ECE 461 - Homework Set #8

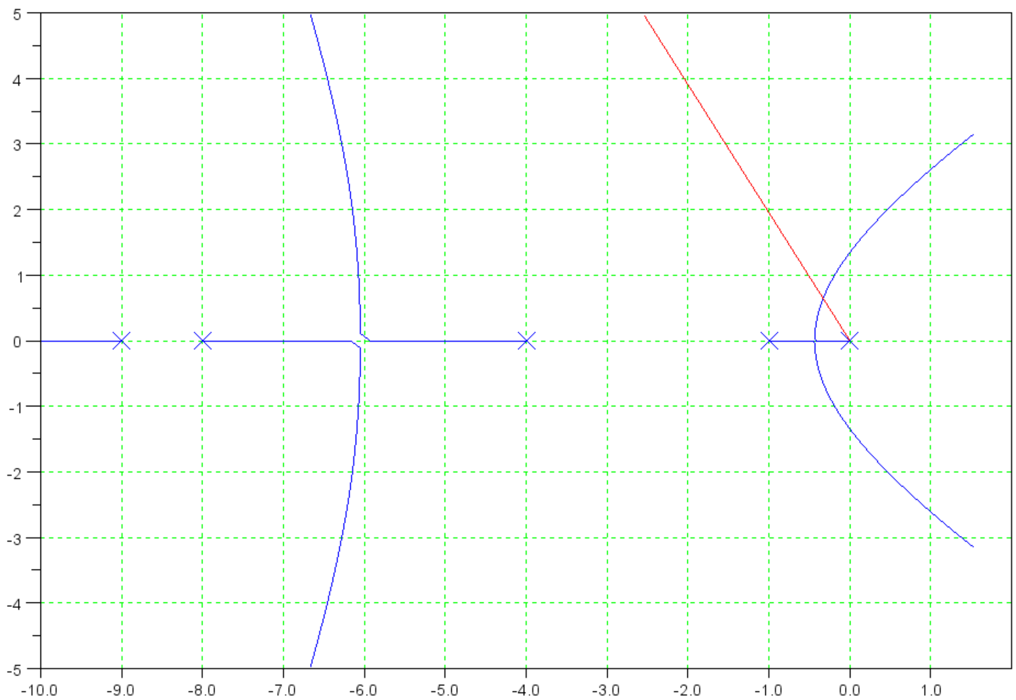
Lead, PID, Meeting Design Specs - Due Monday, November 2nd

Problem 1-4) Let $G(s) = \left(\frac{100}{(s+1)(s+4)(s+8)(s+9)}\right)$

1) Design a I compensator $\left(K(s) = \frac{k}{s}\right)$ which results in 20% overshoot for a step input. Check you answer in VisSim or MATLAB (i.e. take the step response of the closed-loop system).

First, find the point on the root locus with a damping ratio of 0.4559

```
-->GK = zp2ss([],[0,-1,-4,-8,-9],100);
-->k = logspace(-2,2,1000)';
-->R = rlocus(G, k, 0.4559);
```



