ECE 461/661 - Homework Set #8

Gain, Lead, PID Compensators - Due Monday, October 31st 20pt / problem

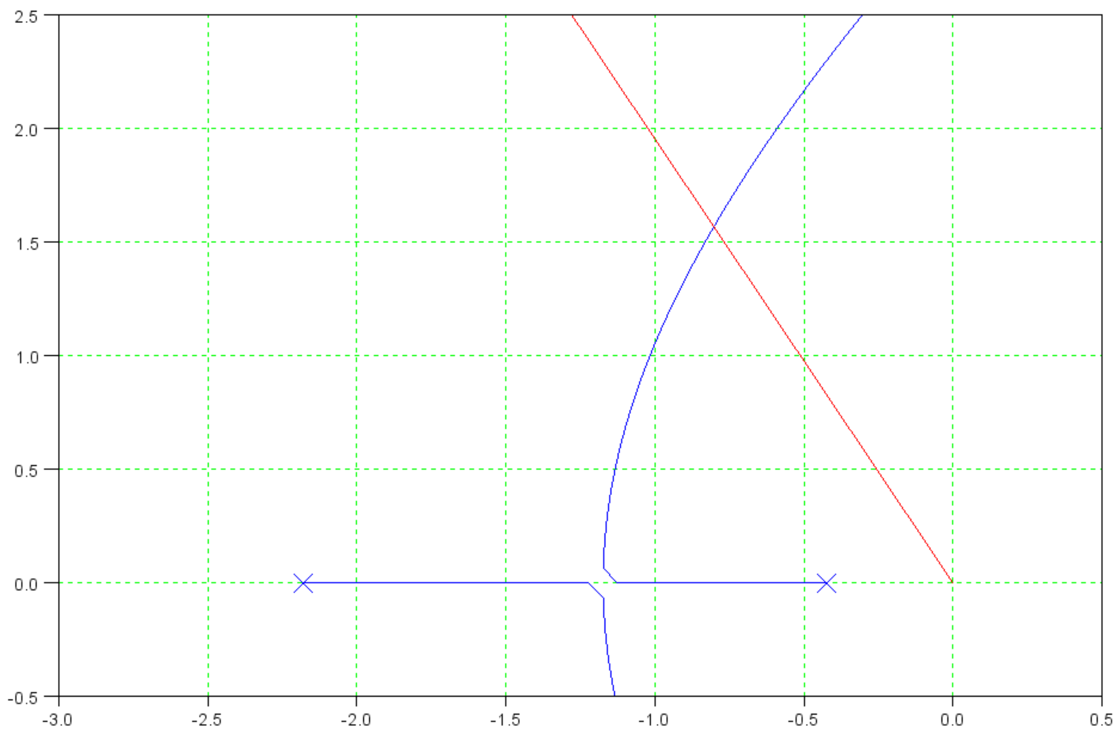
A 4th-order model for the 10-stage RC filter from homework #6 is

$$G(s) \approx \left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)} \right)$$

1) Design a gain compensator, K(s) = k, which results in 20% overshoot for a step input. For this value of K(s), give

- The resulting closed-loop dominant pole(s)
- The error constant, Kp
- The step response of the closed-loop system.

First, draw the root locus of G(s): (zoomed in for the dominant part of the root locus)



Find the point on the root locus that intersects the damping line:

s = -0.7932 + j1.5864

Find k so that GK = -1 at this point

$$\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)} \right)_{s=-0.7932+j1.5864} = 0.1342\angle 180^0$$

$$\boxed{k = \frac{1}{0.1342} = 7.450}$$

The resulting closed-loop dominant pole(s)

