ECE 461 - Homework #9

Systems with delays, lightly damped poles, unstable poles. Due Monday, November 9th

20 points per problem

For each problem, design a compensator, K(s), which results in

- No overshoot for a step input
- 20% overshoot for a step input, and
- A 2% settling time of 4 seconds

Verify your design in VisSim (or lime program)

1) System with a 200ms delay

$$G(s) = \left(\frac{20}{(s+1)(s+2)(s+5)}\right) \cdot e^{-0.2s}$$

To meet the design specs,

- Make the system type-1
- Place the closed-loop dominant pole at -1 + j2

Step 1: Pick the form of the compensator. Try

$$K(s) = k\left(\frac{(s+1)(s+2)}{s(s+a)}\right)$$

Determine 'a' so that the angles add up to 180 degrees at s = -1 + j2

$$GK = \left(\frac{20}{s(s+5)(s+a)}\right) \cdot e^{-0.2s}$$

$$\left(\left(\frac{20}{s(s+5)}\right) \cdot e^{-0.2s}\right)_{s=-1+j2} = 2.4428\angle -166.0484^0$$

(s+a) must subtract another 13.9516 degrees for the angle to add up to 180 degrees

$$\angle(s+a) = 13.9516^0$$

$$a = \frac{2}{\tan\left(13.9516^0\right)} + 1$$

$$a = 9.0505$$

$$K(s) = k\left(\frac{(s+1)(s+2)}{s(s+9.0505)}\right)$$

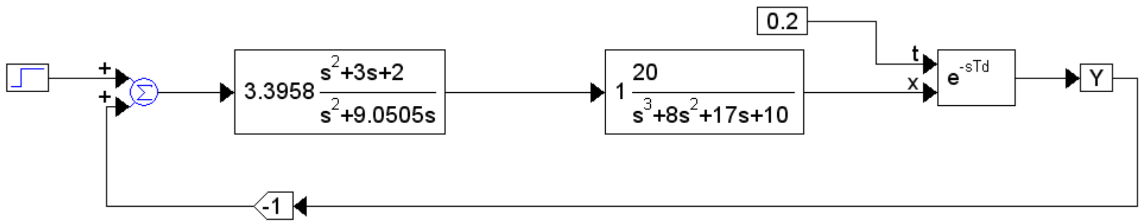
To find k, the gain must be -1

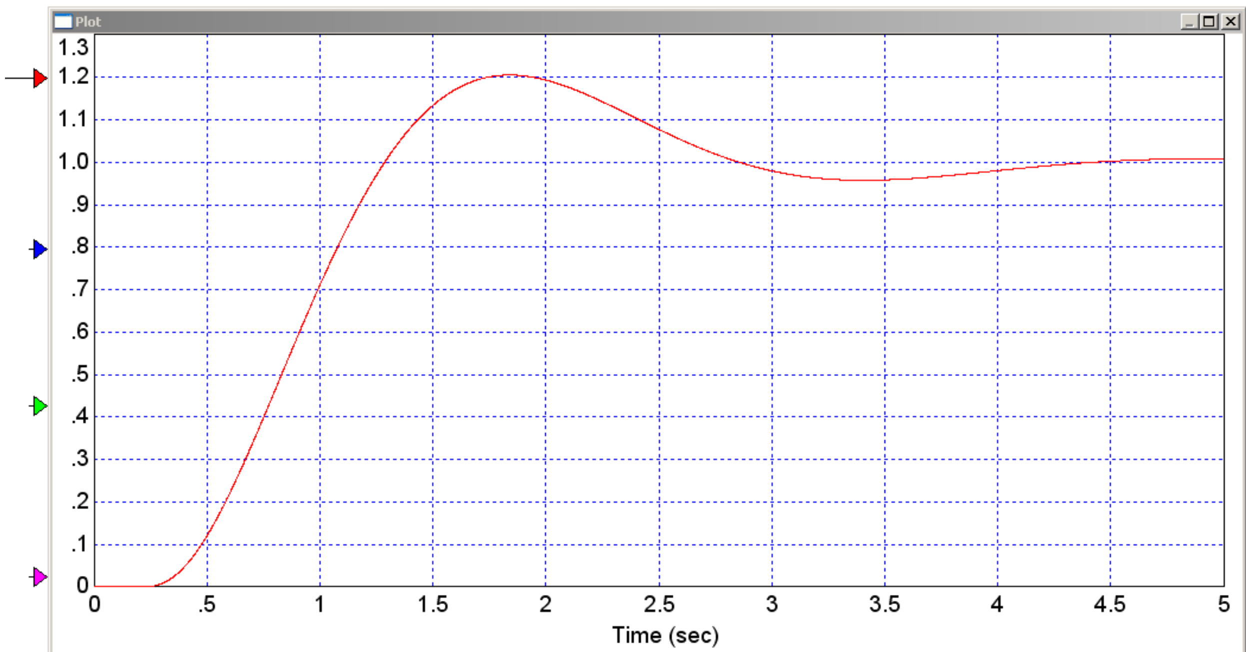
$$GK = \left(\left(\frac{20}{s(s+5)(s+9.0505)}\right) \cdot e^{-0.2s}\right)_{s=-1+j2} = 0.2945\angle 180^0$$