s = -0.3269 + j0.6537

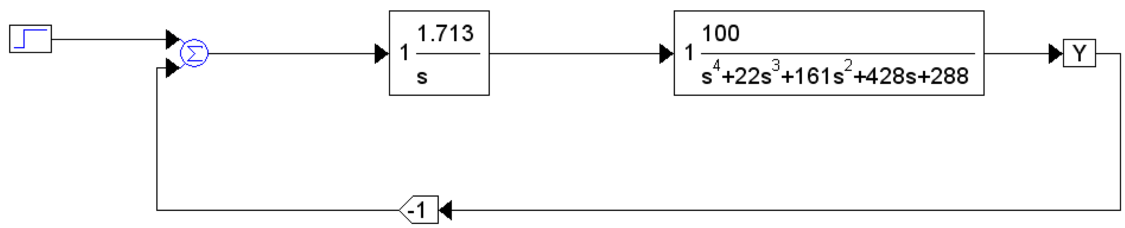
Make G*K = -1 at this point

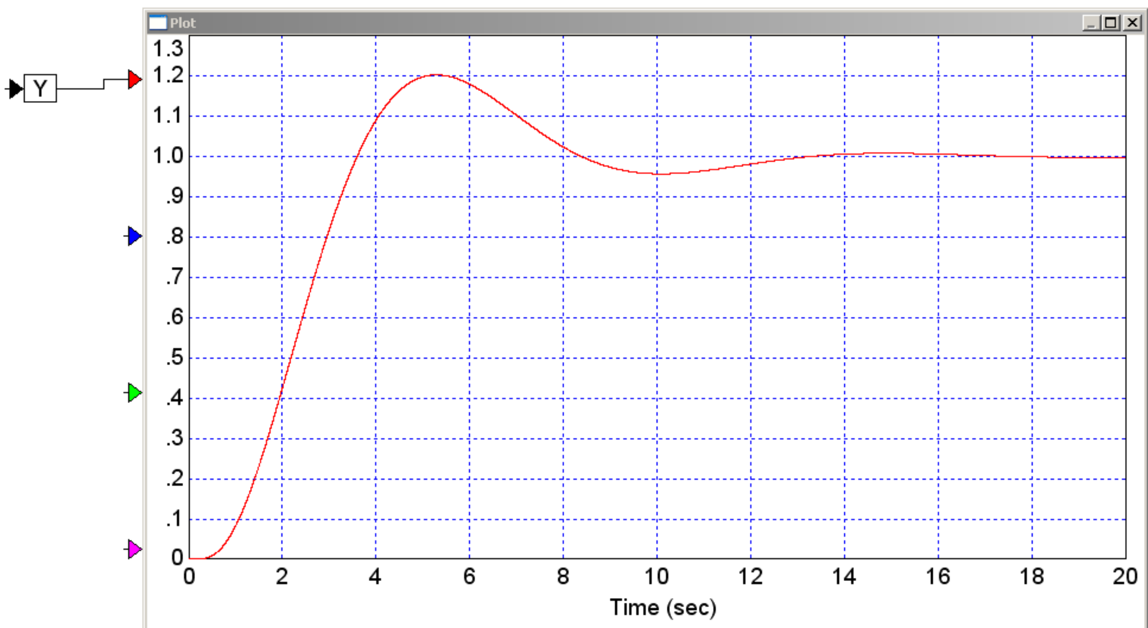
```
-->s = -0.3269 + j*0.6537;

-->evalfr(GK,s)

  - 0.5835349 + 0.0000191i           check:  G(s) is negative real

-->k = 1/abs(ans)

    1.7136934
```

Check in VisSim:

Note:

- The steady-state error is zero (it's a type-1 system)
- The overshoot is 20% - what we designed for





2) Design a PI compensator $\left(K(s) = \left(\frac{as+b}{s}\right)\right)$ which results in 20% overshoot for a step input. Check you answer in VisSim or MATLAB (i.e. take the step response of the closed-loop system).

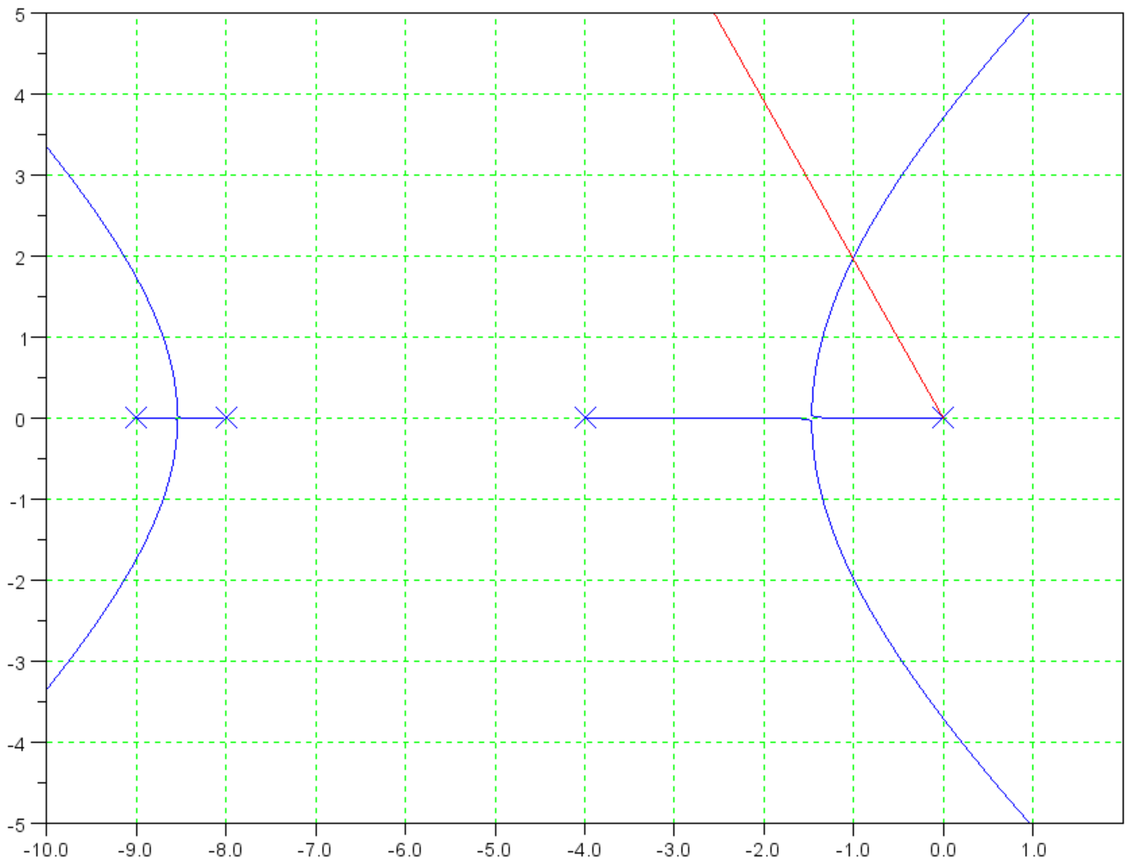
Step 1: Select the form for the compensator

