this voltage directly to the A/D input of the PIC. If the range is small, however, it would help to amplify the voltage. An instrumentation amplifier is a circuit which does this.

Instrumentation Amplifiers:

The basic circuit used to convert a signal to 0 .. 5V is an instrumentation amplifier:



Instrumentation Amplifier: $Y = \left(\frac{R_1}{R_2}\right)(V_a - V_b)$

Example: Assume R is a RTD (temperature sensitive resistor) with the following relationship:

$$R = 1000(1 + 0.004T)\,\Omega$$

Design a circuit which outputs

- 0V at 0C and
- 5V at +50C

Solution: First, determine the resistance at the two endpoints:

0C: R = 1000

50C: R = 1200

Step 2: Convert resistance to voltage. Assume a voltage divider with a 1k resistor. Then

$$V_x = \left(\frac{R}{R+1000}\right)5V$$

0C: Vx = 2.5V

50C: Vx = 2.7273V

Step 3: Determine whether Vx connects to the + input (Va) or the - input (Vb)

The output, Y, is to increase as Vx increases. Connect the + input (Va)

Step 4: Determine the gain.

$$Gain = \frac{\text{change in output}}{\text{change in input}}$$

$$Gain = \left(\frac{5V - 0V}{2.7273V - 2.5V}\right) = 22$$

Pick R1/R2 = 22. Let

- R1 = 10k
- R2 = 220k

Step 5: Determine the offset.

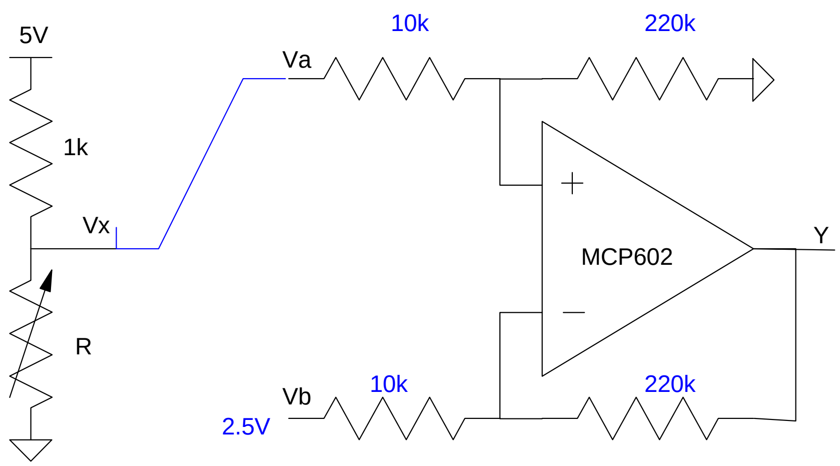
The output should be 0V when the input is 2.5V (0C)

$$V_y = gain(V_a - V_b)$$

$$0V = 22(2.5V - V_b)$$

$$V_b = 2.5V$$

The resulting circuit is as follows:

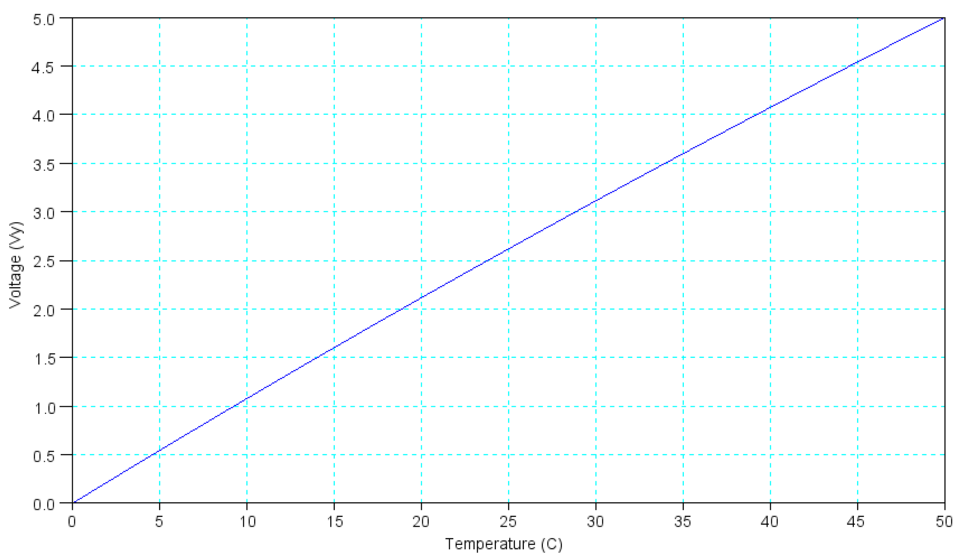


Instrumentation Amplifier: Output is 0V at 0C and 5V at +50C

The output voltage vs. temperature is then:

```
-->T = [0:0.1:50]';
```

```
-->R = 1000 * (1 + 0.004*T);

-->Vx = R ./ (1000+R) * 5;

-->min(Vx)

    2.5

-->max(Vx)

    2.7272727

-->gain = (5 - 0) / (max(Vx) - min(Vx))

    22.

-->Vb = min(Vx)

    2.5

-->Vy = gain*(Vx - 2.5);
-->plot(T,Vy)
-->xlabel('Temperature (C)');
-->ylabel('Voltage (Vy)');
```



Output of the Instrumentation Amplifier

Calibration

Assume the output of the instrumentation amplifier feeds an A/D input, such as RA1. Calibration takes the raw A/D reading (0 .. 1023) and converts this back to what you're trying to measure (temperature in this case). Essentially, calibration is curve fitting.

Assume you want to curve fit the above data with a straight line:

$$T \approx a \cdot A/D + b$$

where A/D is the 0..1023 raw A/D reading. Some types of calibration are:

- Endpoint Calibration: Pass a line through the endpoints
- Lease Squares: Come up with a line that minimizes the mean squared error

To find the least squares solution, rewrite the curve fit in matrix form:

$$Y_{501x1} = X_{501x2}A_{2x1}$$

$$T = \begin{bmatrix} A/D & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

In the previous Matlab code, there are 501 data points, making X a 501x2 matrix. You cannot invert a 501x2 matrix. To solve, multiply both sides by X transpose:

$$X^T Y = X^T X A$$

XTX is a 2x2 matrix, and can usually be inverted. A is then

$$A = (X^T X)^{-1} X^T Y$$

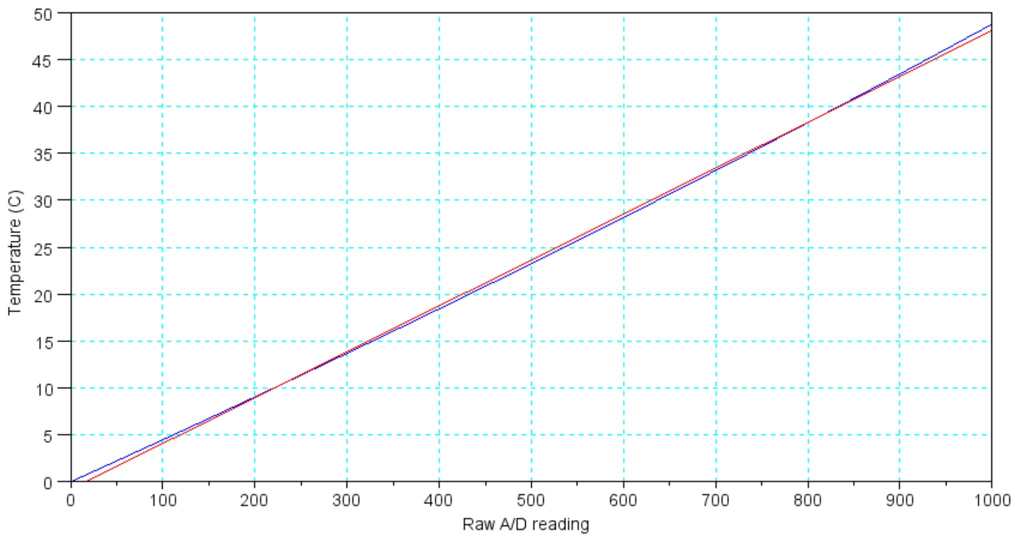
In Matlab:

```
-->A2D = Vy*1023/5;
-->A2D = round(A2D);

-->Y = T;
-->X = [A2D, A2D.^0];

-->A = inv(X'*X)*X'*Y

    0.0488903
  - 0.8000303
```

$$T \approx 0.0488903 \cdot A/D - 0.80003$$

Plotting this in Matlab:

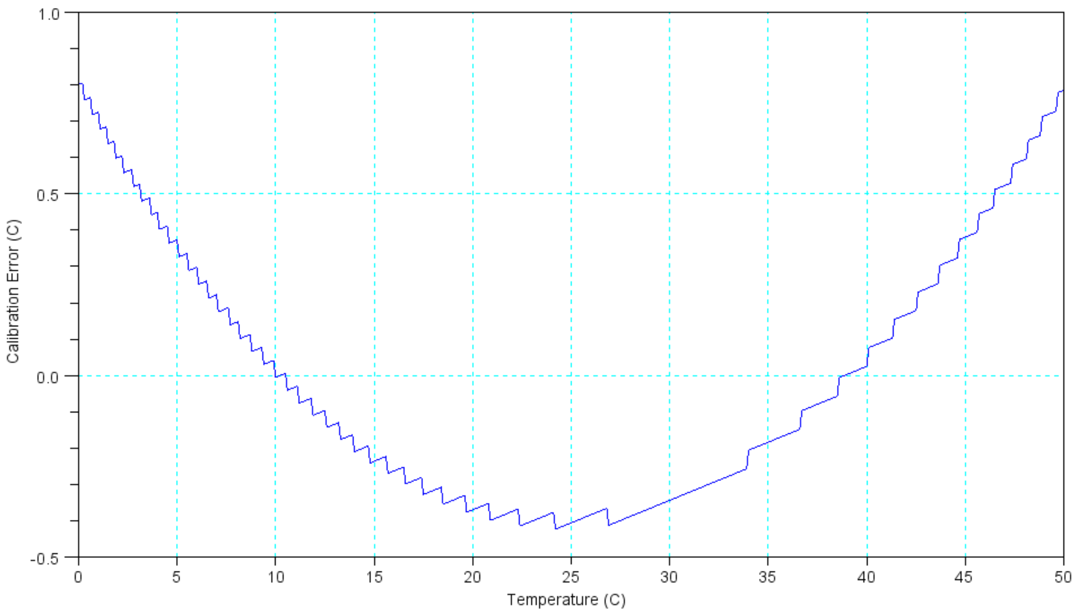
```
-->plot(A2D, T, A2D, X*A)
-->xlabel('Raw A/D reading');
-->ylabel('Temperature (C)');
```



Actual A/D vs Temperature Relationship (blue) and Linear Curve Fit (blue)

To determine the calibration error, plot the difference (the residual)

```
-->plot(T, T-X*A)
-->xlabel('Temperature (C)');
-->ylabel('Calibration Error (C)');
```



Calibration Error for a Linear Curve Fit for Temperature

The estimated temperature based upon the previous calibration function is off by 0.8 degrees.

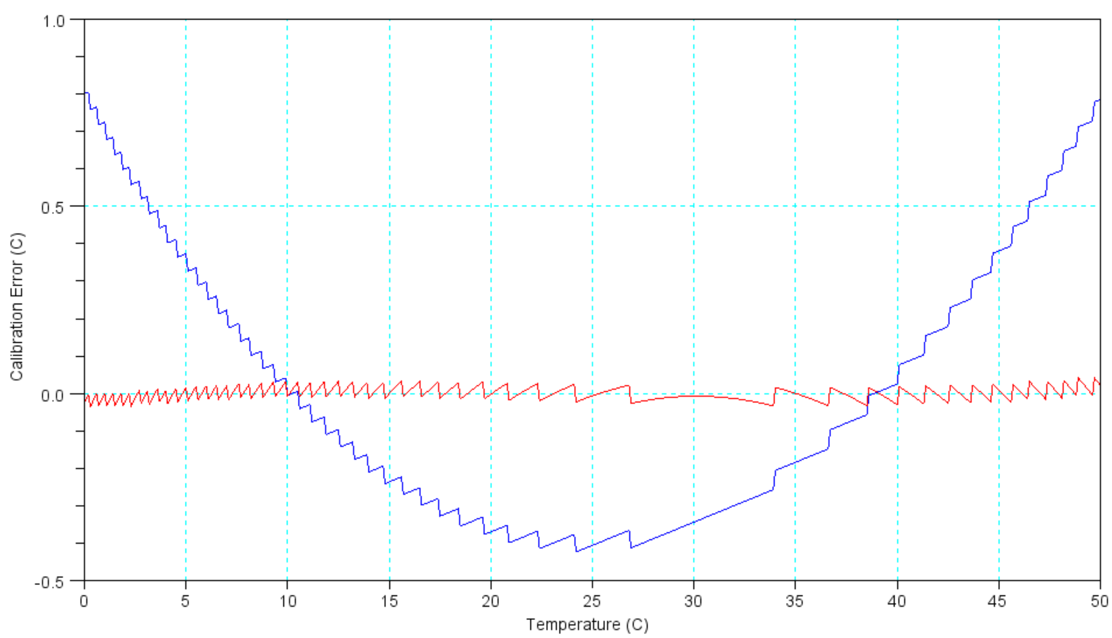
Note that the residual looks like a parabola. You can reduce the error by using a parabolic curve fit. To do this, change the basis:

$$T \approx a \cdot A/D^2 + b \cdot A/D + c$$

```
-->X = [A2D.^2, A2D, A2D.^0];

-->A = inv(X'*X)*X'*Y
Warning :
matrix is close to singular or badly scaled. rcond =    0.0000D+00

 A  =

a =     0.0000046
b =     0.0441556
c =     0.0258837
```

The resulting residual error is then within 0.04C

```
-->max(T - X*A)
 ans  =

    0.0424377

-->min(T - X*A)
 ans  =

  - 0.0351971
```



Calibration Error for a Linear Curve Fit (blue) and Parabolic Curve Fit (red)

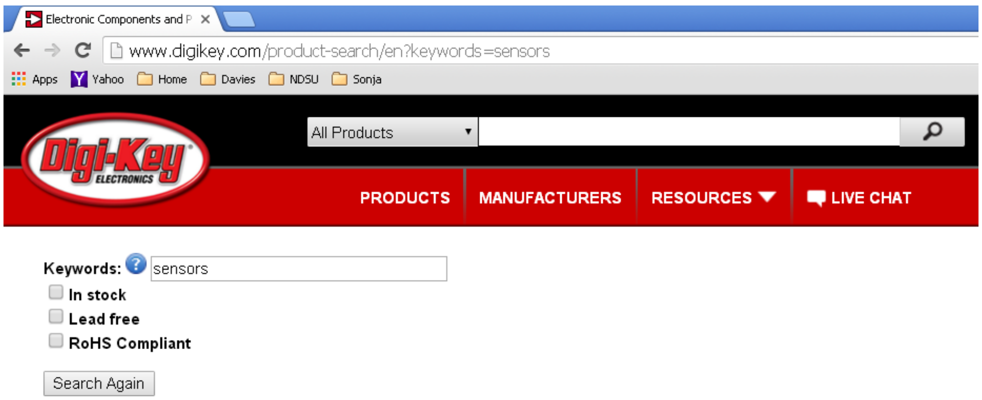
At this point, the quantization noise (the steps above) are dominating the error. There is no point in adding more terms.

More Sensors:

Several sites contain toys for electrical and computer engineers. My favorites are

- www.Digikey.com Electronics parts with a very good search engine and fast delivery
- www.Jameco.com Also a good supplier
- www.AdaFruit.com Good spot for microcontrollers
- www.SparkFun.com Good spot for GPS, sensors, micrcontrollers, and custom interface boards

If you go to Digikey and search "Sensors", you get over 19,000 hits: (results on 12/23/15)

