ECE 461 - Homework Set #7

Root Locus an dGain Compensation - Due Monday, October 19th

Problem 1-4)

$$G(s) = \left(\frac{100}{(s+1)(s+4)(s+8)(s+9)}\right)$$

1) Sketch the root locus of G(s) including real axis loci, asymptotes, and jw rossing.

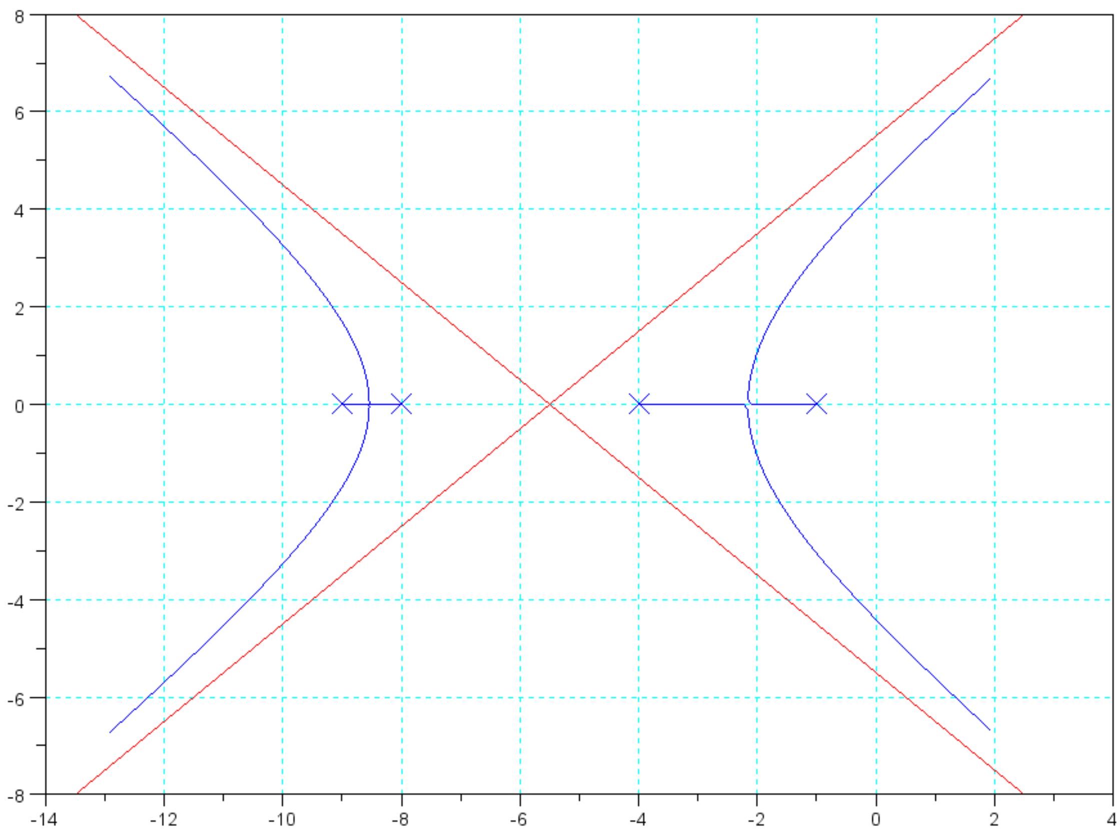
Real Axis Loci: (-1, -4), (-8, -9)

