ECE 461 - Homework Set #6

Rotational Systems, Error Constants, Routh Criteria - Due Monday, October 12th

1) The parameters for a 750W (1hp) DC motor as follows ($1126 ea)

Data Sheets:

http://servosystems.com/pdf/amp/m0750-102-5-000.pdf

What is the transfer function for this motor from Volts to Speed (rad/sec)?

Armature Resistance: 1.3 Ohms

Armature Inductance: 6.3mH

Torque Constant: 4.4 inch pounds / amp

$$4.4\left(\frac{\text{inch}\cdot\text{pounds}}{amp}\right)\left(\frac{1m}{39.37in}\right)\left(\frac{1kg}{2.204lb}\right)\left(\frac{9.8N}{kg}\right) = 0.4969\frac{Nm}{A}$$

Rotor Intertia: 0.0153 oz in sec2

$$0.0153(\text{oz}\cdot\text{in}\cdot s^2)\left(\frac{1lb}{16oz}\right)\left(\frac{1kg}{2.204lb}\right)\left(\frac{9.8N}{kg}\right)\left(\frac{1m}{39.37in}\right) = 0.000108\ kg\cdot m^2$$

Friction: No data. Assume zero

Plugging all this in

$$\omega = \left(\frac{K_t}{(Js+B)(Ls+R)+K_t^2}\right)V_a$$

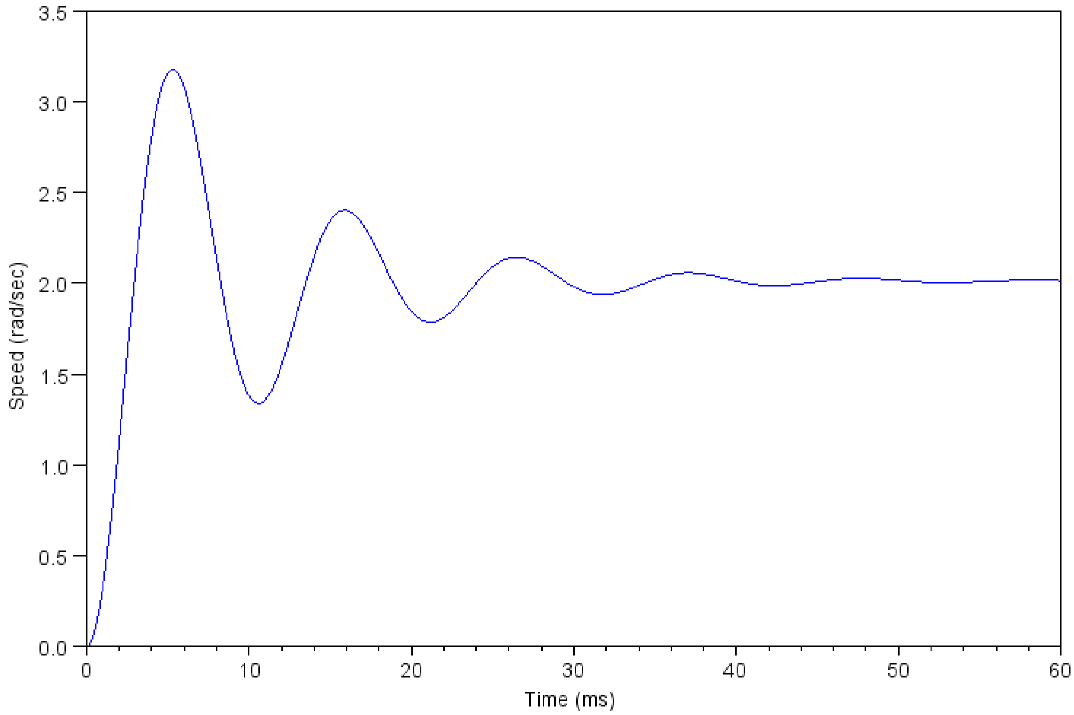
$$\omega = \left(\frac{0.4969}{(0.000108s+0)(0.0063s+1.3)+0.4946^2}\right)V_a$$

