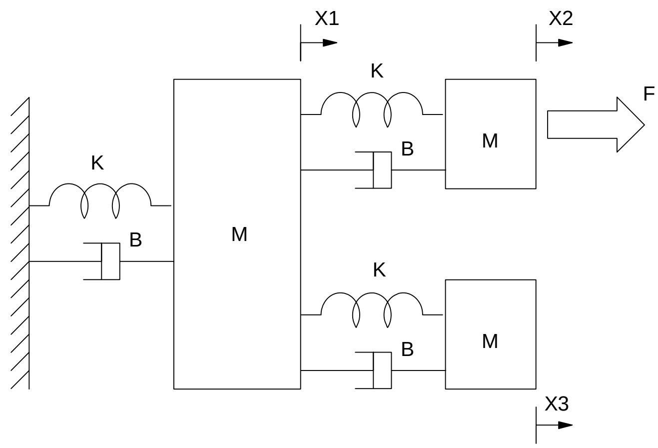
ECE 461/661 - Homework Set #6

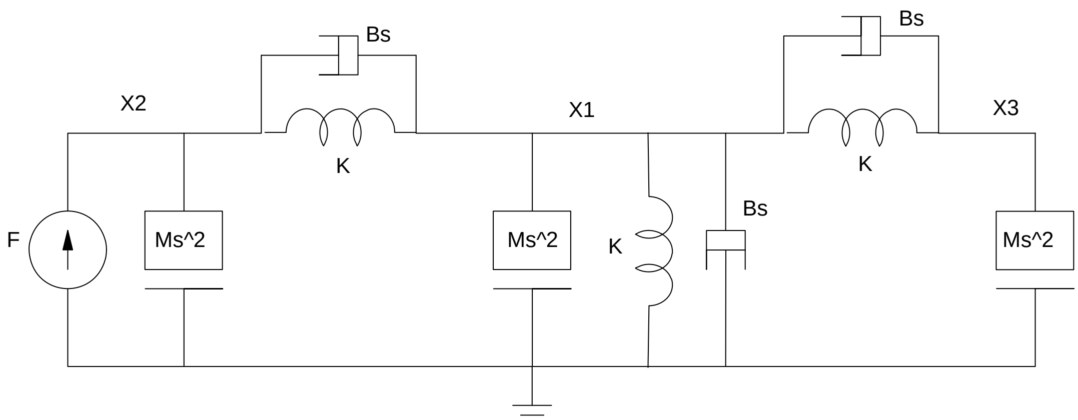
Mass-Spring Systems, Rotational Systems, DC Servo Motors - Due Monday, October 10th

Mass-Spring Systems.



Problem 1-4: M = 1kg, K = 10 N/m, B = 0.1 Ns/m

1) Draw the circuit equivalent for the following mass-spring system



2) Write the dynamics for this system in state-space form

First, write the voltage node equations:

$$(Ms^2 + 3Bs + 3K)X_1 - (Bs + K)X_2 - (Bs + K)X_3 = 0$$

$$(Ms^2 + Bs + K)X_2 - (Bs + K)X_1 = F$$

$$(Ms^2 + Bs + K)X_3 - (Bs + K)X_1 = 0$$

Solve for the highest derivative and plugging in numbers:

$$s^2X_1 = (-0.3s+30)X_1 + (0.1s+10)X_2 + (0.1s+10)X_3$$

$$s^2X_2 = (0.1s+10)X_1 - (0.1s+10)X_2 + F$$

$$s^2X_3 = (0.1s+10)X_1 - (0.1s+10)X_3$$

Put in matrix form (state-space)

$$s\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \cdots \\ sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots \\ -30 & 10 & 10 & \vdots & -0.3 & 0.1 & 0.1 \\ 10 & -10 & 0 & \vdots & 0.1 & -0.1 & 0 \\ 10 & 0 & -10 & \vdots & 0.1 & 0 & -0.1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \cdots \\ sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix} F$$