Asymptotes: four @ $\pm 45^0$, $\pm 135^0$

Interesect = -5.5

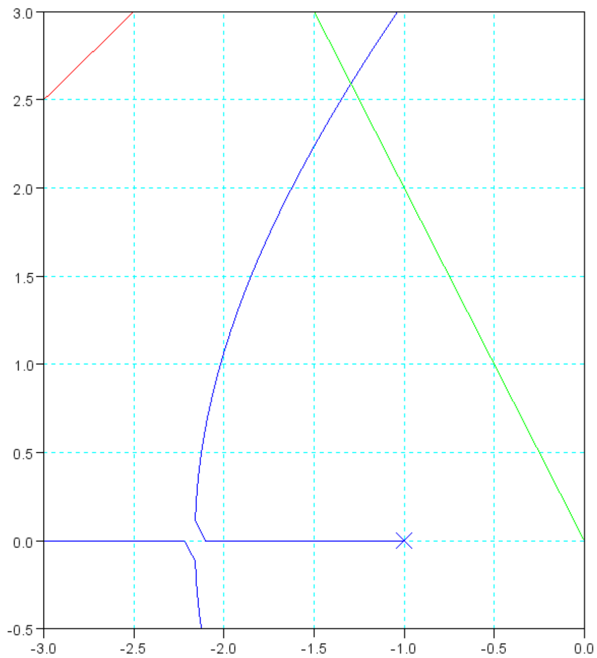
jw crossing: j4.4107

Breakaway Points: -2.1611, -8.5438



2) Determine the feedback gain, k, which results in 20% overshoot for a step input.

Search along the 0.4554 damping line until the angles add up to 180 degrees



ans:

$$s = -1.2956 + j2.5911$$

At any point on the root locus

$$GK = -1$$

At this point

$$\left(\frac{100}{(s+1)(s+4)(s+8)(s+9)} \right)_{s=-1.2956+j2.5911} = 0.1752\angle 180^0$$

meaning

$$k = \frac{1}{0.1752} = 5.7068$$

3) For this value of k, determine the closed-loop system's

Closed-loop dominant poles

$s = -1.2956 + j2.5911$

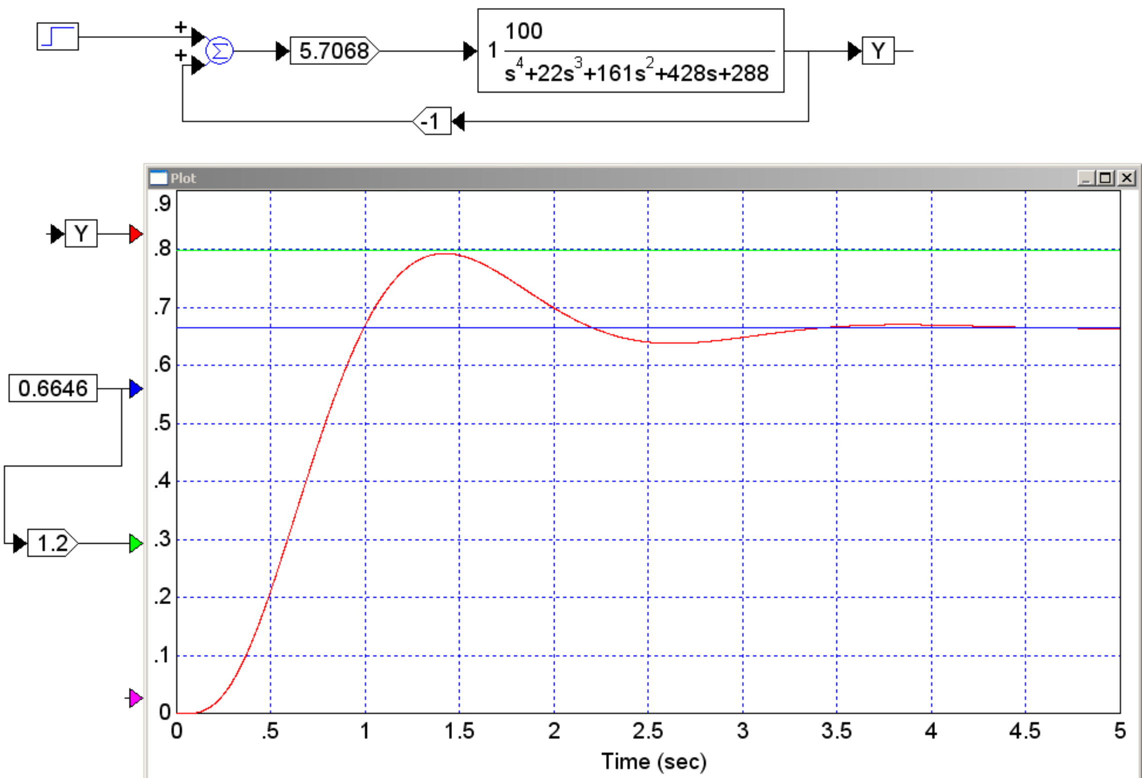
Steady-state error for a step input

$$K_p = (G \cdot k)_{s=0} = \left(\frac{100k}{(s+1)(s+4)(s+8)(s+9)}\right)_{s=0} = (0.3472)(5.7068)$$

$$K_p = 1.9815$$

$$E = \frac{1}{K_p+1} = 0.3354$$

4) Check your design in VisSim or MATLAB



Problem 5-8)

$$G(s) = \left(\frac{100(s+6)}{(s-1)(s+5)(s+10)(s+12)}\right)$$

5) Sketch the root locus of G(s) including real axis loci, asymptotes, and jw rossing.