$$\omega = \left(\frac{730{,}305.7}{s^2+206.34s+362{,}888}\right)V_a$$

$$\omega = \left(\frac{730{,}305.7}{(s+103.17\pm j593.5)}\right)V_a$$

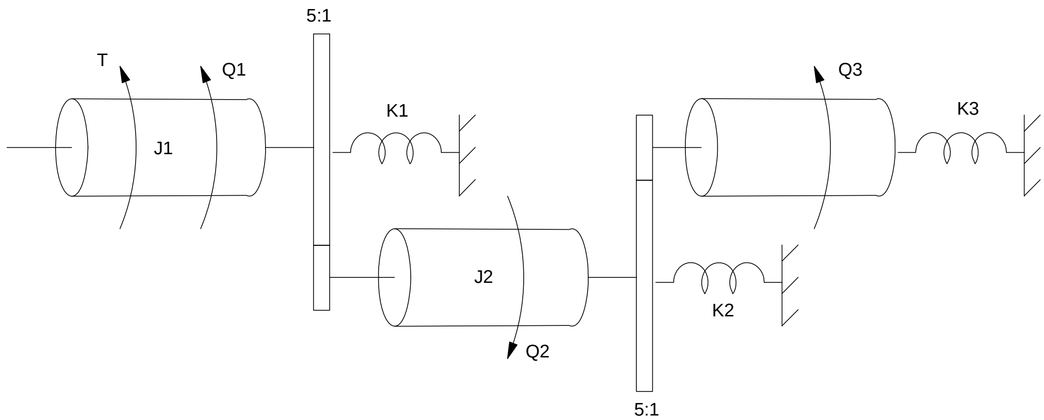
Taking the step response. In MATLAB

```
G = tf(730305, [1, 206.34, 362888]);
t = [0:0.001:1]' * 0.06;
y = step(G, t);
plot(t,y)
```