- Add a pole at s = 0 to make it type-1
- Add a zero at s = -1 to cancel the slowest stable pole

$$K(s) = k\left(\frac{s+1}{s}\right)$$

$$GK = \left(\frac{100}{s(s+4)(s+8)(s+9)}\right)$$

Step 2: Draw the root locus of GK. Find the point on the root locus which has a damping ratio of 0.4559



s = -0.9965 + j1.9930

Step 3: Find k so that GK = -1 at this point

```
-->s = -0.9965 + j*1.9930;

-->evalfr(GK,s)
  - 0.2073069 - 0.0000010i

-->k = 1/abs(ans)
```

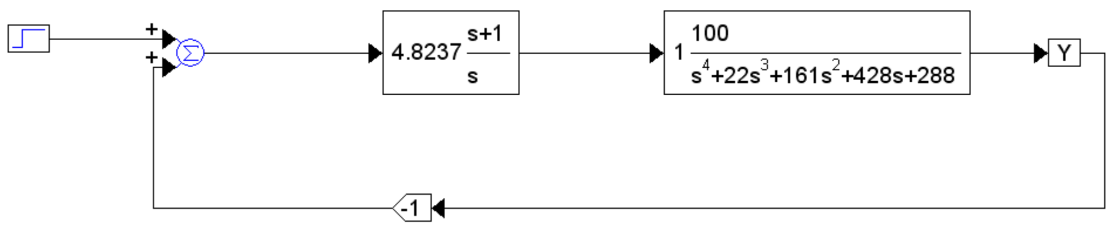
```
k  =

    4.8237668
```

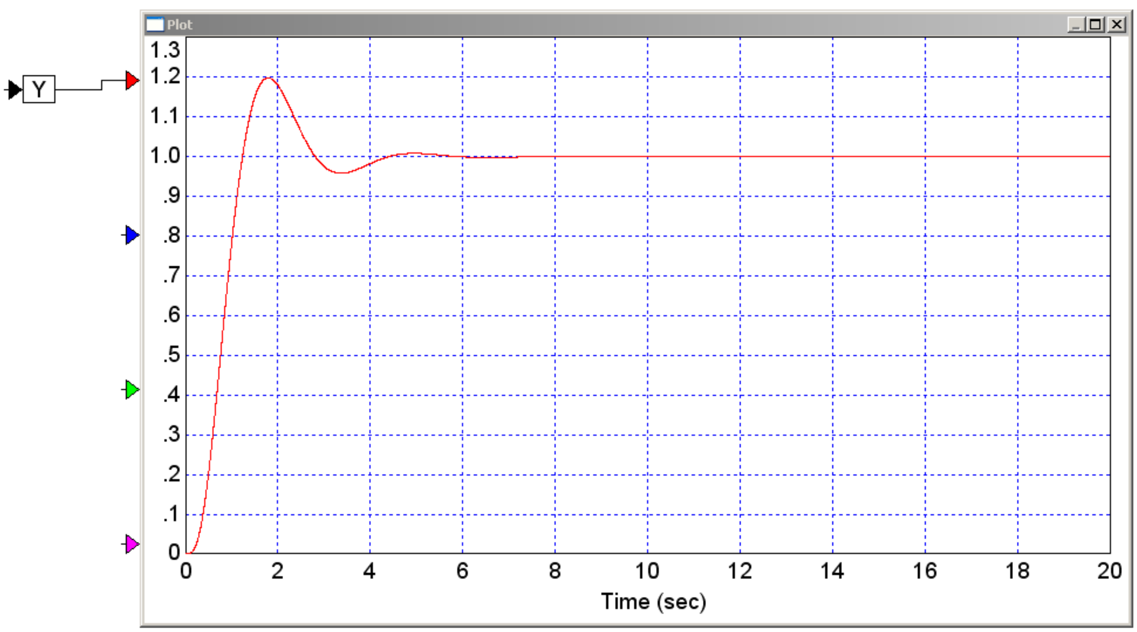
meaning

$$K(s) = 4.8237\left(\frac{s+1}{s}\right)$$

Check in VisSim. Note

• The system is quicker than before: cancelling the pole at -1 pulled the root locus left
• The overshoot is 20% - as expected
• The settling time is 4 seconds (the real part of the closed-loop dominant pole is -1)





3) Design a compensator which results in

- No error for a step input
- 20% overshoot for a step input, and
- A 2% settling time of 2 seconds.

Check you answer in VisSim or MATLAB (i.e. take the step response of the closed-loop system).

Translation:

- Make it a type-1 system
- Place the closed-loop dominant pole at -2 +j4

Step 1: Choose the form of K(s).

- Add a pole at s = 0 to make it type 1.
- Cancel the pole at -1 to speed up the system
- Cancel another pole at -4 to speed it up further
