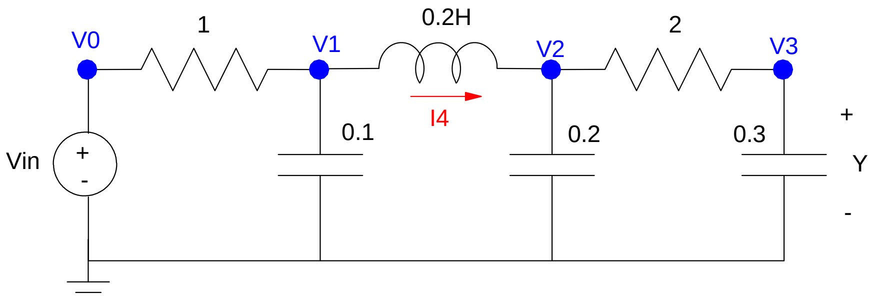
ECE 461/661 - Homework Set #5

State-Space, Matlab, Electric circuits - Due Monday, October 3rd



Problem 1-5

Problem 1) Write the differential equations which describe the following circuit.

$$0.1\dot{V}_1 = \left(\frac{V_0 - V_1}{1}\right) - I_4$$

$$0.2\dot{V}_2 = I_4 + \left(\frac{V_3 - V_2}{2}\right)$$

$$0.3\dot{V}_3 = \left(\frac{V_2 - V_3}{2}\right)$$

$$0.2\dot{I}_4 = V_1 - V_2$$

Problem 2) Express the dynamics for this system in state-space form

Solve for the highest derivatives:

$$\dot{V}_1 = 10V_0 - 10V_1 - 10I_4$$

$$\dot{V}_2 = -2.5V_2 + 2.5V_3 + 5I_4$$

$$\dot{V}_3 = 1.667V_2 - 1.667V_3$$

$$\dot{I}_4 = 5V_1 - 5V_2$$

Place in matrix form

$$s\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 & -10 \\ 0 & -2.5 & 2.5 & 5 \\ 0 & 1.667 & -1.667 & 0 \\ 5 & -5 & 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$Y = V_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I_4 \end{bmatrix}$$

Problem 3) Find the transfer function from X to Y

```
>> A = [-10,0,0,-10 ; 0, -2.5, 2.5, 5 ; 0, 1.667, -1.667, 0 ; 5, -5, 0, 0]

  -10.0000         0         0  -10.0000
         0   -2.5000    2.5000    5.0000
         0    1.6670   -1.6670         0
    5.0000   -5.0000         0         0

>> B = [10;0;0;0];
>> C = [0,0,1,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> tf(G)

Transfer function:
                   416.7
-------------------------------------------
s^4 + 14.17 s^3 + 116.7 s^2 + 500 s + 416.7

>> zpk(G)

Zero/pole/gain:
                 416.75
----------------------------------------
(s+6.53) (s+1.067) (s^2 + 6.57s + 59.79)
```