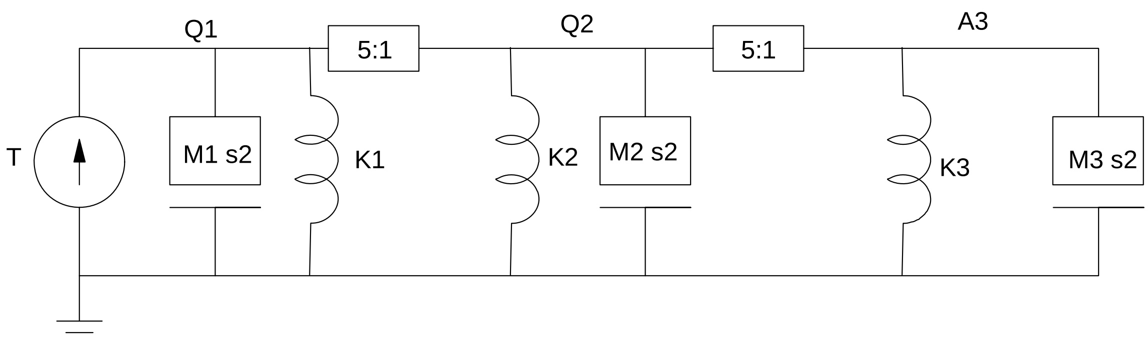


Step response of m0750-102-5-000 motor

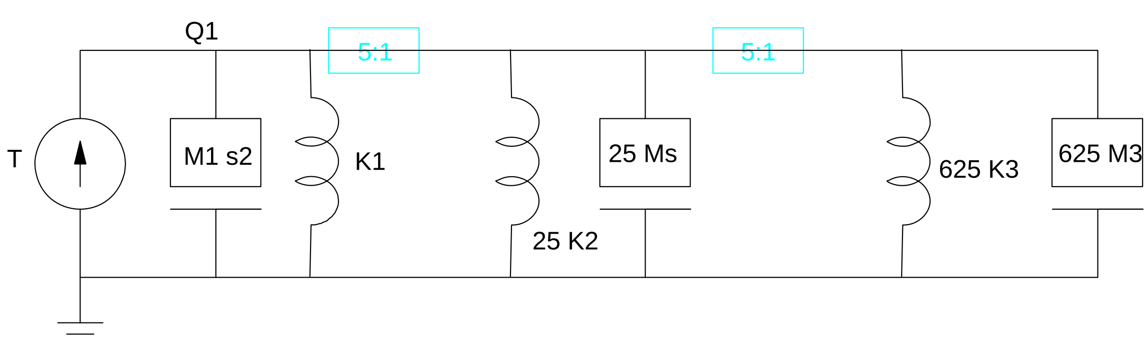
2) For the following rotational system



2a) Draw the circuit equivalent



2b) Remove the gears and bring everything to node Q1



2c) Determine the dynamics assuming J = 0.01 kg m2/s2 and K = 5Nm/rad

$$((M_1 + 25M_2 + 625M_3)s^2 + (K_1 + 25K_2 + 625K_3))\theta_1 = T$$

$$(6.51s^2 + 3255)\theta_1 = T$$

$$\theta_1 = \left(\frac{0.1536}{s^2+5}\right)T$$

3) The Determine the system type and stead-state error for a step and ramp input

G(s)	System Type	Kp	Kv	Error for a Step Input
$\left(\frac{100}{(s+3)(s+10)}\right)$	**0**	**3.33**	**0**	**0.23**
$\left(\frac{100}{(s-3)(s+10)}\right)$	**0**	**-3.33**	**0**	**-0.4**
$\left(\frac{100}{s(s+3)(s+10)}\right)$	**1**	**infinity**	**3.33**	**0**
$\left(\frac{100}{s^2(s+3)(s+10)}\right)$	**2**	**infinity**	**infinity**	**0**

4) Determine the range of k that results in a stable system using a Routh table

$$(s+1)(s+5)(s+10)+10k=0$$

$$s^3+16s^2+65s+50+10k=0$$

1	**65**	**0**	
16	**50 + 10k**	**0**	
$\frac{-\begin{vmatrix} 1 & 65 \\ 16 & 50+10k \end{vmatrix}}{16}=61.875-0.625k$	$\frac{-\begin{vmatrix} 1 & 0 \\ 16 & 0 \end{vmatrix}}{16}=0$	**0**	**k < 99**
50 + 10k	**0**	**0**	**k > -5**
0	**0**	**0**	

result:

-5 < k < 99

5) Determine the range of k that results in a stable system using a Routh table

$$(s-1)(s+5)(s+10)(s+15)+10k=0$$

1. 29. 245. 475. - 750.

$$s^4+29s^3+245s^2+475s-750+10k=0$$

1	**245**	**10k-750**	
29	**475**	**0**	
$\frac{-\begin{vmatrix} 1 & 245 \\ 29 & 475 \end{vmatrix}}{29}=228.62$	$\frac{-\begin{vmatrix} 1 & 10k-750 \\ 29 & 0 \end{vmatrix}}{29}=10k-750$	**0**	
$\frac{-\begin{vmatrix} 29 & 475 \\ 228.62 & 10k-750 \end{vmatrix}}{228.62}=570.1-1.27k$	**0**	**0**	k<449
10k-750	**0**	**0**	k>75
0	**0**	**0**	

result

75 < k < 449

Lab (Friday)

Determine the dymamics of the DC servo motor used in lab

$$\omega = \left(\frac{K_t}{(Js+B)(Ls+R)+K_t^2}\right) V_{in}$$

6) Measure the resistance and inductance

R	L
11.6 Ohms	**12.5 mH**

7) Apply a DC voltage to the motor (such as 10V). Measure the speed and current. From this compute the torque constant (Kt)

Vin	Iin	w (rad/sec)	Kt
10.00V	93.86 mA	72.33 rad/sec	**0.1232 Nm/A**

At constant speed: $V_{in} = I_{in} R + K_t \omega$

8) Measure the step response (apply a 10V step input and measure the speed vs. time). From this, compute a first-order model for the motor. (L = 0)

$$\frac{K_t}{(Js+B)(R+K_t^2} = \frac{a}{s+b}$$

Match terms to get J and B

1st Order Model	J	B
$\left(\frac{43}{s+6}\right)$	0.000 254 kg m^2	0.000 138 Nm / (rad/sec)

$$\left(\frac{K_t}{(Js+B)(R)+K_t^2}\right) = \frac{43}{s+6}$$

$$\left(\frac{\left(\frac{K_t}{JR}\right)}{s+\left(\frac{BR+K_t^2}{JR}\right)}\right) = \frac{43}{s+6}$$

$$\left(\frac{K_t}{JR}\right) = 43$$

J = 0.000 254 kg m2

$$\left(\frac{BR+K_t^2}{JR}\right) = 6$$

B = 0.000 138 Nm / (rad/sec)