Real Axis Loci: (+1, -5) (-6, -10), (-12, -infinity)

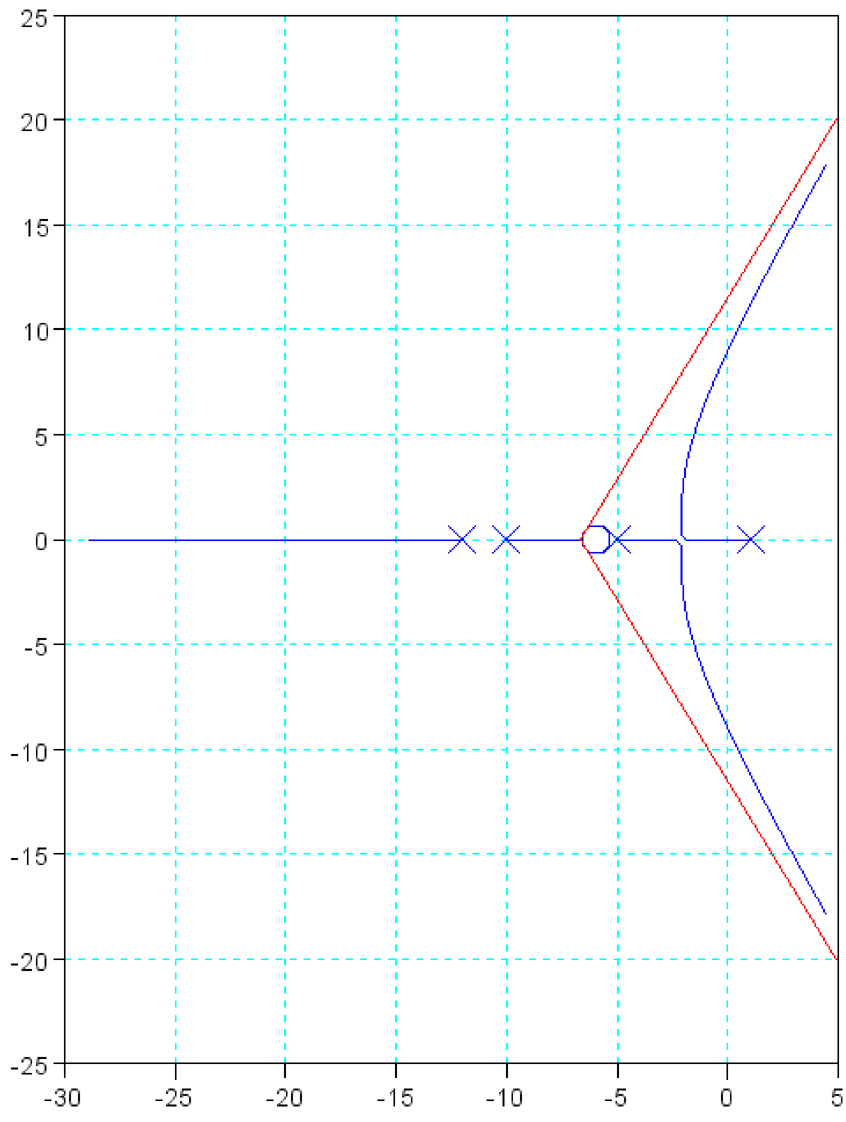
Asymptotes three

$\pm 60^0, 180^0$

$$\text{Intersect} = \left(\frac{(1-5-10-12)-(-6)}{4-1}\right) = -6.6666$$

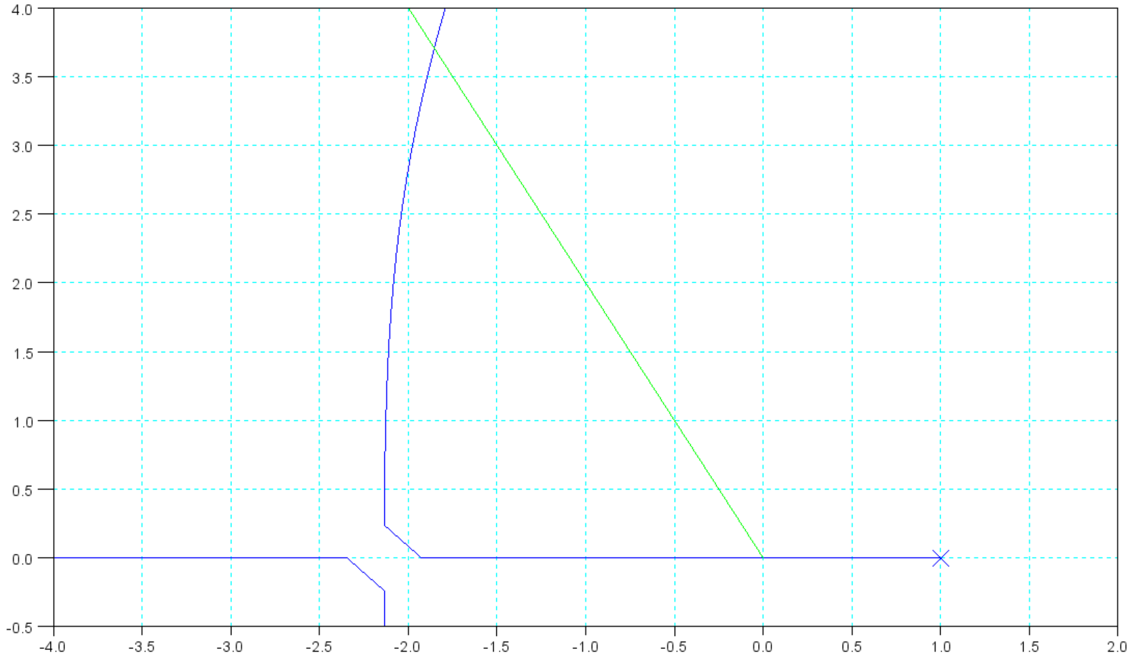
Breakaway Points: -2.1358

jw Crossing: j9.0317



6) Determine the feedback gain, k, which results in 20% overshoot for a step input.

Search along the 0.4559 damping line until the angles add up to 180 degrees