3) Find the transfer function from F to X1

```
>> a11 =  zeros(3,3);
>> a12 = eye(3,3);
>> a21 = [-30,10,10;10,-10,0;10,0,-10];
>> A = [a11,a12 ; a21, a22]

         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
  -30.0000   10.0000   10.0000   -0.3000    0.1000    0.1000
   10.0000  -10.0000         0    0.1000   -0.1000         0
   10.0000         0  -10.0000    0.1000         0   -0.1000

>> B = [0;0;0;0;1;0]

     0
     0
     0
     0
     1
     0

>> C = [1,0,0,0,0,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)


                  0.1 (s+100) (s^2 + 0.1s + 10)
-----------------------------------------------------------------
(s^2 + 0.02679s + 2.679) (s^2 + 0.1s + 10) (s^2 + 0.3732s + 37.32)
```

4) Find the transfer function from F to X2

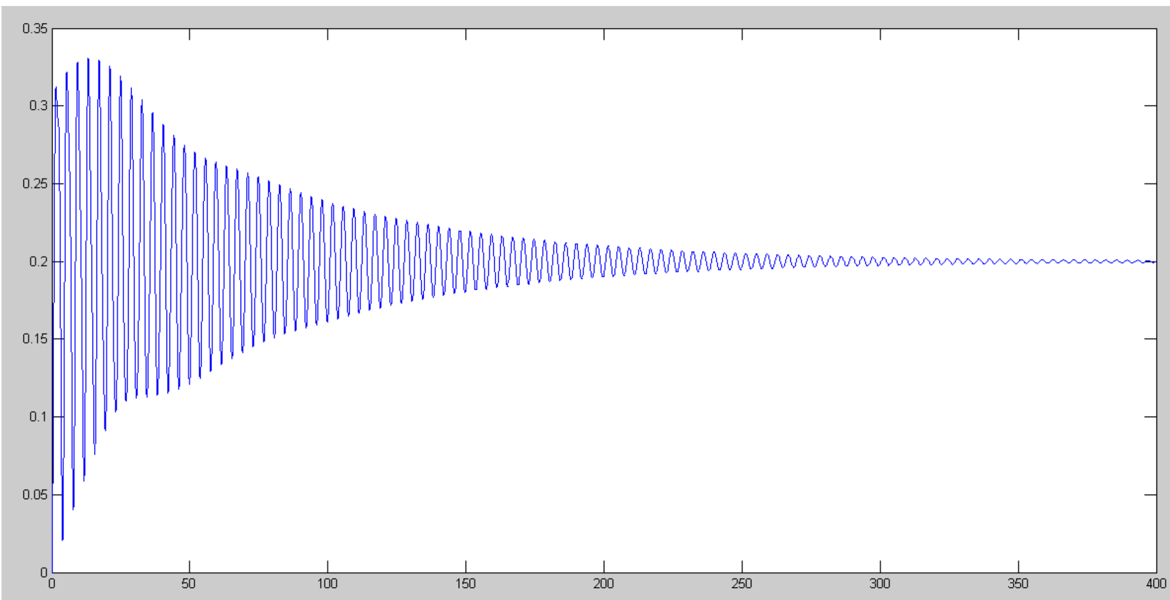
```
>> C = [0,1,0,0,0,0];
>> D = [0];
>> G = ss(A,B,C,D);
>> zpk(G)


         (s^2 + 0.05858s + 5.858) (s^2 + 0.3414s + 34.14)
  ------------------------------------------------------------------
  (s^2 + 0.02679s + 2.679) (s^2 + 0.1s + 10) (s^2 + 0.3732s + 37.32)
```

Note: When you change what you're measuring

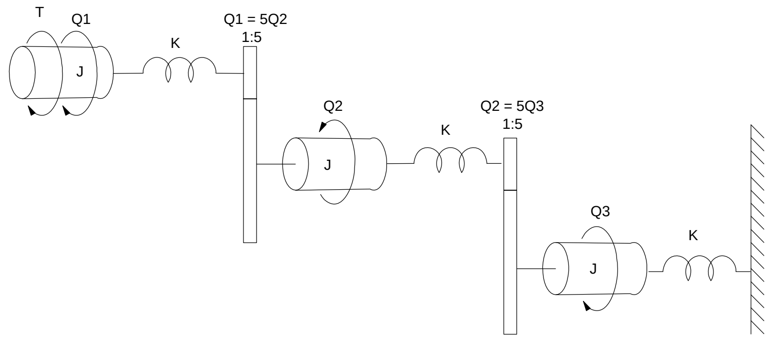
- The zeros change
- The poles do not change