Problem 4) Find a 2nd-order approximation for this transfer function

The DC gain is

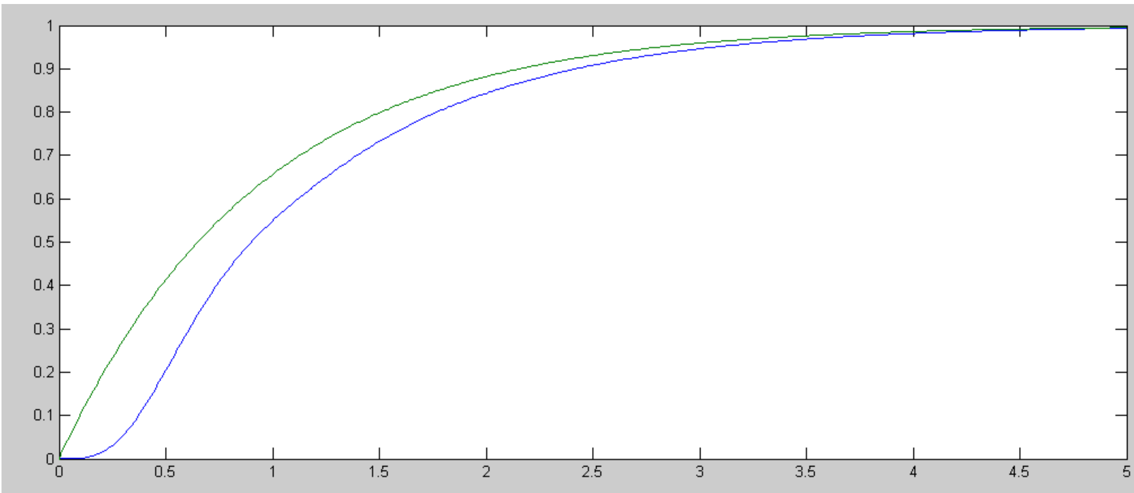
```
>> DC = evalfr(G,0)

     1
```

Keeping the dominant pole

$$G(s) \approx \left(\frac{1.067}{s+1.067}\right)$$

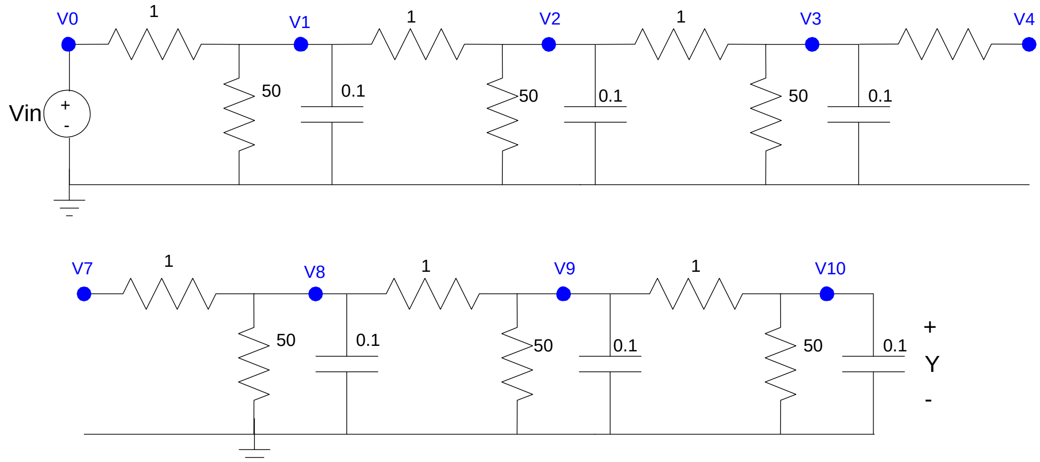
Problem 5) Plot the step response of the 4th-order system and its 2nd-order approximation

```
>> G1 = zpk([],-1.067,1.067)

Zero/pole/gain:
  1.067
---------
(s+1.067)

>> t = [0:0.01:5]';
>> y1 = step(G1,t);
>> y = step(G,t);
>> plot(t,y,t,y1);
```



Step response of the 4th-order system (blue) and its 1st-order approxiamtion (green)

Problem 6-10



Problem 6-10: 10-Stage RC Filter (nodes 5-7 repeat the pattern - not shown)

Problem 6) Write the differential equations which describe the following 10-stage RC filter at node V2 (i.e. write the voltage node equation at V2)

Nove V2: (same pattern for nodes 1..9)

$$0.1sV_2 = \left(\frac{V_1 - V_2}{1}\right) + \left(\frac{V_3 - V_2}{1}\right) + \left(\frac{0 - V_2}{50}\right)$$

$$sV_2 = 10V_1 - 20.2V_2 + 10V_3$$

Nove V10:

$$0.1sV_{10} = \left(\frac{V_9 - V_{10}}{1}\right) + \left(\frac{0 - V_{10}}{50}\right)$$

$$sV_{10} = 10V_9 - 10.2V_{10}$$

Problem 7) Express the dynamics for this system in state-space form

$$s\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} = \begin{bmatrix} -20.2 & 10 & - & - & - & - & - & - & - & - \\ 10 & -20.2 & 10 & - & - & - & - & - & - & - \\ - & 10 & -20.2 & 10 & - & - & - & - & - & - \\ - & - & 10 & -20.2 & 10 & - & - & - & - & - \\ - & - & - & 10 & -20.2 & 10 & - & - & - & - \\ - & - & - & - & 10 & -20.2 & 10 & - & - & - \\ - & - & - & - & - & 10 & -20.2 & 10 & - & - \\ - & - & - & - & - & - & 10 & -20.2 & 10 & - \\ - & - & - & - & - & - & - & 10 & -20.2 & 10 \\ - & - & - & - & - & - & - & - & 10 & -10.2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_x$$