- Add a pole to make the angle add up to 180 degrees at -2 + j4

$$K(s) = k\left(\frac{(s+1)(s+4)}{s(s+a)}\right)$$

$$GK = \left(\frac{100k}{s(s+a)(s+8)(s+9)}\right)$$

Step 2: Find 'a' so that the angles add up to 180 degrees at s = -2 + j4

$$\left(\frac{100}{s(s+8)(s+9)}\right)_{s=-2+j4} = 0.3846\angle 180^0$$

Ummmm. Talk about dumb luck. What's the change I'd pick a point on the root locs.....

This is actually a probem since is says the angle is already 180 degrees - meaning (s+a) adds zero degrees. You need an angle > 0 degrees.

Take 2:

Step 1: Choose the form of K(s). Cancelling two poles didn't work - so cancel another pole.

$$K(s) = k\left(\frac{(s+1)(s+4)(s+8)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{100k}{s(s+a)^2(s+9)}\right)$$

Step 2: Find 'a' so that the angles add up to 180 degrees at s = -2 + j4

$$\left(\frac{100}{s(s+9)}\right)_{s=-2+j4} = 2.7735\angle -146.3099^0$$

For the angle to add up to 180 degrees

$$\angle(s+a)^2 = 33.6901^0$$

$$\angle(s+a) = 16.8450^0$$

$$a = \frac{4}{\tan\left(16.84^0\right)} + 2$$

$$a = 15.2111$$

and

$$K(s) = k\left(\frac{(s+1)(s+4)(s+8)}{s(s+15.2111)^2}\right)$$

$$GK = \left(\frac{100k}{s(s+9)(s+15.2111)^2}\right)$$

Step 3: Find k so that GK = -1 at s

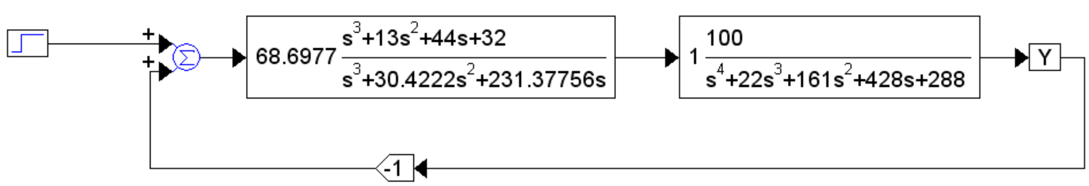
$$\left(\frac{100k}{s(s+9)(s+15.2111)^2}\right)_{s=-2+j4} = 0.0146\angle 180^0$$