s = -1.8534 + j3.7067

At this point, make GK = -1

$$\left(\frac{100(s+6)}{(s-1)(s+5)(s+10)(s+12)}\right)_{s=-1.8534+j3.7067} = 0.2529\angle 180^0$$

$$k = \frac{1}{0.2529} = 3.9538$$

7) For this value of k, determine the closed-loop system's

Closed-loop dominant poles

s = -1.8534 + j3.7067

Steady-state error for a step input

$$K_p = (GK)_{s=0}$$

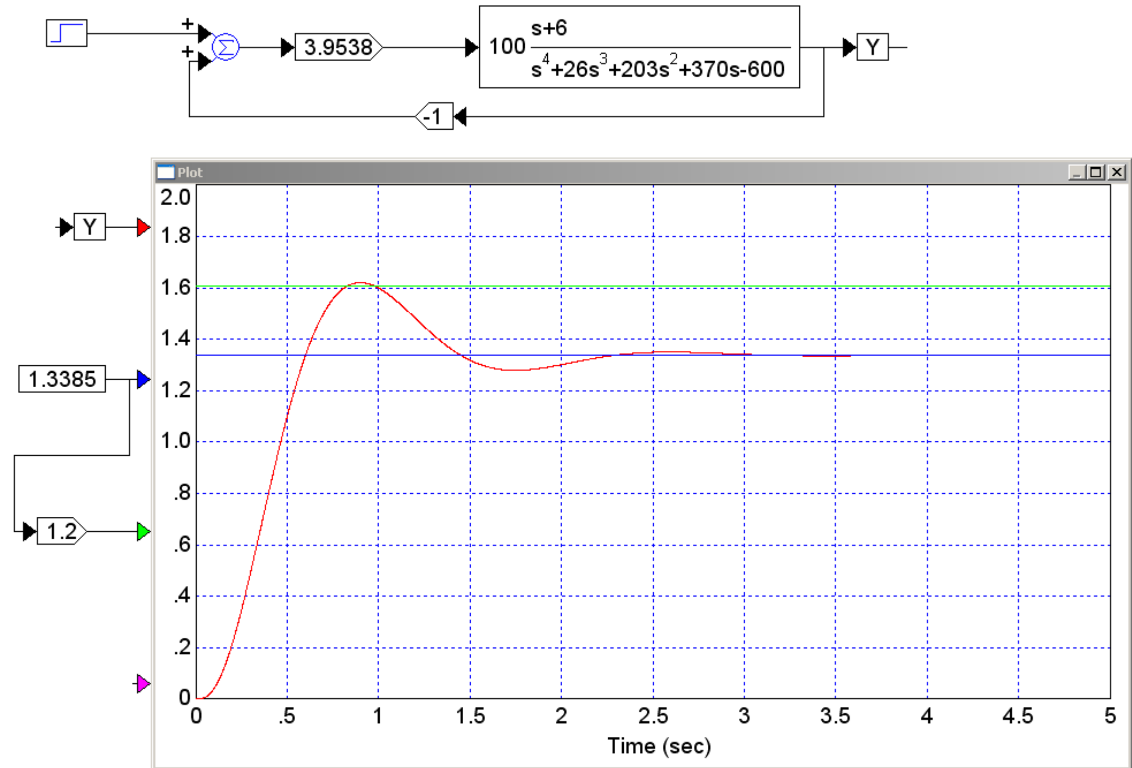
$$K_p = \left(\frac{100(s+6)}{(s-1)(s+5)(s+10)(s+12)}\right)_{s=0} \cdot 3.9538$$

