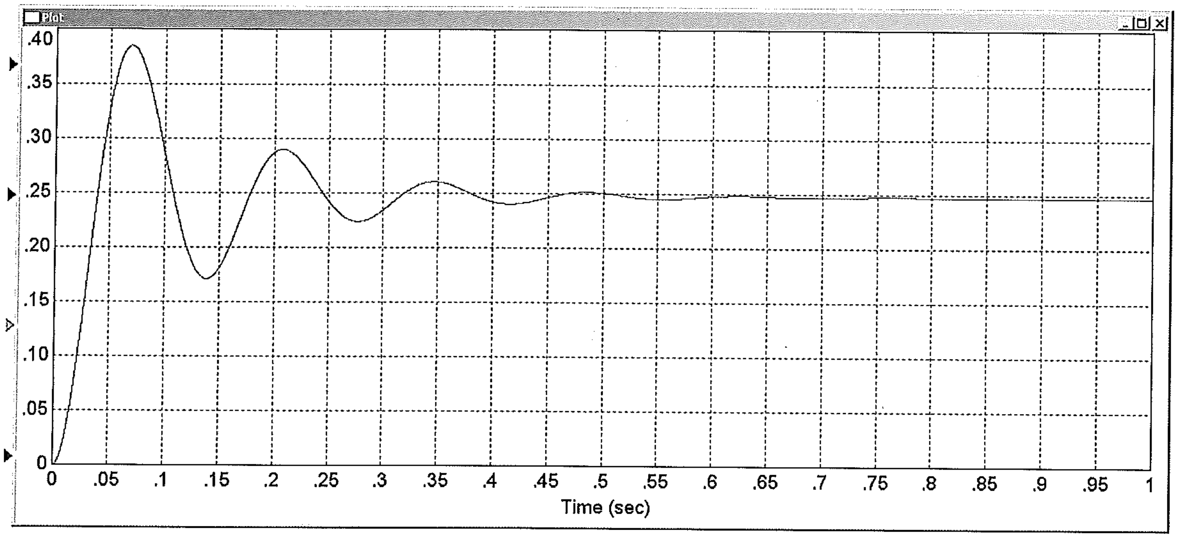
ECE 461 - Test #2: Name ____________________

Modeling Dynamics, Root Locus - October 23, 2015

1) Find the transfer function for the system which has the following step response:



DC = 0.24 3

48% OS

$\omega_d = \frac{3\text{ cycles}}{0.42\text{ sec}} \cdot 2\pi = 44.88$ 3

$\zeta = .2275$

$\theta = 76.8°$

$T_s \approx 0.5$ sec 3

$$\frac{497}{(s+8+j44.8)(s+8-j44.8)}$$

3   3   3

2a) What is the differential equation relating X and Y?

$$Y = \left(\frac{80{,}000}{(s+4)(s+8)(s+20)(s+40)}\right)X = \left(\frac{80{,}000}{s^4+72s^3+1552s^2+11{,}520s+25{,}600}\right)X$$

$y'''' + 72y''' + 1552y'' + 11520y' + 25600y = 80000x$

2b) Give the transfer function for a 2nd-order system with approximately the same step response as this system

DC gain = 3.125

$$\frac{100}{(s+4)(s+8)}$$

3) Determine the range of k for stability using a Routh Table for the following system