$$Y = V_{10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix}$$

Problem 8) Find the transfer function from X to Y

```
>> A10 = zeros(10,10);
>> for i=1:9
   A10(i,i) = -20.2;
   A10(i+1,i) = 10;
   A10(i,i+1) = 10;
   end
>> A10(10,10) = -10.2

A10 =

  -20.2000   10.0000         0         0         0         0         0         0         0         0
   10.0000  -20.2000   10.0000         0         0         0         0         0         0         0
         0   10.0000  -20.2000   10.0000         0         0         0         0         0         0
         0         0   10.0000  -20.2000   10.0000         0         0         0         0         0
         0         0         0   10.0000  -20.2000   10.0000         0         0         0         0
         0         0         0         0   10.0000  -20.2000   10.0000         0         0         0
         0         0         0         0         0   10.0000  -20.2000   10.0000         0         0
         0         0         0         0         0         0   10.0000  -20.2000   10.0000         0
         0         0         0         0         0         0         0   10.0000  -20.2000   10.0000
         0         0         0         0         0         0         0         0   10.0000  -10.2000

>> B10 = zeros(10,1);
>> B10(1) = 10

B10 =

    10
     0
     0
     0
     0
     0
     0
     0
     0
     0

>> C10 = zeros(1,10);
>> C10(10) = 1

C10 =

     0     0     0     0     0     0     0     0     0     1

>> D10 = 0;


>> G10 = ss(A10,B10,C10,D10);
```

The transfer funciton is a 10th-order system (doesn't help much)

```
>> tf(G10)

Transfer function:
                                                              1e010
-----------------------------------------------------------------------------------------------------------------------------

s^10 + 192 s^9 + 1.564e004 s^8 + 7.048e005 s^7 + 1.917e007 s^6 + 3.227e008 s^5 + 3.314e009 s^4 + 1.969e010 s^3

     + 6.054e010 s^2 + 7.696e010 s + 2.312e010
```

Zeros-poles-gain tells you a lot more:

```
>> zpk(G10)

Zero/pole/gain:
                                           10000000000
------------------------------------------------------------------------------------------
(s+39.31)(s+36.72)(s+32.67)(s+27.51)(s+21.69)(s+15.75)(s+10.2)(s+5.539)(s+2.181)(s+0.4234)

>>
```

Problem 9) Find a 2nd-order approximation for this transfer function

Match the DC gain

```
>> DC = evalfr(G10,0)

DC =

    0.4325
```

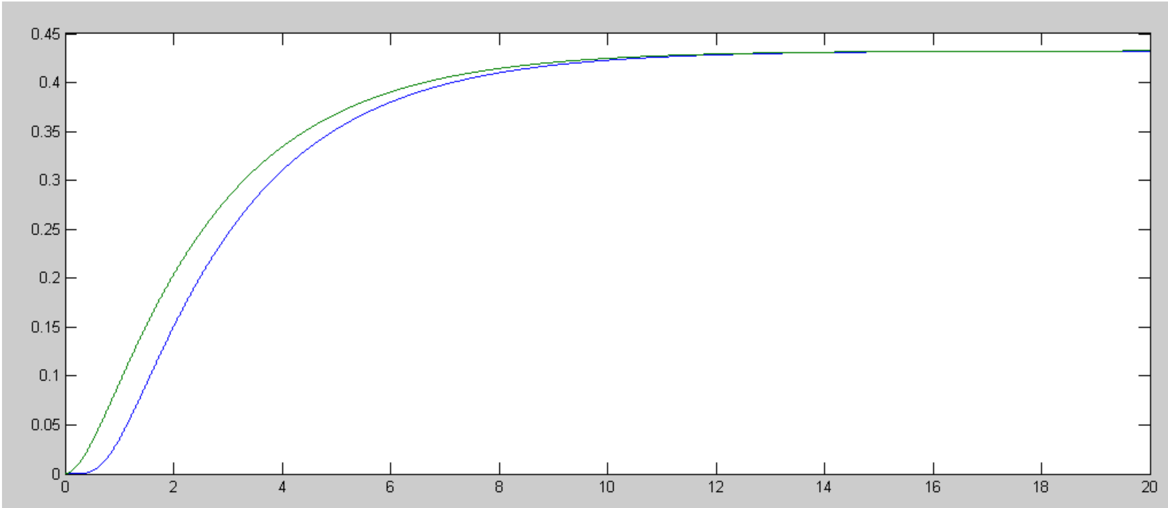
Keep the two most dominant poles (shown in bold)

$$G(s) \approx \left(\frac{0.3994}{(s+0.4234)(s+2.181)} \right)$$

Problem 10) Plot the step response of the 4th-order system and its 2nd-order approximation

The 2% settling time is about 10 seconds (4 / .4234), so plot the step response out to 20 seconds.

```
>> t = [0:0.001:20]';
>> y10 = step(G10,t);
>> y2 = step(G2,t);
>> plot(t,y10,t,y2);
```



Step Response of the 10th Order System (blue) and 2nd-Order Approximation (green)