$$k = \frac{1}{0.2945} = 3.3958$$

and

$$K(s) = 3.3958\left(\frac{(s+1)(s+2)}{s(s+9.0505)}\right)$$



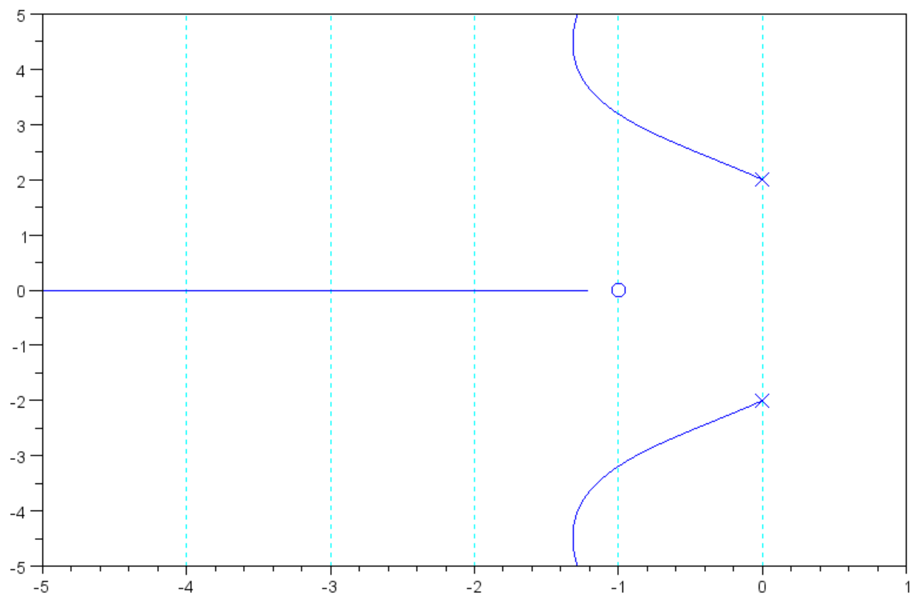


2) Lightly damped system

$$G(s) = \left(\frac{20}{(s+1)(s+j2)(s-j2)}\right)$$

First, stabilize the system. Add a compensator

$$K_1(s) = k\left(\frac{(s+1)^2}{(s+10)^2}\right)$$



Pick a point on the root locus

```
-->GK1 = zp2ss(-1,[-10,-10,j*2,-j*2],20);
-->k = logspace(-2,2,1000)';
-->R = rlocus(G,k);

-->s = -1.2 + j*4

  - 1.2 + 4.i
```