Sidelight: By inspection, the step response

- Settles out in 298 seconds (dominant pole at -0.01339 + j1.63)
- And oscillate at 1.63 rad/sec

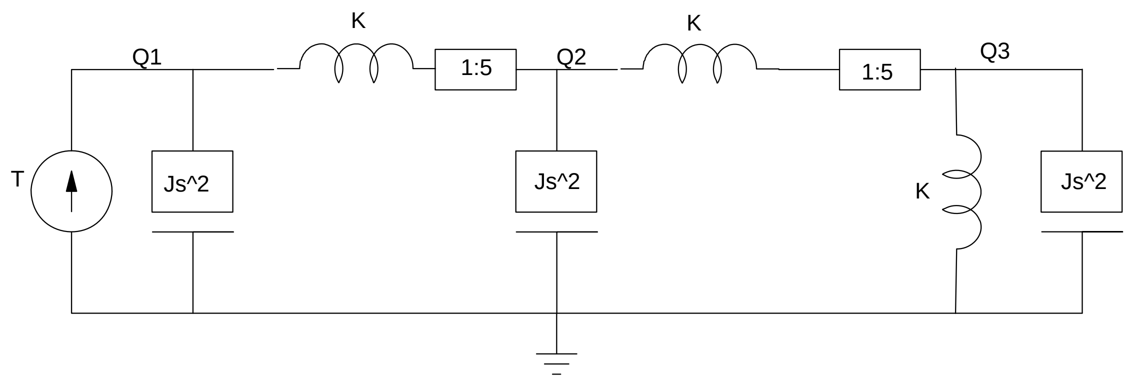


Rotational Systems:



Problem 5-7: J = 1 Kg m^2, K = 10 Nm/rad

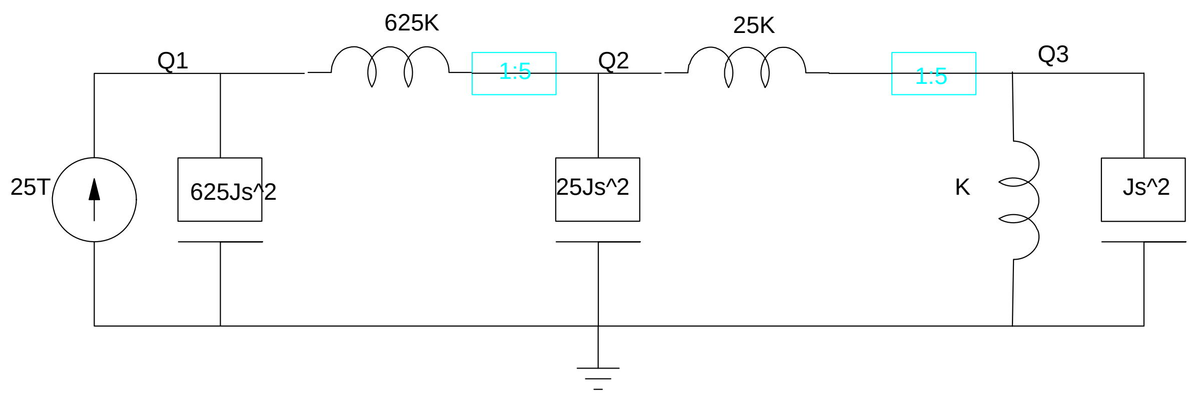
5) Draw the circuit equivalent for the following rotational system



6) Write the dynamics for this system in state-space form

Remove the gears and take everything to node Q3 (since that's where we're measuring):

- Torque increases as the turn ratio
- Impedances change as the squre of the turn ratio.