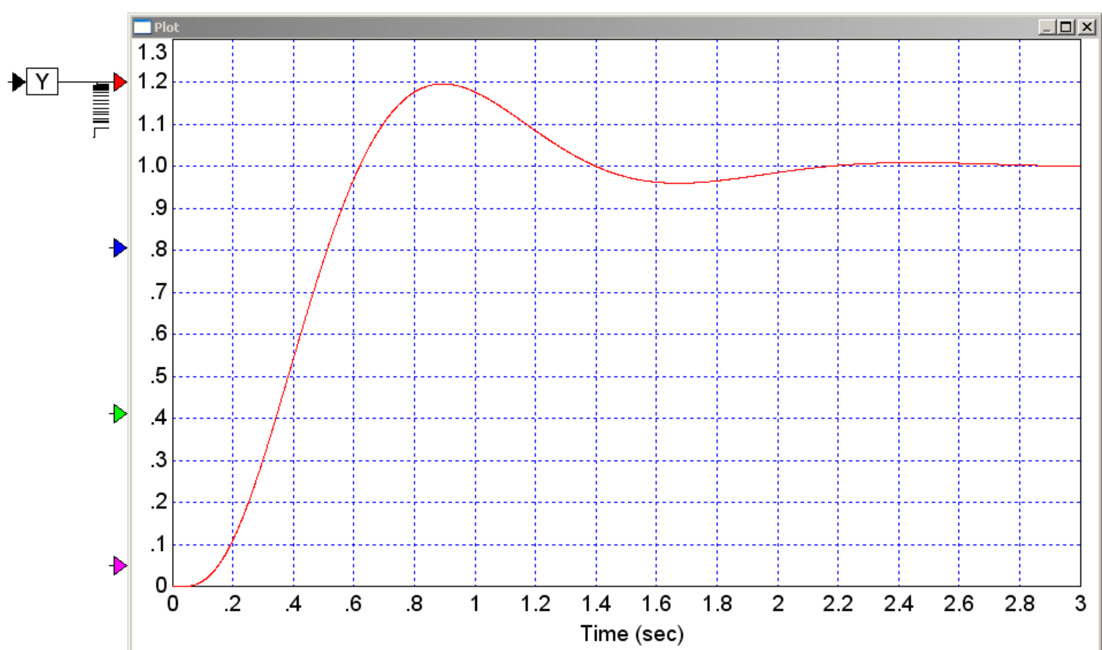
$$k = \frac{1}{0.0146} = 68.6977$$

and

$$\boxed{K(s) = 68.6977\left(\frac{(s+1)(s+4)(s+8)}{s(s+15.2111)^2}\right)}$$

Check in VisSim



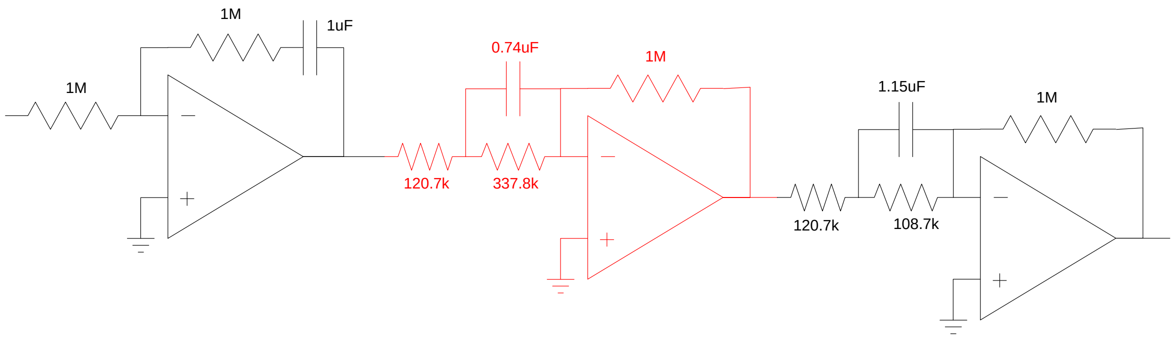


4) Design an op-amp circui to impliment K(s) for problem 3.

$$K(s) = 68.6977\left(\frac{(s+1)(s+4)(s+8)}{s(s+15.2111)^2}\right)$$

Rewrite this as

$$K(s) = \left(\frac{s+1}{s}\right)\left(\frac{8.29(s+4)}{s+15.2111}\right)\left(\frac{8.29(s+8)}{s+15.2111}\right)$$



Problem 5-8) Let $G(s) = \left(\frac{100}{s(s+1)(s+4)(s+8)}\right)$

5) Design a lead compensator $\left(K(s) = k\left(\frac{s+a}{s+10a}\right)\right)$ which results in 20% overshoot for a step input. Check you answer in VisSim or MATLAB (i.e. take the step response of the closed-loop system).