$$K_p = (-1)(3.9538) = -3.9538$$

$$E_{step} = \frac{1}{K_p+1}$$

$$E_{step} = -0.3386$$

8) Check your design in VisSim or MATLAB



Problem 9-12)

$$G(s) = \left(\frac{10(s+1+j3)(s+1-j3)}{s(s+5)(s+10)}\right)$$

9) Sketch the root locus of G(s) including real axis loci, asymptotes, and approach angle to the zero at -1 + j3.

Real Axis Loci: (0, -5), (-10, -infinity)

Asymptotes: one @ 180 degrees

Breakaway Point -1.7260

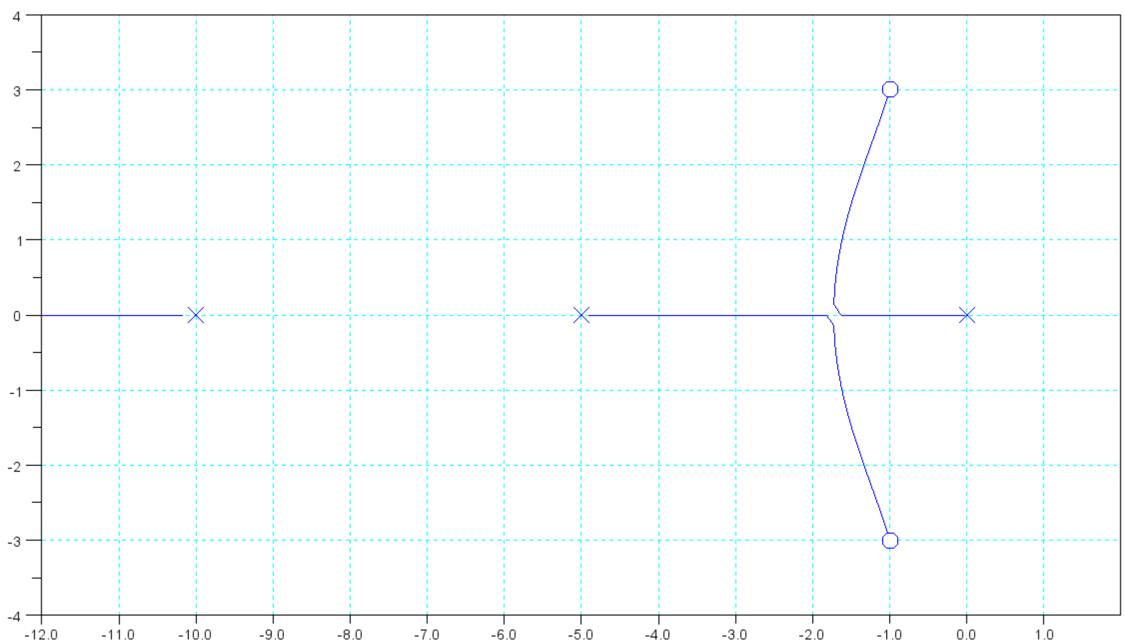
Approach Angle:

$$\angle\left(\frac{10(s+1+j3)(s+1-j3)}{s(s+5)(s+10)}\right)_{s=-1+j3+\varepsilon} = 180^0$$

$$\angle\left(\frac{10(s+1+j3)}{s(s+5)(s+10)}\right)((s+1-j3))_{s=-1+j3+\varepsilon} = 180^0$$