$$(s+2)(s+4)(s+6)(s+8)+10k=0$$

$$s^4+20s^3+140s^2+400s+384+10k=0$$

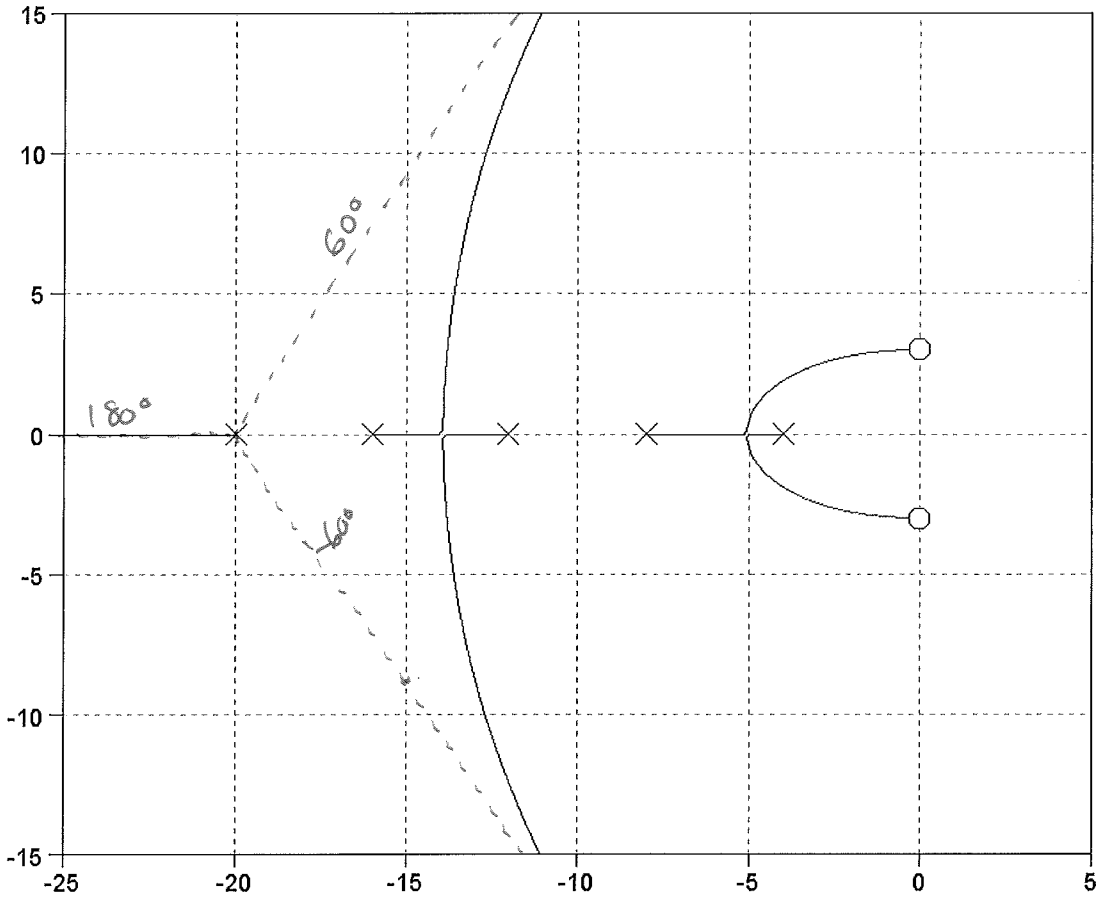
1	140 (2)	384+10k (2)
20 (2)	400 (2)	0
120 (2)	384+10k (2)	
336 − 1.667k (2 ✓)	0	
384+10k (2 ✓)	0	

$-38.4 < K < 201.6$ (2) (✓2) (✓2)

4) The root locus for G(s) is shown below. Compute and show on the root locus plot the following:

$$G(s) = \left(\frac{8(s^2+9)}{(s+4)(s+8)(s+12)(s+16)(s+20)} \right)$$

Real Axis Loci	(-4, -8) (-12, -16) (-20, -∞)
Breakaway Point(s)	-5.0657 -13.9342
Asymptotes	show on graph
Approach Angle to the zero at +j3	-179.38°



$$(\quad)(s-j3)$$

$-0.612°$ $-179.38°$

5) The root locus for G(s) is shown below. Determine the gain, k, which results in a damping ratio of 0.4. For this value of k, determine the following:

k for a damping ratio of 0.4	11.39
closed-loop dominant poles for this value of k	$-1.6791 + j\,3.8473$
resulting 2% settling time	2.38 sec
resulting overshoot for a step input	25.38%
resulting DC gain of the closed-loop system	0.6704

$$G(s) = \left(\frac{10}{(s+2)(s+4)(s+7)}\right)$$

$K_p = 2.03$

$e = \frac{1}{K_p+1} = 0.3296$