ECE 461 - Solution to Homework Set #4

LaPlace Transforms, 1st and 2nd Order Approximations, Block Diagrams. - Due Monday, September 28th

1) For the following system

$$Y = \left(\frac{100}{(s+2)(s+5)(s+20)}\right) U$$

1a) Find a 1st order approximation which has almost the same step response

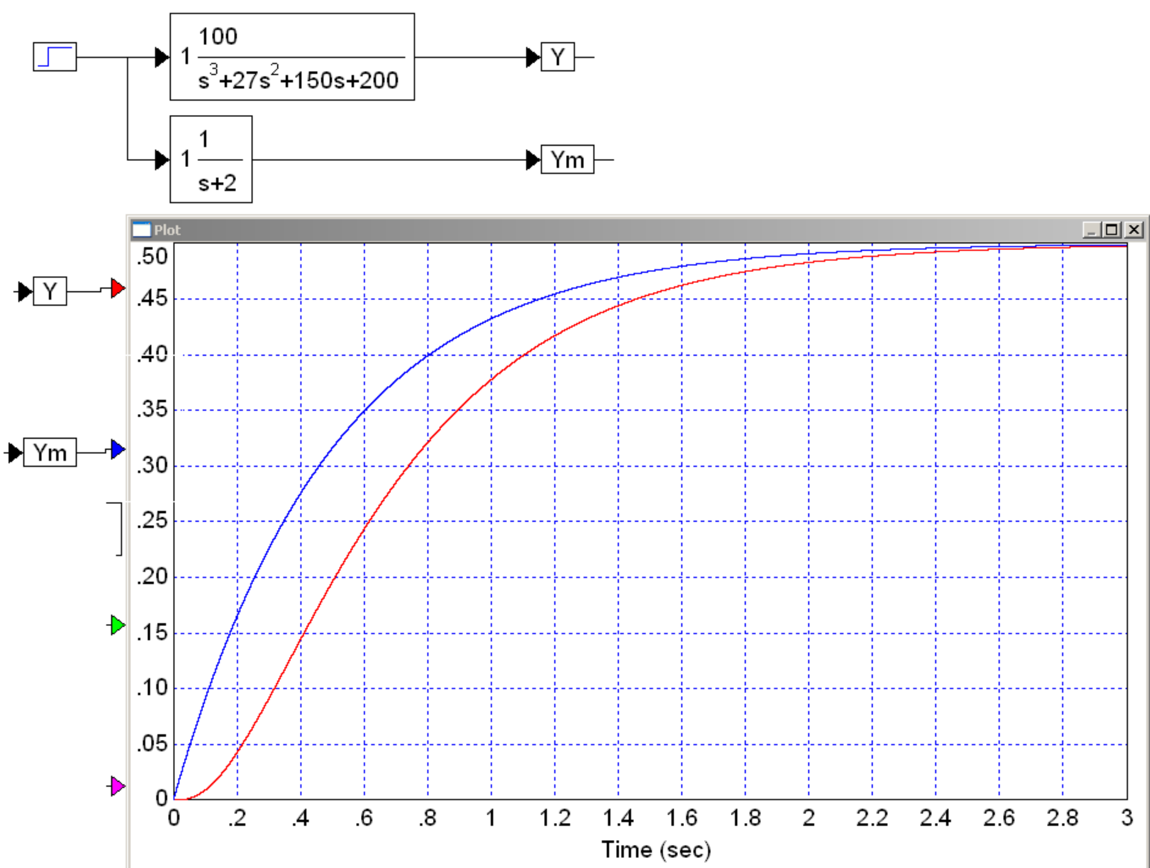
Dominant pole: s = -2

DC Gain: 0.5

$$Y = \left(\frac{1}{s+2}\right)\left(\frac{5}{s+5}\right)\left(\frac{20}{s+20}\right) U$$

$$Y \approx \left(\frac{1}{s+2}\right) U$$

1b) Plot the step response of the 3rd-order system and its 1st-order approximation.