Step 1: Choose the form of K(s).

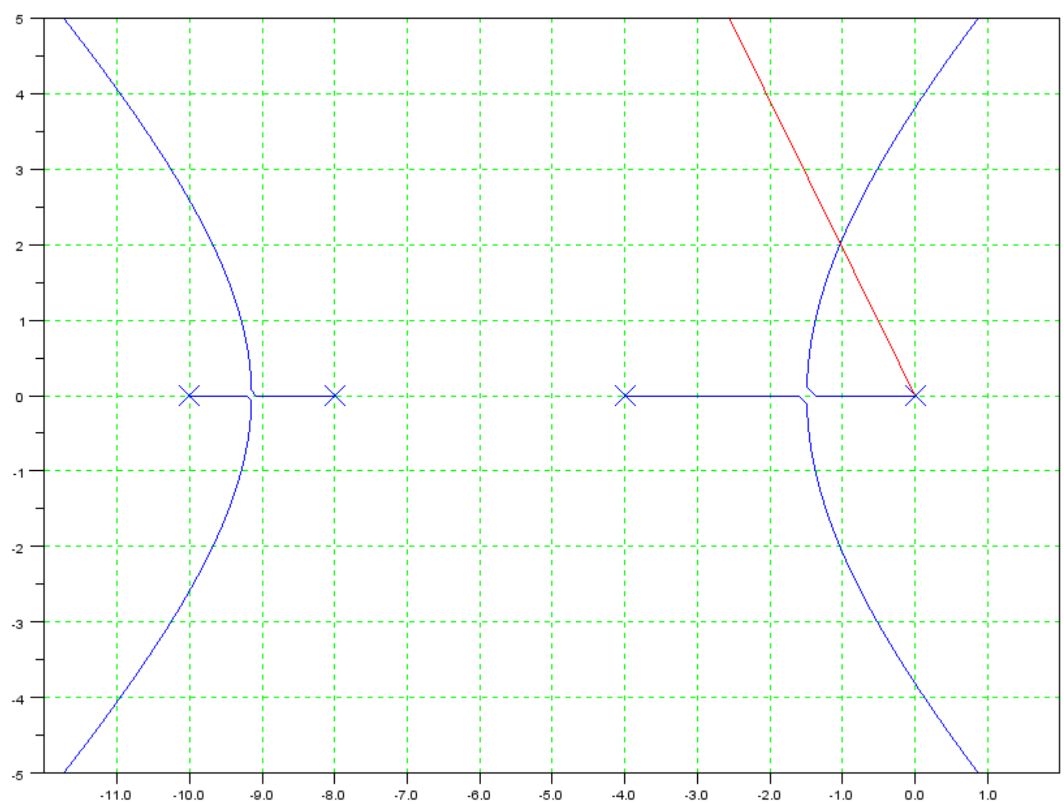
- Add a zero to cancel the slowest stable pole (s = -1)
- Add a pole 10x left of the zero

$$K(s) = k\left(\frac{s+1}{s+10}\right)$$

$$GK = \left(\frac{100}{s(s+10)(s+4)(s+8)}\right)$$

Step 2: Find the point on the root locus where the damping ratio is 0.4559

```
-->GK = zp2ss([],[0,-4,-8,-10],100);
-->k = logspace(-2,2,1000)';
-->R = rlocus(GK, k, 0.4559);
```



s = -1.0192 + j2.0385

Step 3: Find k so that at this point, GK = -1

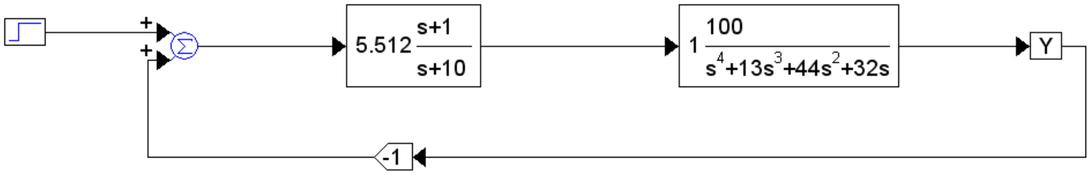
```
-->evalfr(GK,s)

  - 0.1814220 - 0.0000051i
```