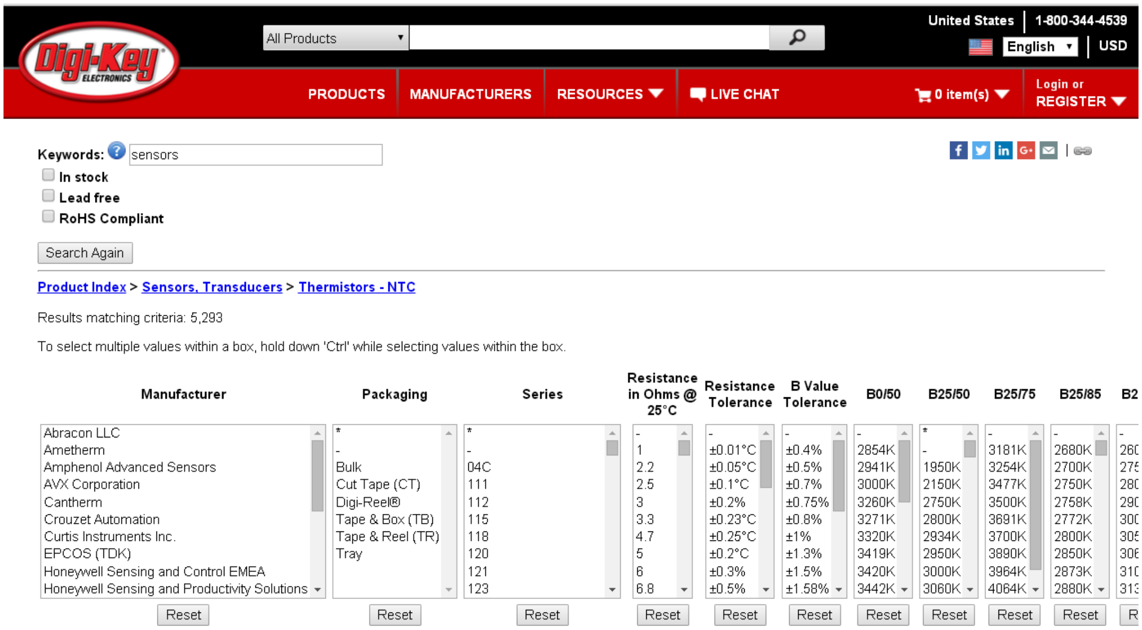


Sensors, Transducers - 1017 New Products
Accelerometers (1135 items)
Accessories (3898 items)
Amplifiers (313 items)
Capacitive Touch Sensors, Proximity Sensor ICs (606 items)
Color Sensors (135 items)
Current Transducers (1503 items)
Dust Sensors (16 items)
Encoders (4364 items)
Flex Sensors (1 items)
Float, Level Sensors (622 items)
Flow Sensors (191 items)
Force Sensors (104 items)
Gas Sensors (78 items)
Gyroscopes (248 items)
Image Sensors, Camera (555 items)
Inclinometers (55 items)
IrDA Transceiver Modules (295 items)
LVDT Transducers (Linear Variable Differential Transformer) (8 items)
Magnetic Sensors - Compass, Magnetic Field (Modules) (24 items)
Magnetic Sensors - Hall Effect, Digital Switch, Linear, Compass (ICs) (3747 items)
Magnetic Sensors - Position, Proximity, Speed (Modules) (3288 items)
Magnets (145 items)
Moisture Sensors, Humidity (376 items)
Motion Sensors, Detectors (290 items)
Multifunction (147 items)
Optical Sensors - Ambient Light, IR, UV Sensors (736 items)
Optical Sensors - Distance Measuring (41 items)
Optical Sensors - Mouse (118 items)
Optical Sensors - Photo Detectors - CdS Cells (59 items)
Optical Sensors - Photo Detectors - Logic Output (134 items)
Optical Sensors - Photo Detectors - Remote Receiver (1188 items)
Optical Sensors - Photodiodes (1062 items)
Optical Sensors - Photoelectric, Industrial (11149 items)
Optical Sensors - Photointerrupters - Slot Type - Logic Output (1063 items)
Optical Sensors - Photointerrupters - Slot Type - Transistor Output (1171 items)
Optical Sensors - Phototransistors (808 items)
Optical Sensors - Reflective - Analog Output (332 items)
Optical Sensors - Reflective - Logic Output (134 items)
Position Sensors - Angle, Linear Position Measuring (1290 items)
Pressure Sensors, Transducers (26790 items)
Proximity Sensors (3762 items)
Proximity/Occupancy Sensors - Finished Units (240 items)
RTD (Resistance Temperature Detector) (87 items)
Shock Sensors (15 items)
Solar Cells (103 items)
Specialized Sensors (610 items)

Strain Gauges (22 items)
Temperature Regulators (Mechanical) (3947 items)
Temperature Sensors, Transducers (3457 items)
Temperature Switches (917 items)
Thermistors - NTC (5293 items)
Thermistors - PTC (1251 items)
Thermocouple, Temperature Probe (431 items)
Tilt Sensors (55 items)
Ultrasonic Receivers, Transmitters (96 items)
Vibration Sensors (58 items)

Most of these have a resistance output. What this means is that if you can measure resistance, you can measure acceleration, color, dust, flex, gas, incline, magnetic fields, motion, light, pressure, etc.

For example, suppose you want to find a temperature sensor:



www.Digikey.com search for NTC Thermistor

What this tells you is you have 5,293 thermistors to choose from. To whittle this down a bit, you can narrow the search using

- Resistance @ 25C: Pretty much what is says. Low values are good for measuring wind speed (low R produces more self heating. The wind provides cooling. Temperature is thus a measurement of the cooling or wind speed). High values are good for low self-heating and low-power consumption.
- Resistance Tolerance: Smaller is better. The variation in the resistance at 25C (manufacturing tolerance.) Sort of a measure of the standard deviation - only most people don't know what standard deviation is.
- B Value Tolerance: Smaller is better. How accurate you know the temperature / resistance relationship, The lower the number, the more precise the measurement (and the more it costs.)
- B0/50, B25/50, etc. The temperature-resistance relationship parameter (more on this later)
- Operating Temperature (off the page to the right): The range the sensor can operate
- Mounting Type (off the page to the right): Through hole (good for us) or surface mount (good for industry).