s = -0.7932 + j1.5864

The error constant, Kp

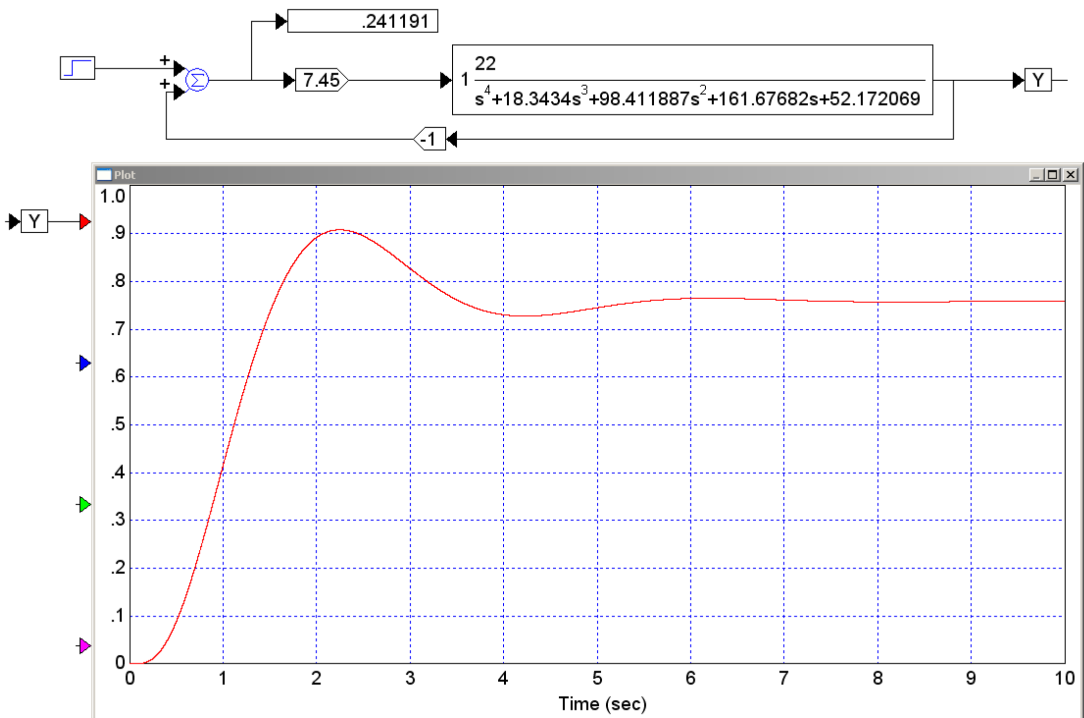
$$(GK)_{s=0} = (0.42168)(7.450) = 3.1415$$

Kp = 3.1415

The Steady-State Error for a Step Input:

$$\frac{1}{K_p+1} = 0.2414$$

The step response of the closed-loop system.



2) Design a lead compensatro, $K(s) = k\left(\frac{s+a}{s+10a}\right)$, which results in 20% overshoot for a step input.