2) For the following system

$$Y = \left(\frac{1000}{(s+1+j4)(s+1-j4)(s+30)(s+50)} \right) U$$

2a) Find a 2nd order approximation which has almost the same step response

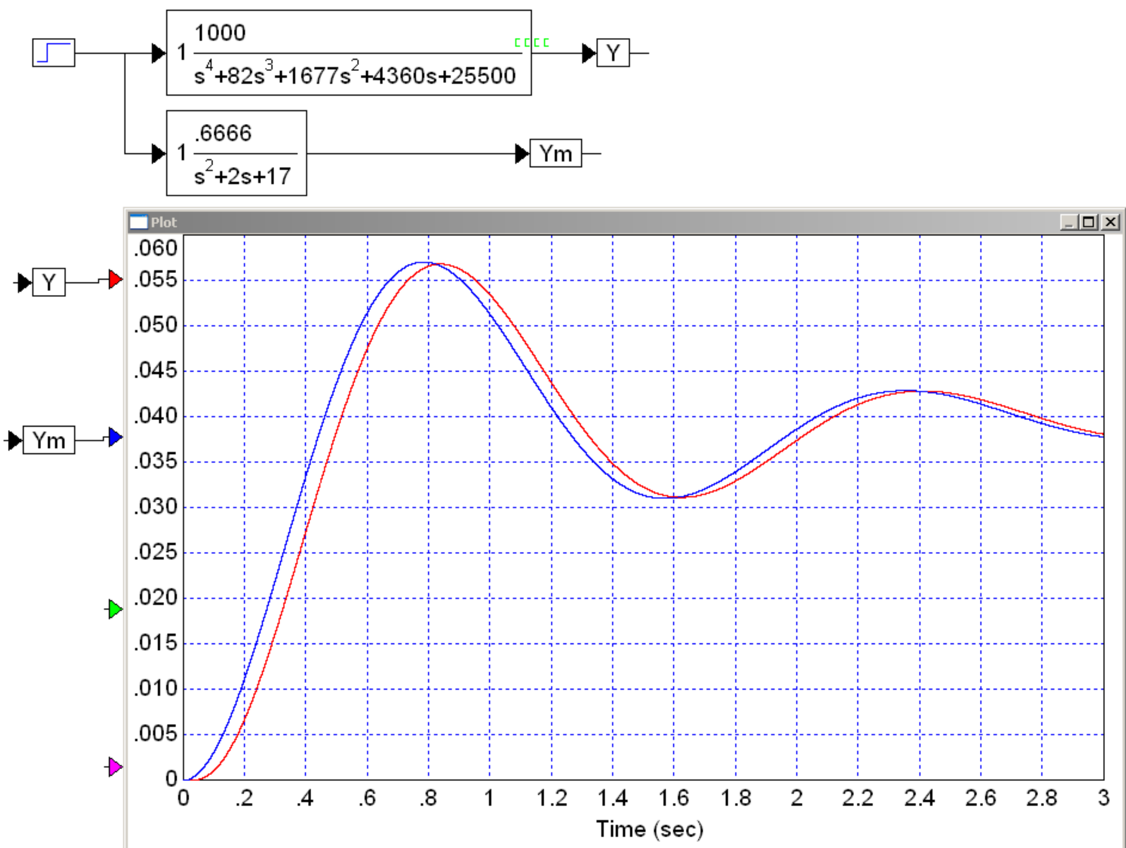
Dominant Pole: s = -1 + j4, -1 - j4

DC Gain: 0.03921

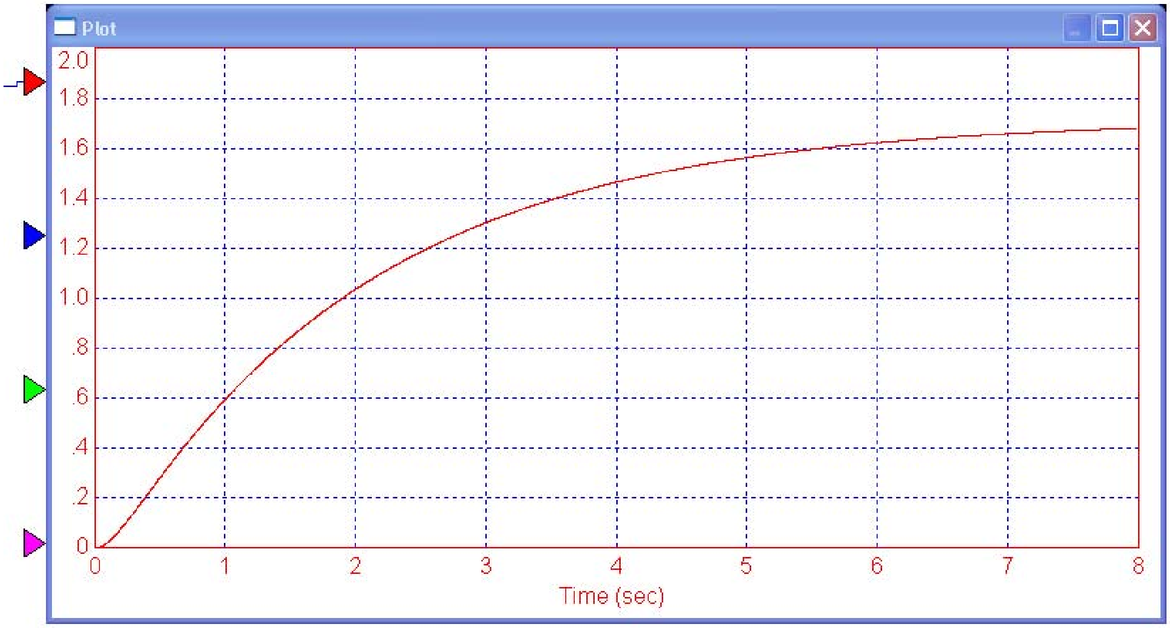
$$Y = \left(\frac{0.6666}{(s+1+j4)(s+1-j4)} \right) \left(\frac{30}{s+30} \right) \left(\frac{50}{s+50} \right) U$$

$$Y \approx \left(\frac{0.6666}{(s+1+j4)(s+1-j4)} \right) U$$

2b) Plot the step response of the 4th-order system and its 2nd-order approximation.



