# ECE 461/661 Handout #10

LaPlace Transforms

1) Determine the differential equation relating X and Y

$$Y = \left(\frac{20s+50}{s^3+8s^2+20s+40}\right)X$$

2) Determine y(t)

$$Y = \left(\frac{10(s+5)}{(s+1)(s+4)}\right)X$$

$$x(t) = 2\cos(3t) + 4\sin(3t)$$

3) Determine the step response of the following system

$$Y = \left(\frac{10(s+5)}{(s+1)(s+4)}\right)X$$

4) Determine the step response of the following system

$$Y = \left(\frac{10(s+5)}{(s+1)(s+2+j4)(s+2-j4)}\right)X$$

1) Determine the differential equation relating X and Y

$$Y = \left(\frac{20s+50}{s^3+8s^2+20s+40}\right)X$$

Cross multiply

$$(s^3 + 8s^2 + 20s + 40)Y = (20s + 50)X$$

meaning

$$y''' + 8y'' + 20y' + 40y = 20x' + 50x$$

2) Determine y(t)

$$Y = \left( \frac{10(s+5)}{(s+1)(s+4)} \right) X$$

$$x(t) = 2\cos(3t) + 4\sin(3t)$$

Solution: This is actually a phasor problem (x(t) was on for infinite time)

- $x(t) = 2\cos(3t) + 4\sin(3t)$ phasors: $-\infty < t < \infty$
- $x(t) = (2\cos(3t) + 4\sin(3t))u(t)$ LaPlace $t > 0$

G(s) tells you the gain everywhere.

$$Y = \left( \frac{10(s+5)}{(s+1)(s+4)} \right) X$$

All we care about is the gain at s = j3

$$s = j3$$

$$X = 2 - j4$$

( real = cosine, -imag = sine )

$$Y = \left( \frac{10(s+5)}{(s+1)(s+4)} \right)_{s=j3} \cdot (2 - j4)$$

$$Y = -12.8 - j10.4$$

meaning

$$y(t) = -12.8\cos(3t) + 10.4\sin(3t)$$

3) Determine the step response of the following system

$$Y = \left(\frac{10(s+5)}{(s+1)(s+4)}\right)X$$

$$Y = \left(\frac{10(s+5)}{(s+1)(s+4)}\right)\left(\frac{1}{s}\right)$$

Use partial fraction

$$Y = \left(\frac{A}{s}\right) + \left(\frac{B}{s+1}\right) + \left(\frac{C}{s+4}\right)$$

$$A = \left(\frac{10(s+5)}{(s+1)(s+4)}\right)_{s=0} = 12.5$$

$$B = \left(\frac{10(s+5)}{s(s+4)}\right)_{s=-1} = -13.333$$

$$C = \left(\frac{10(s+5)}{s(s+1)}\right)_{s=-4} = 0.8333$$

$$Y = \left(\frac{12.5}{s}\right) + \left(\frac{-13.333}{s+1}\right) + \left(\frac{0.8333}{s+4}\right)$$

Take the inverse LaPlace transform

$$y(t) = 12.5 - 13.3333e^{-t} + 0.8333e^{-4t} \quad \text{for } t > 0$$

4) Determine the step response of the following system

$$Y = \left(\frac{10(s+5)}{(s+1)(s+2+j4)(s+2-j4)}\right)X$$

$$Y = \left(\frac{10(s+5)}{(s+1)(s+2+j4)(s+2-j4)}\right)\left(\frac{1}{s}\right)$$

Use partial fractions

$$Y = \left(\frac{A}{s}\right) + \left(\frac{B}{s+1}\right) + \left(\frac{C}{s+2+j4}\right) + \left(\frac{D}{s+2-j4}\right)$$

$$A = \left(\frac{10(s+5)}{(s+1)(s+2+j4)(s+2-j4)}\right)_{s=0} = 2.5$$

$$B = \left(\frac{10(s+5)}{s(s+2+j4)(s+2-j4)}\right)_{s=-1} = -2.3529$$

$$C = \left(\frac{10(s+5)}{s(s+1)(s+2-j4)}\right)_{s=-2-j4} = -0.0735 - j0.3309$$

$$D = \left(\frac{10(s+5)}{s(s+1)(s+2+j4)}\right)_{s=-2+j4} = -0.0735 + j0.3309$$

meaning

$$Y = \left(\frac{2.5}{s}\right) + \left(\frac{-2.3529}{s+1}\right) + \left(\frac{-0.0735-j0.3309}{s+2+j4}\right) + \left(\frac{-0.0735+j0.3309}{s+2-j4}\right)$$

$$Y = \left(\frac{2.5}{s}\right) + \left(\frac{-2.3529}{s+1}\right) + \left(\frac{0.3390\angle -102^0}{s+2+j4}\right) + \left(\frac{0.3390\angle 102^0}{s+2-j4}\right)$$

meaning

$$y(t) = 2.5 - 2.3929e^{-t} + 0.6779e^{-2t}\cos(4t + 102^0) \quad \text{for } t > 0$$