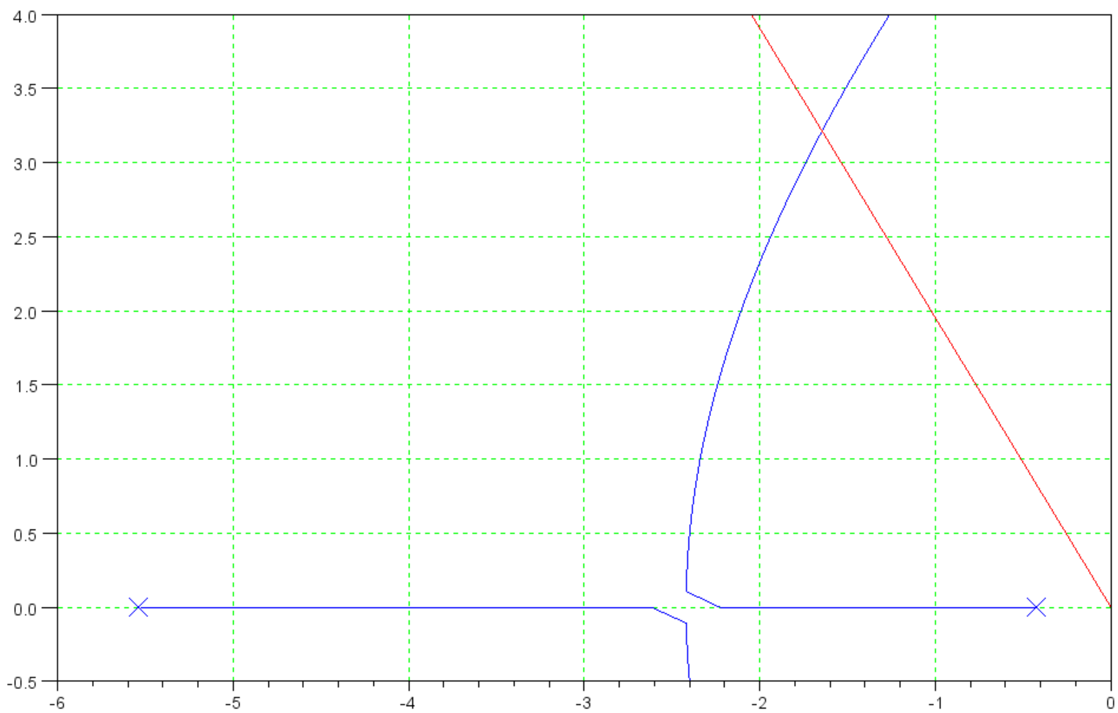
Keep the pole at s = 0.4234

Cancel the pole at -2.181

$$K(s) = k\left(\frac{s+2.181}{s+21.81}\right)$$

$$GK = \left(\frac{22}{(s+10.2)(s+5.539)(s+21.81)(s+0.4234)}\right)$$

Sketch the root locus of GK (zoomed in)



Find the point that intersects the damping line:

s = - 1.6271 + j3.2542

At this point, pick 'k' so that GK = -1

$$\left(\frac{22}{(s+10.2)(s+5.539)(s+21.81)(s+0.4234)}\right)_{s=-1.6271+j3.2542} = 0.0066\angle 180^0$$

$$k = \frac{1}{0.0066} = 150.44$$

and

$$\boxed{K(s) = 150.44\left(\frac{s+2.181}{s+21.81}\right)}$$

For this value of K(s), give

The resulting closed-loop dominant pole(s)