3) Find the transfer function for a system with the following step response.



This behaves like a 1st order system (no oscillation), so

$$G(s) \approx \frac{a}{s+b}$$

You need to get two pieces of information from this graph:

1) DC gain = 1.666

$$\left(\frac{a}{s+b}\right)_{s=0} = 1.666$$

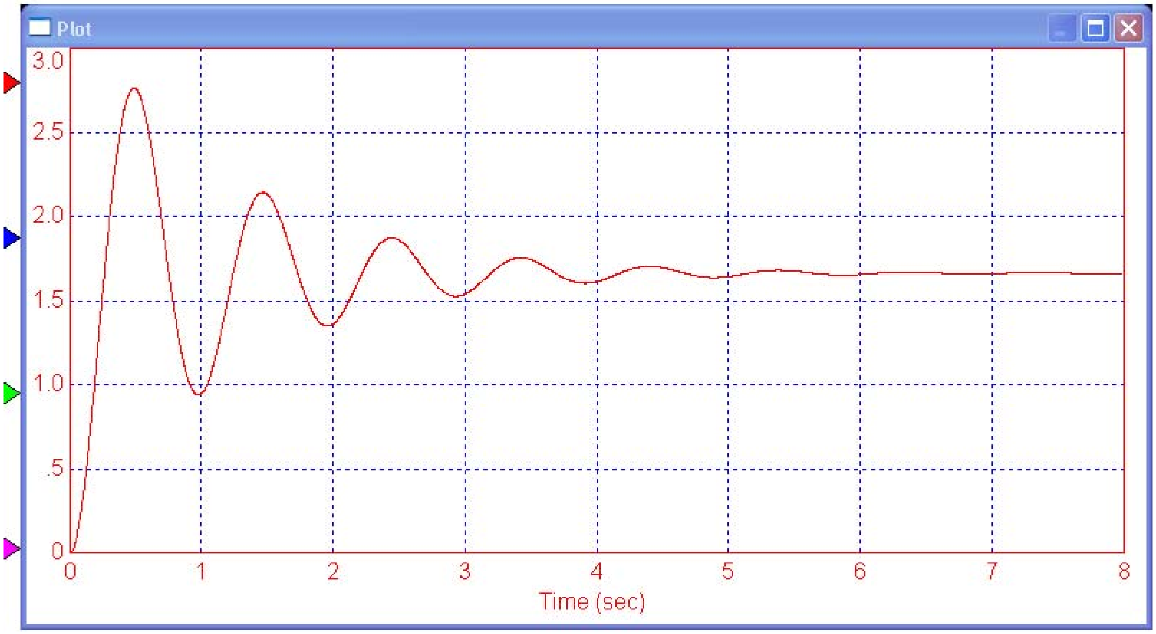
2) 2% settling time = 7 seconds (approx)

$$\frac{4}{b} = 7$$

Resulting in

$$G(s) \approx \left(\frac{0.952}{s+0.571}\right)$$

4) Find the transfer function for a system with the following step response.