Let's narrow the search to

- 1000 Ohms at 25C
- Through Hole
- In Stock

This narrows the selection down to 13 thermistors. (note: through-hole is mostly for students. High-volume applications, like cell phones, use surface mount parts - which is where the bulk of electronics parts go.)

Compare Parts		Image	Digi-Key Part Number	Manufacturer Part Number	Manufacturer	Description	Quantity Available	Unit Price USD	Minimum Quantity	Packaging	Series	Resistance in Ohms @ 25°C	Resistance Tolerance
			BC2519-ND	NTCLE100E3102JB0	Vishay BC Components	THERMISTOR NTC 1.0K 5% RADIAL	8,535 - Immediate	0.39000	1	Bulk	2381	1k	±5%
			BC2394-ND	NTCLE100E3102HB0	Vishay BC Components	THERMISTOR NTC 1.0K 3% RADIAL	2,954 - Immediate	0.45000	1	Bulk	-	1k	±3%
			BC2393-ND	NTCLE100E3102GB0	Vishay BC Components	THERMISTOR NTC 1.0K 2% RADIAL	1,428 - Immediate	0.67000	1	Bulk	-	1k	±2%
			KC016N-ND	RL2004-582-97-D1	Amphenol Advanced Sensors	THERMISTOR NTC 1K OHM @ 25C	1,233 - Immediate	2.07000	1	Bulk	RL2004	1k	±10%
			480-3157-ND	192-102DEW-A01	Honeywell Sensing and Productivity Solutions	THERMISTOR NTC 1KOHM RADIAL	483 - Immediate	7.29000	1	Bulk	192	1k	±1%
			495-2156-ND	B57891M102J	EPCOS (TDK)	THERMISTOR NTC 1.0K OHM 5% RAD	2,067 - Immediate	0.95000	1	Bulk	-	1k	±5%

Selecting the bottom one on the list (fairly low price and its what we have in stock) tells you the price in quantities of 1 to 10,000

Product Index > Sensors, Transducers > Thermistors - NTC > EPCOS (TDK) B57891M102J

Digi-Key Part Number	495-2156-ND
Quantity Available	Digi-Key Stock: 2,067 Can ship immediately
Manufacturer	EPCOS (TDK)
Manufacturer Part Number	B57891M102J
Description	THERMISTOR NTC 1.0K OHM 5% RAD
Lead Free Status / RoHS Status	Lead free / RoHS Compliant
Moisture Sensitivity Level (MSL)	1 (Unlimited)

All prices are in US dollars.

Price Break	Unit Price	Extended Price
1	0.95000	0.95
10	0.72200	7.22
100	0.51680	51.68
500	0.40014	200.07
1,000	0.34457	344.57
5,000	0.30566	1,528.31
10,000	0.29455	2,945.48

Quantity | Item Number | Customer Reference

1 | 495-2156-ND | | Add to Cart

Image shown is a representation only. Exact specifications should be obtained from the product data sheet.

Clicking on the data sheet brings up the specifications for the thermistor:

General technical data

Climatic category	(IEC 60068-1)		40/125/56	
Max. power	(at 25 °C)	P_{25}	200	mW
Resistance tolerance		$\Delta R_R/R_R$	±5, ±10	%
Rated temperature		T_R	25	°C
Dissipation factor	(in air)	δ_{th}	approx. 3.5	mW/K
Thermal cooling time constant	(in air)	τ_c	approx. 12	s
Heat capacity		C_{th}	approx. 40	mJ/K

Electrical specification and ordering codes

R_{25} Ω	No. of R/T characteristic	$B_{25/100}$ K	Ordering code
1 k	1009	3930 ±3%	B57891M0102+000
1.5 k	1008	3560 ±3%	B57891M0152+000
2.2 k	1013	3900 ±3%	B57891M0222+000
3.3 k	2003	3980 ±3%	B57891M0332+000
4.7 k	2003	3980 ±3%	B57891M0472+000
6.8 k	2003	3980 ±3%	B57891M0682+000
10 k	4901	3950 ±3%	B57891M0103+000
15 k	2004	4100 ±3%	B57891M0153+000
22 k	2904	4300 ±3%	B57891M0223+000