Pick 'k' so that the gain is one at this point:

```
-->evalfr(GK1,s)

  - 0.0600400 - 0.0017828i

-->1/abs(ans)

    16.648228
```

meaning

$$K_1(s) = 16.64\left(\frac{(s+1)^2}{(s+10)^2}\right)$$

Find the closed-loop transfer function for this compensator

```
-->G2 = intcon(GK1,16.65)
-->[z,p,k] = ss2zp(G2)
 k  =

    333. + 1.951D-13i
 p  =

  - 14.570764 + 8.882D-16i
  - 1.2810646 + 3.9880898i
  - 1.2810646 - 3.9880898i
  - 2.8671066 - 5.641D-16i
 z  =

  - 1. - 1.317D-22i
```

$$G_2 = \left(\frac{GK_1}{1+GK_1}\right) = \left(\frac{333(s+1)}{)s+2.86)(s+1.28+j3.99)(s+1.28-j3.99))s+14.57)}\right)$$

Step 2: Now that the system is stable, add a compensator, K2, to meet the design specs. Choose K2 of the form:

$$K_2(s) = k\left(\frac{(s+2.86)(s+1.28+j3.99)(s+1.28-j3.99)}{s(s+1)(s+a)}\right)$$

and

$$G_2K_2 = \left(\frac{333k}{s(s+a)(s+14.57)}\right)$$

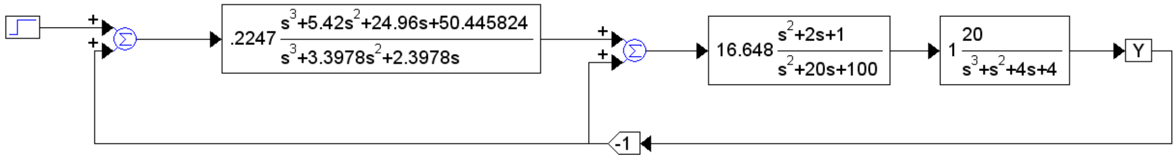
Pick 'a' to make the angles add up to 180 degrees at s = -1 + j2. This results in

a = 2.3978

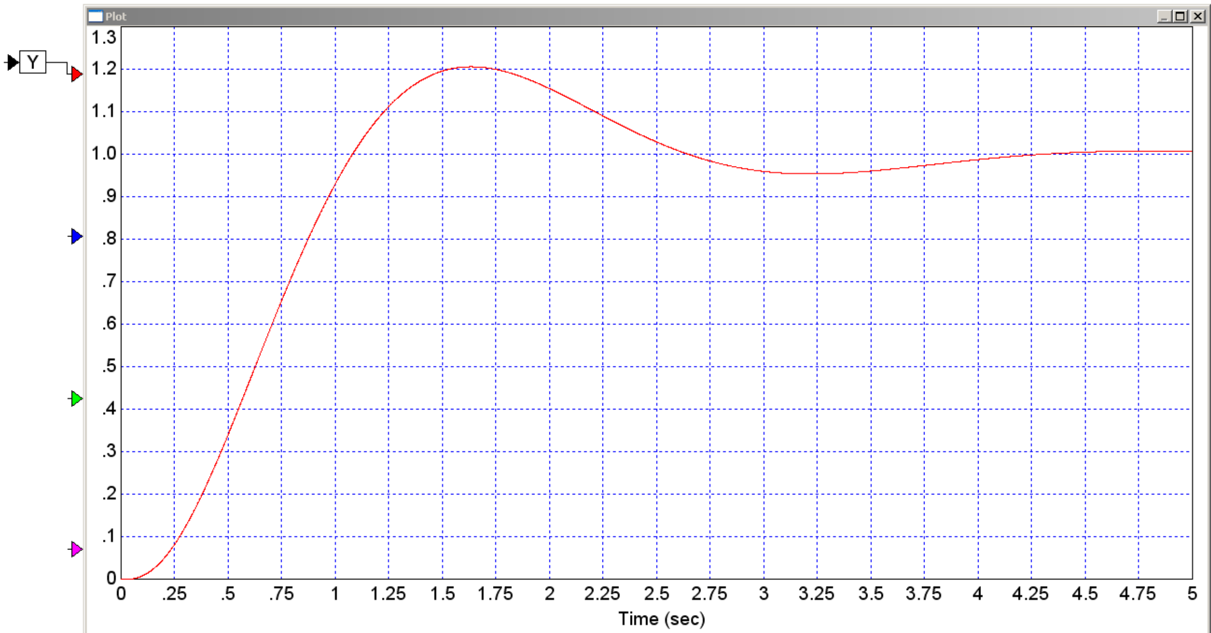
Pick 'k' to make the gain one at this point. This results in

$$K_2(s) = 0.2247\left(\frac{(s+2.86)(s+1.28+j3.99)(s+1.28-j3.99)}{s(s+1)(s+2.3978)}\right)$$