This behaves like a 2nd-order system (oscillation). You need to pull three pieces of information from this graph.

1) DC gain = 1.666

2) Frequency of oscillation = $\left(\frac{4 \text{ cycles}}{4 \text{ seconds}}\right) = 1Hz$

$$\omega_d = 2\pi f = 6.28\frac{rad}{sec}$$

3) 2% Settling time = 5 seconds

$$\frac{4}{\sigma} = 5$$

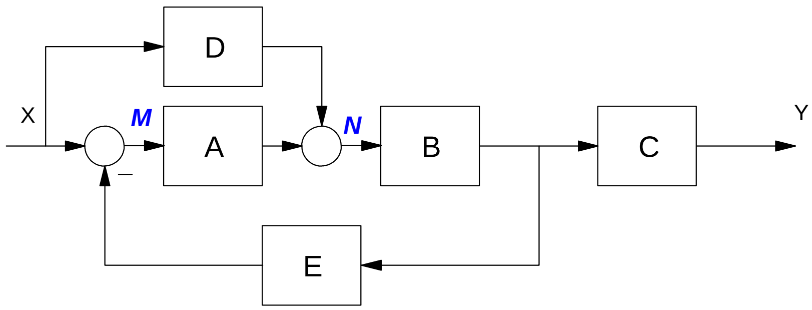
Resulting in

$$G(s) \approx \left(\frac{66.77}{(s+0.8+j6.28)(s+0.8-j6.28)}\right)$$

or

$$G(s) \approx \left(\frac{66.77}{s^2+1.6s+40.07}\right)$$

5) For the following block diagram, determine the transfer function from X to Y



Shortcut:

$$G(s) = \left(\frac{\text{gain from X to Y}}{1 + \text{loop gains}}\right)$$

$$Y = \left(\frac{ABC+BCD}{1+ABE}\right)X$$

Long Way:

Add a dummy variable at the output of each summing junction.

Write N equations

M = X - EBN

N = AM + DX

Y = CBN

Solve

N = A(X - EBN) + DX

$$(1 + ABE)N = (A + D)X$$

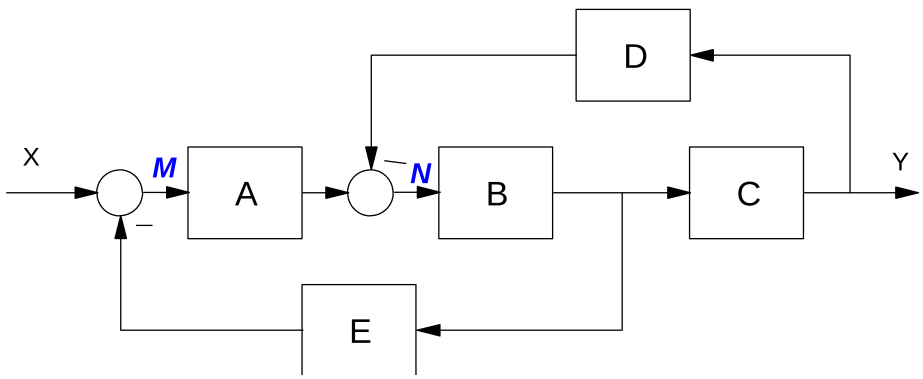
$$N = \frac{(A+D)}{(1+ABE)}X$$

$$Y = CBN = \left(\frac{CB(A+D)}{(1+ABE)}\right)X$$

$$Y = \left(\frac{ABC+BCD}{1+ABE}\right)X$$

which is what we got before...

6) For the following block diagram, determine the transfer function from X to Y



Shortcut:

$$G(s) = \left(\frac{\text{gain from X to Y}}{1 + \text{loop gains}}\right)$$

$$Y = \left(\frac{ABC}{1+ABE+BCD}\right)X$$

Long Version: Add a dummy variable at the output of each summing junction. Write N equations

M = X - EBN

N = AM - DCBN

Y = CBN

Solve

AM = AX - AEBN

N = (AX - AEBN) - DCBN

$$(1 + ABE + BCD)N = AX$$

$$N = \left(\frac{A}{1+ABE+BCD}\right)X$$

$$Y = CBN$$

$$Y = \left(\frac{ABC}{1+ABE+BCD}\right)X$$