Writing the node equations:

$$(625Js^2 + 625K)\theta_1 - (625K)\theta_2 = 25T$$

$$(25Js^2 + 650K)\theta_2 - (625K)\theta_1 - (25K)\theta_3 = 0$$

$$(Js^2 + 26K)\theta_3 - (25K)\theta_2 = 0$$

Plug in numbers and solve for the highest derivative:

$$s^2\theta_1 = (-10)\theta_1 + (10)\theta_2 + 0.04T$$

$$s^2\theta_2 = (250)\theta_1 + (-260)\theta_2 + (10)\theta_3$$

$$s^2\theta_3 = (250)\theta_2 + (-260)\theta_3$$

Put in matrix form

$$s\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdots \\ s\theta_1 \\ s\theta_2 \\ s\theta_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \vdots & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -10 & 10 & 0 & \vdots & 0 & 0 & 0 \\ 250 & -260 & 10 & \vdots & 0 & 0 & 0 \\ 0 & 250 & -260 & \vdots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdots \\ s\theta_1 \\ s\theta_2 \\ s\theta_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdots \\ 0.04 \\ 0 \\ 0 \end{bmatrix} T$$

7) Find the transfer function from T to Q3

```
a11 =  zeros(3,3);
a12 = eye(3,3);
a21 = [-10,0,0;250,-260,10;0,250,-260];
a22 = zeros(3,3);
A = [a11,a12 ; a21, a22]

     0     0     0     1     0     0
     0     0     0     0     1     0
     0     0     0     0     0     1
   -10     0     0     0     0     0
   250  -260    10     0     0     0
     0   250  -260     0     0     0

>> B = [0;0;0;0.04;0;0];
>> C = [0,0,1,0,0,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

               2500
  ---------------------------------
  (s^2 + 10) (s^2 + 210) (s^2 + 310)
```

DC Servo Motors

ebay listing: Baldor MTB-3363-BLYCN servo motor servomotor w/brake
Date Sheets: http://www.baldor.com/mvc/DownloadCenter/Files/BR1202-F

8) Determine the transfer function and step response for the following DC servo motor:

Baldour MT-3363-B DC Servo Motor: (476W)

$$J = 3.67 \text{ kg cm}^2 \cdot \left(\frac{1m}{100cm}\right)^2 = 367 \cdot 10^6 \text{ kg m}^2$$

$$D = 7.8e - 3Nm/krpm \cdot \left(\frac{krev}{1000rev}\right)\left(\frac{1rev}{2\pi rad}\right)\left(\frac{60\text{s}}{\text{min}}\right) = 74.48 \cdot 10^{-6} \frac{Nm}{rad/\text{sec}}$$

$$K_t = 0.297\frac{Nm}{A}$$

- Resistance: 2.4 Ohms
- Inductance: 6.1mH
- Total Weight: 5kg (11 lb)
- Price: $625 on ebay

```
>> J = 367e-6;
>> D = 74.48e-6;
>> Kt = 0.297;
>> R = 2.4;
>> L = 6.1e-3;
```