Checking the step response of the resulting system:



+
+
.2247 s^3+5.42s^2+24.96s+50.445824 / s^3+3.3978s^2+2.3978s
+
+
16.648 s^2+2s+1 / s^2+20s+100
1 20 / s^3+s^2+4s+4
Y
-1



Plot
Y
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0 .25 .5 .75 1 1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75 5
Time (sec)

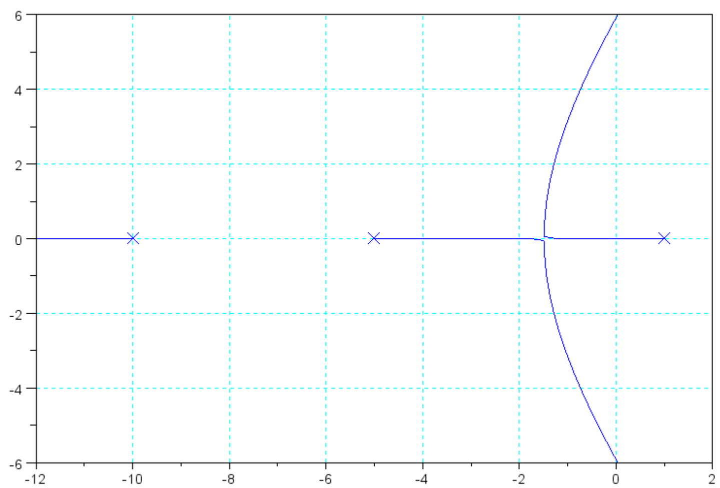
3) Unstable System

$$G(s) = \left(\frac{20}{(s-1)(s+2)(s+5)}\right)$$

First, add a compensator, K1, to stabilize the system. Let

$$K_1 = k\left(\frac{s+2}{s+10}\right)$$

The root locus of GK1 is



Find a spot that's stable. s = -1 works, so place the dominant pole at s = -1

```
-->s = -1
-->evalfr(GK1,s)

  - 0.2777778

-->k1 = 1/abs(ans)

    3.6
```

meaning

$$\boxed{K_1 = 3.6\left(\frac{s+2}{s+10}\right)}$$

$$G_2 = \left(\frac{GK_1}{1+GK_1}\right) = \left(\frac{72}{(s+1)(s+2)(s+11)}\right)$$

Now, add a second compensator to meet the design specs. Choose K2 of the form

$$K_2 = \left(\frac{(s+1)(s+2)}{s(s+a)}\right)$$

$$G_2K_2 = \left(\frac{72k}{s(s+a)(s+11)}\right)$$

Pick 'a' so that the angles add up to 180 degrees at s = -1 + j2

$$\left(\frac{72}{s(s+11)}\right)_{s=-1+j2} = 3.1574\angle -127.875^0$$

To make the angles add up to 180 degrees, (s+a) must subtract another 52.125 degrees

$$a = \frac{2}{\tan(52.125^0)} + 1 \;=\; 2.5556$$

Pick 'k' so that the gain of G2K2 is -1 at s = -1 + j2

$$\left(\frac{72}{s(s+2{,}5556)(s+11)}\right)_{s=-1+j2} = 1.2461\angle 180^0$$

$$k_2 = \frac{1}{1.2461} = 0.8025$$

so

$$K_2 = 0.8025\left(\frac{(s+1)(s+2)}{s(s+2.5556)}\right)$$

The net system is then

$$\left(\frac{G_2K_2}{1+G_2K_2}\right) = \left(\frac{57.779}{(s+1+j2)(s+1-j2)(s+11.555)}\right)$$

Checking in VisSim