This tells you

- Limit the self-heating to 200mW $(I^2R < 200mW)$. For a 1k thermistor, limit the voltage to less than 14.14V across the thermistor at 25C
- Dissipation factor (in air): 3.5mW/K. At equilibrium, power out = power in. Power in is self heating (I^2R). Power out is from cooling (3.5mW/K).
- Thermal cooling time constant (in air): 12 seconds. It takes some time for the thermistor to warm up to air temperature. The thermistor will be within 5% of equilibrium in 3 time constants, or 36 seconds.
- B25/100: 3930K. This is the temperature - resistance relationship where T is the temperature in degrees Kelvin (Celsius + 273)

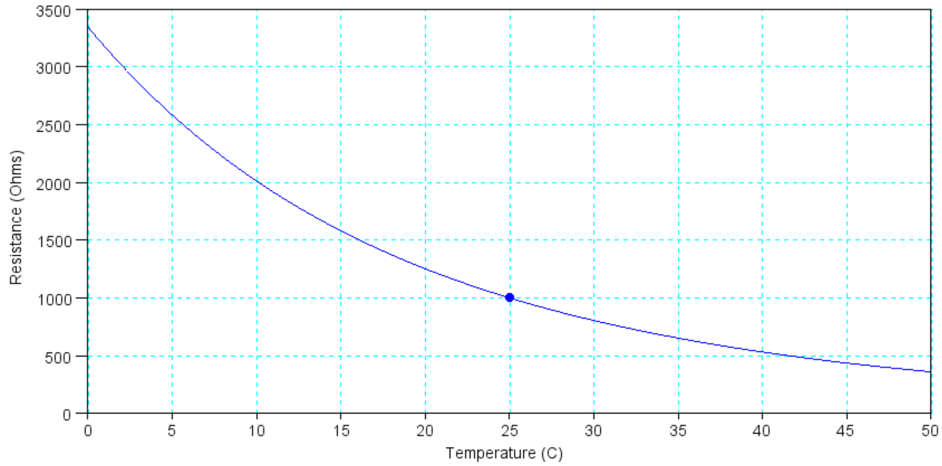
$$R \approx 1000 \cdot \exp\left(\frac{3930}{T} - \frac{3930}{298K}\right)$$

Modeling in Matlab

Suppose this thermistor is used with a 1k resistor in a voltage divider. Determine the following relationships from 0C to 50C

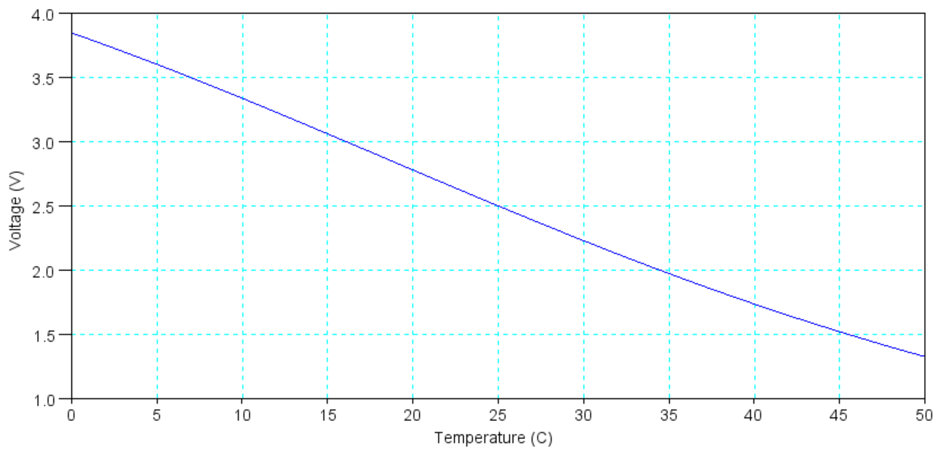
- Resistance vs. temperature
- Voltage vs. temperature
- A/D reading vs. temperature

```
-->T = [0:0.1:50]';
-->K = T + 273;

-->R = 1000 * exp((3930 ./ K)  -  (3930 / 298) );

-->plot(T,R);
-->xlabel('Temperature (C)');
-->ylabel('Resistance (Ohms)');
```



Temperature vs. Resistance Relationship. Note the resistance is 1000 Ohms at 25C.