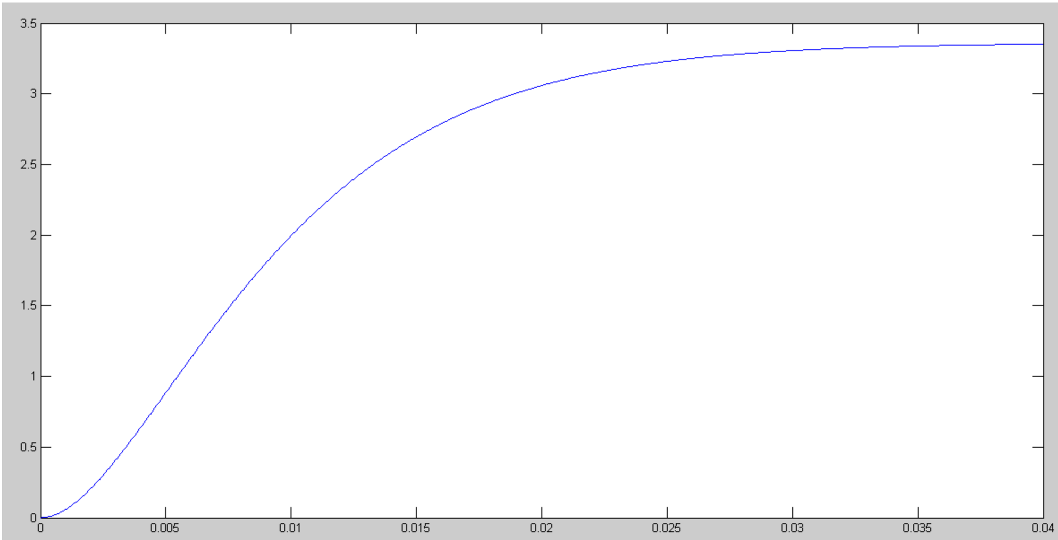
The transfer function for a DC motor is

$$s\theta = \left(\frac{K_t}{(Js+D)(Ls+R)+K_t^2}\right)V_a$$

$$s\theta = \left(\frac{K_t}{(JL)s^2+(JR+DL)s+(DR+K_t^2)}\right)V_a$$

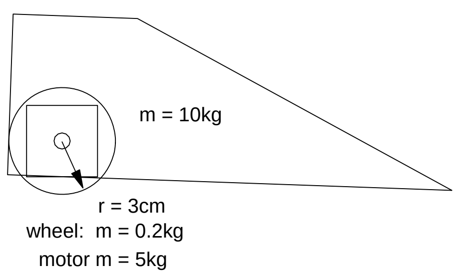
```
>> num = Kt;
>> den = [J*L, J*R + D*L, D*R + Kt^2];
>> G = tf(num,den);
>> zpk(G)

        132666.2795
--------------------------
(s^2 + 393.6s + 3.948e004)
```

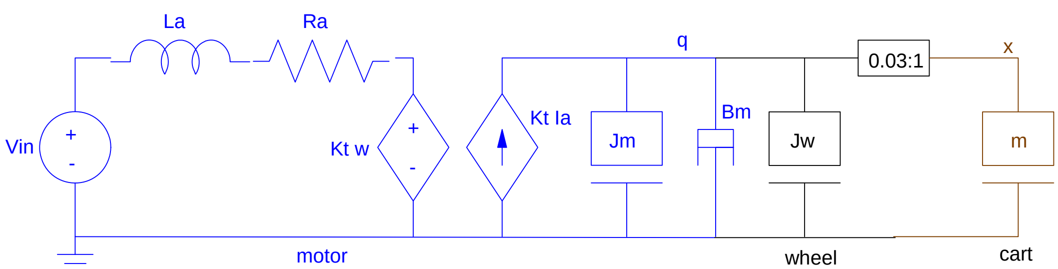
```
>> t = [0:0.001:1]' * 0.04;
>> y = step(G,t);
>> plot(t,y);
```



9) Determine the transfer function and step response for this DC servo motor if it is attached to a Battle Bot with the following specs

- Cart Mass: 10kg
- Wheel Mass: 0.2kg
- Wheel Diameter: 3cm





The inertia increases to

J(total) = J(motor) + J(wheel) + J(cart)

Wheel:

$$J = \frac{1}{2}mr^2$$

$$J = \frac{1}{2} \cdot (0.2kg) \cdot (0.015m)^2$$

$$J = 22.5 \cdot 10^{-6} \text{ kg m}^2$$

Cart: Total mass (motor + wheel + cart) = 5kg + 0.2kg + 10kg = 15.2kg

$$J = \left(\frac{0.03}{1}\right)^2 15.2\, kg = 0.0137 \text{ kg m}^2$$

Adding them all together:

$$J_{net} = J_{motor} + J_{wheel} + J_{total\ mass}$$

$$J_{net} = 0.000167 + 0.0000225 + 0.0137 \text{ kg m}^2$$

```
>> J=0.0137;
>> num = Kt
>> den = [J*L, J*R + D*L, D*R + Kt^2]
>> G = tf(num,den)

                0.297
-----------------------------------
8.357e-005 s^2 + 0.03288 s + 0.08839

>> zpk(G)

      3553.9069
-------------------
(s+390.7) (s+2.707)
```

```
>>
```