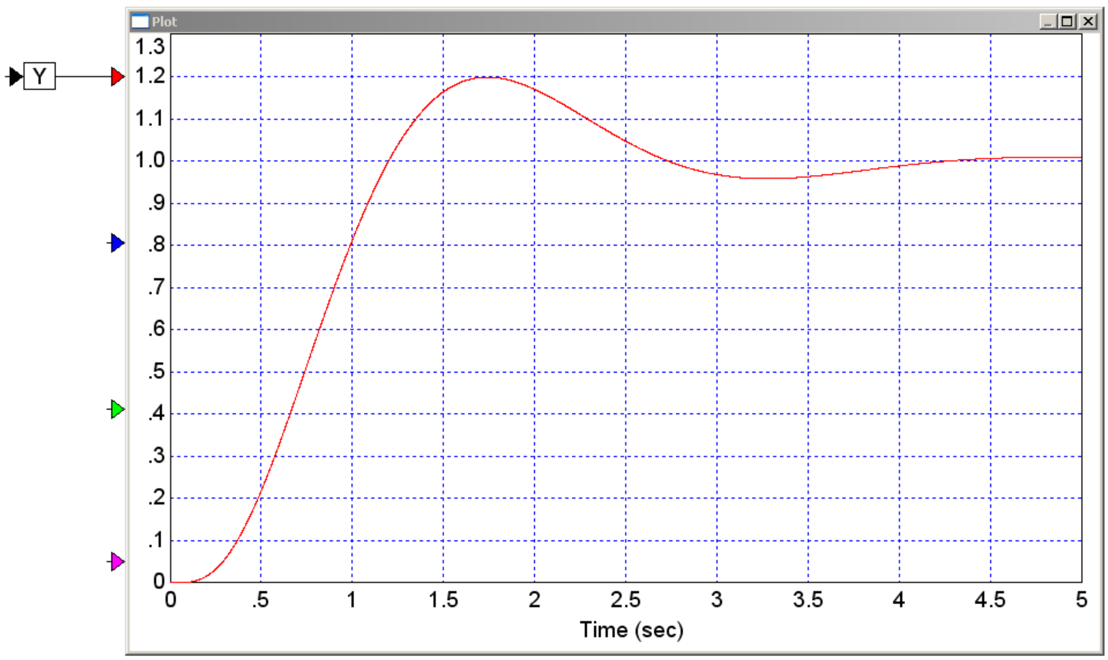
```
-->k = 1/abs(ans)

     5.5120093
```

meaning

$$K(s) = 5.512\left(\frac{s+1}{s+10}\right)$$

Check in VisSim





6) Design a compensator which results in

- No error for a step input
- 20% overshoot for a step input, and
- A 2% settling time of 2 seconds.

Check you answer in VisSim or MATLAB (i.e. take the step response of the closed-loop system).

Translation:

- Make it a type-1 system
- Place the closed-loop dominant pole at -2 +j4

Step 1: Choose the form of K(s).

- Cancel the pole at -1 to speed up the system
- Cancel another pole at -4 to speed it up further
- Add a pole to make the angle add up to 180 degrees at -2 + j4