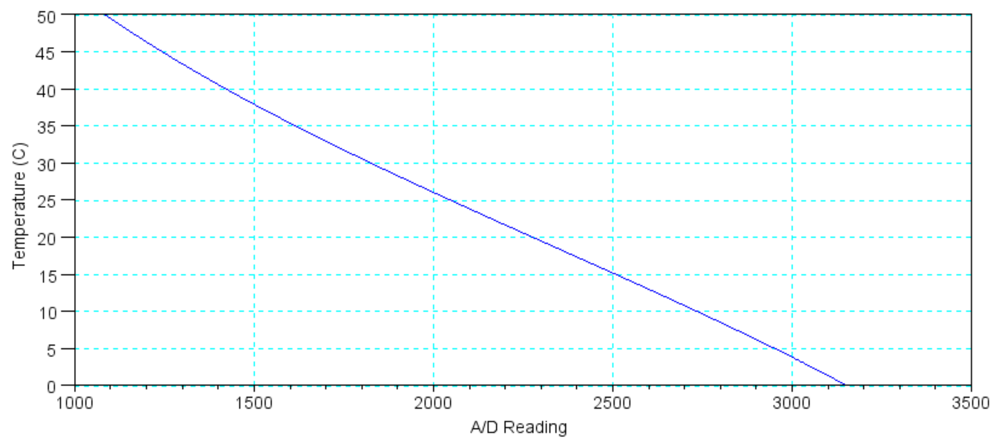
```
-->V = R ./ (1000 + R) * 5;
-->plot(T,V);
-->xlabel('Temperature (C)');
-->ylabel('Voltage (V)');
```



Temperature vs. Voltage Relationship.

```
-->A2D = V/5 * 4095;
-->plot(A2D,T)
```

```
-->xlabel('A/D Reading');
-->ylabel('Temperature (C)');
```



A/D Reading (what you know) vs. Temperature (what you want to measure)

Once you have the A/D reading vs. temperature (or light, pressure, gas, etc), you can use least squares to come up with a calibration function like we did before.

Self Heating:

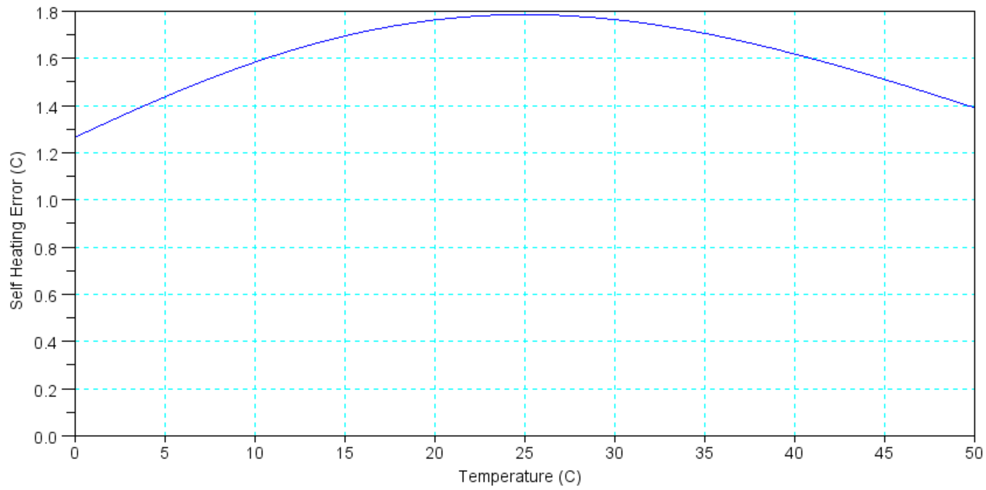
As a sidelight, with thermistors you need to worry about self heating. By powering the thermistor, it warms up as

$$P = I^2 R = \frac{V^2}{R}$$

If exposed to air, the thermistor is able to dissipate heat as 3.5mW/K. This means, at equilibrium, the thermistor will be slightly warmer than the air as

$$dT \approx \left(\frac{V^2/R}{0.0035 W/K} \right)$$

```
-->P = V.^2 ./ R;
-->dT = P / 0.0035;
-->plot(T,dT);
-->xlabel('Temperature (C)');
-->ylabel('Self Heating Error (C)');
```



Self-Heating from 0..50C. This self-heating results in the measured temperature being off

Assuming you know you are measuring air temperature with no wind, you could compensate for this bias in your calibration function.

If you use a thermistor with a larger resistance, the self-heating will be smaller. If you use a thermistor with a smaller resistance, the self heating will be larger.

If there is wind, the heat dissipation will increase. This allows to measure wind speed by comparing the measured temperature of a low R (high self-heating) and high R thermistor (low self-heating) circuit.