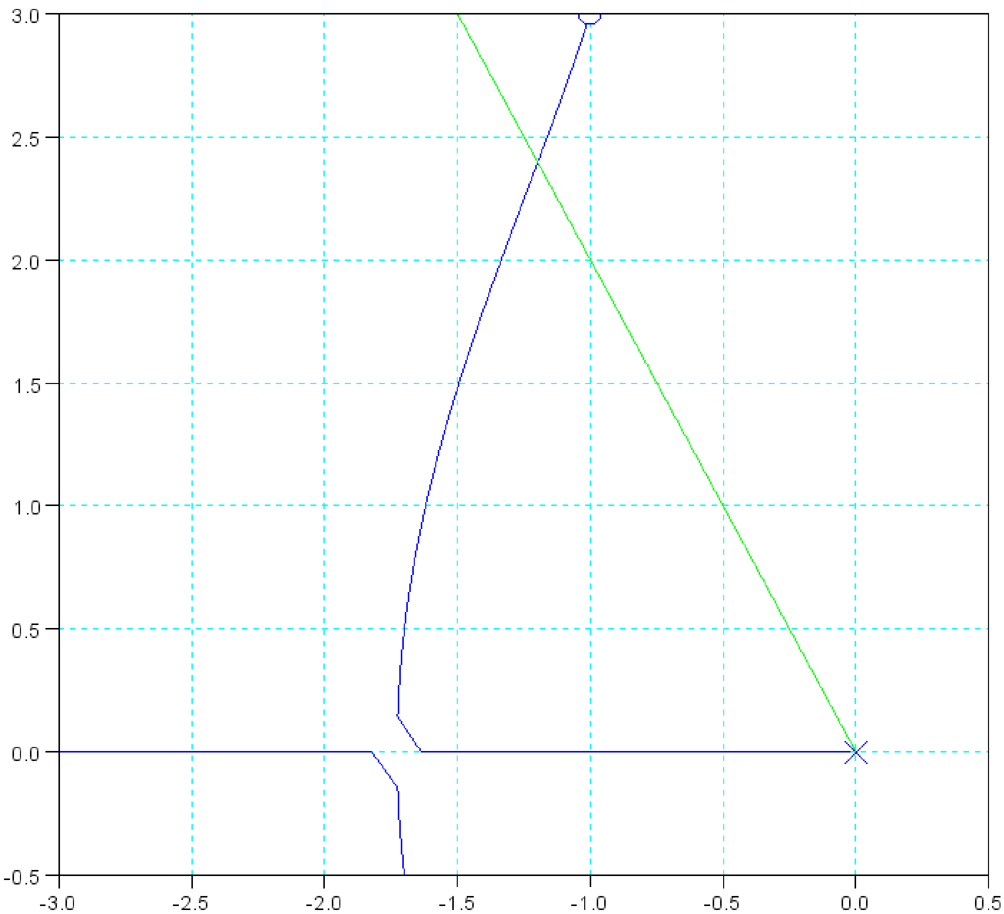
$$(-73.7398^0) + \angle(s+1-j3) = 180^0$$

$$\angle(s+1-j3) = 253.73^0 = -106.26^0$$



10) Determine the feedback gain, k, which results in 20% overshoot for a step input.

Find the point which has a damping ratio of 0.4559



s = -1.1977 + j2.3953

At this point

$$GK = -1$$

$$\left(\frac{10(s+1+j3)(s+1-j3)}{s(s+5)(s+10)}\right)_{s=-1.1977+j2.39053} = 0.3128\angle 180^0$$

$$k = \frac{1}{0.3128}$$

$$k = 3.1965$$

11) For this value of k, determine the closed-loop system's

Closed-loop dominant poles

$s = -1.1977 + j2.3953$

Steady-state error for a step input

None. This is a type-1 system

12) Check your design in VisSim or MATLAB