$$K(s) = k\left(\frac{(s+1)(s+4)}{(s+a)^2}\right)$$

$$GK = \left(\frac{100k}{s(s+8)(s+a)^2}\right)$$

Checking the angle

$$\left(\frac{100}{s(s+8)}\right)_{s=-2+j4} = 3.1009\angle -150.2551^0$$

The angle is off by 29.7449 degrees. For the angles to add up to 180 degrees

$$\angle(s+a)^2 = 29.7449^0$$

$$\angle(s+a) = 14.8724^0$$

$$a = \frac{4}{\tan\left(14.8724^0\right)} + 2$$

$$a = 17.0623$$

and

$$GK = \left(\frac{100k}{s(s+8)(s+17.0623)^2}\right)$$

Checking at s = -2 + j4

$$\left(\frac{100}{s(s+8)(s+17.0623)^2}\right)_{s=-2+j4} = 0.0128\angle 180^0$$

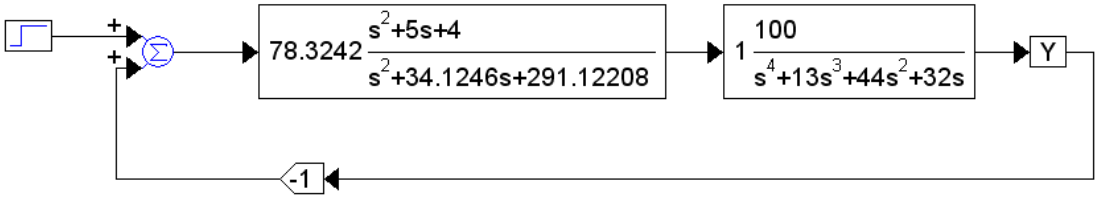
so

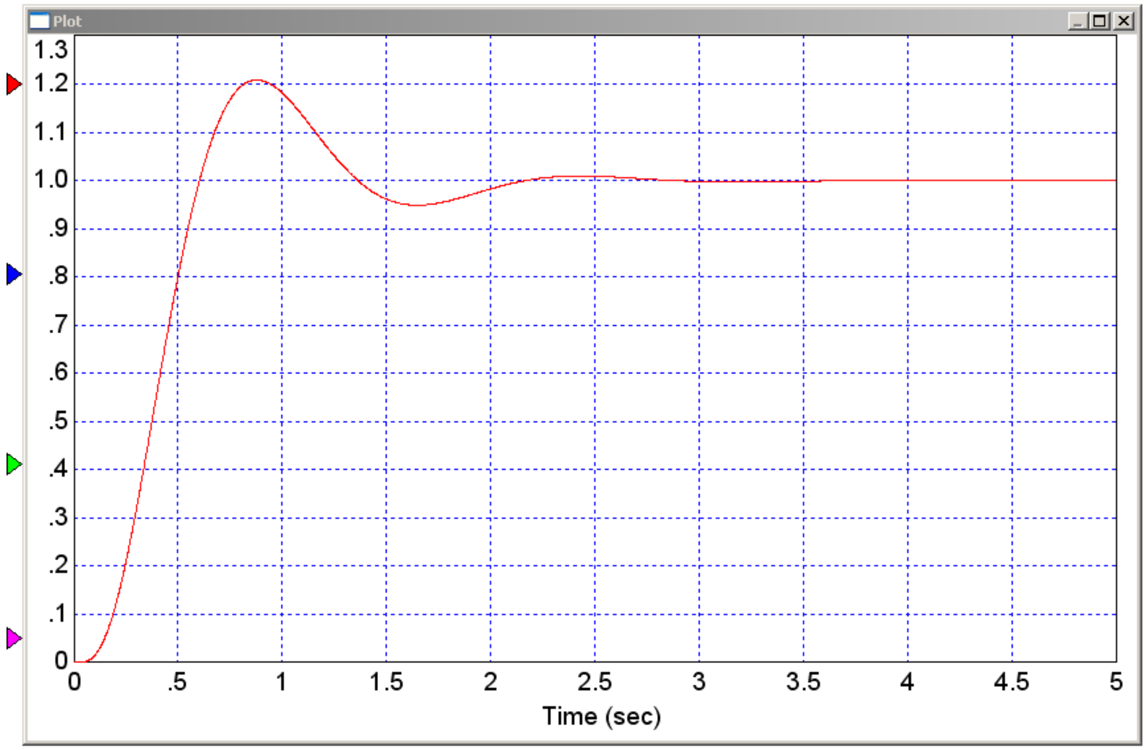
$$k = \frac{1}{0.0128} = 78.3242$$

and

$$\boxed{K(s) = \left(\frac{78.3242(s+1)(s+4)}{(s+17.0623)^2}\right)}$$

Check in VisSim





7) Design an op-amp circui to impliment K(s) for problem 6.

$$K(s) = \left(\frac{78.3242(s+1)(s+4)}{(s+17.0623)^2}\right)$$

Rewrite this as

$$K(s) = \left(\frac{7.8242(s+1)}{s+17.0623}\right)\left(\frac{10(s+4)}{s+17.0623}\right)$$