s = - 1.6271 + j3.2542

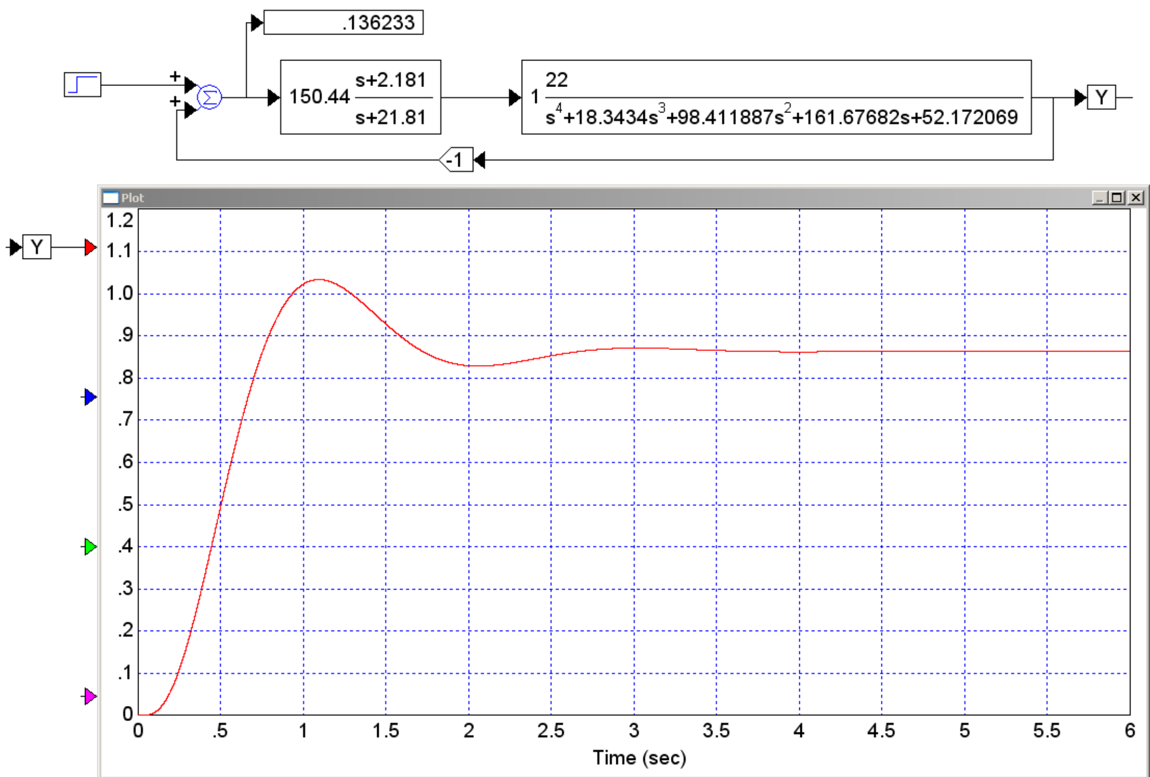
The error constant, Kp

$$K_p = (GK)_{s \to 0} = (0.42168)(15.044) = 6.34$$

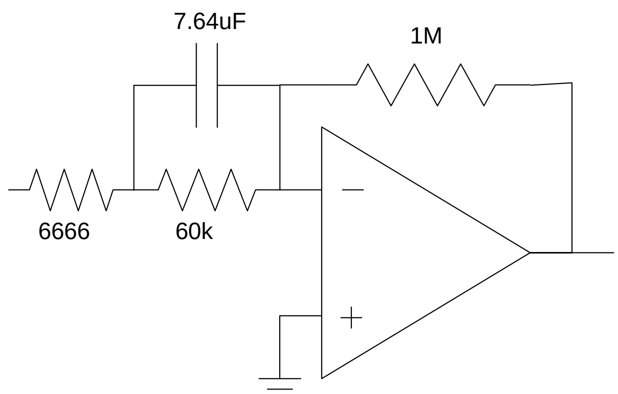
The steady-state error for a step input is smaller

$$\frac{1}{K_p + 1} = 0.1362$$

The step response of the closed-loop system, and



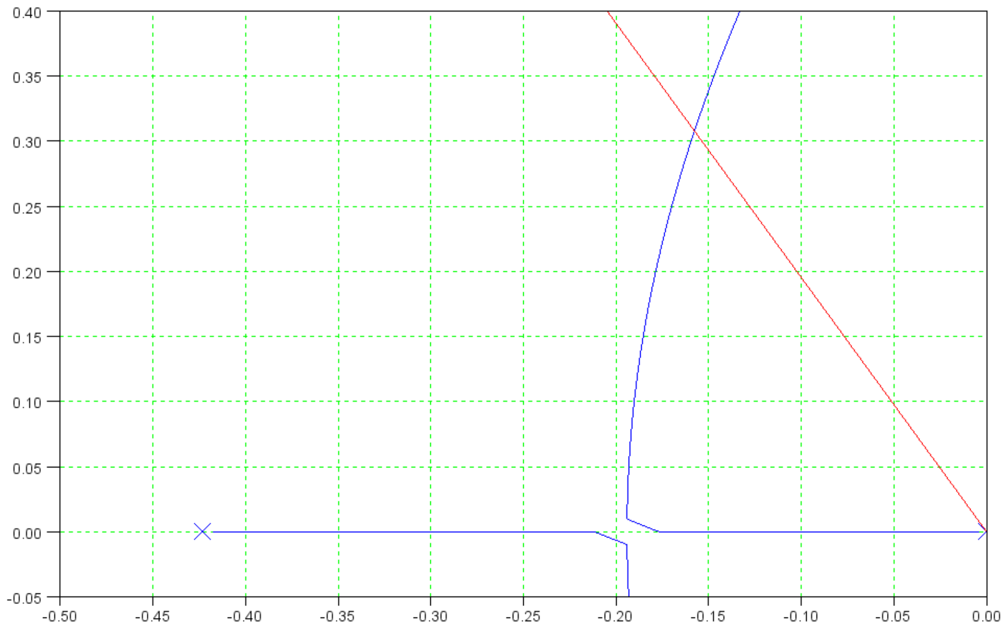
A circuit to implement K(s)



3) Design a I compensator, $K(s) = \left(\frac{k}{s}\right)$, which results in 20% overshoot for a step input.

$$GK = \left(\frac{22k}{s(s+0.4234)(s+2.181)(s+5.539)(s+10.2)}\right)$$

Sketch the root locus of GK



Find the spot on the root locus that intercepts the damping line

s = -0.1565 + j0.3129

Find 'k' so that GK = -1 at this point

$$\left(\frac{22}{s(s+0.4234)(s+2.181)(s+5.539)(s+10.2)}\right)_{s=-0.1565+j0.3129} = 1.377\angle 180^0$$

$$k = \frac{1}{1.377} = 0.7260$$

and

$$K(s) = \left(\frac{0.7260}{s}\right)$$

For this value of K(s), give

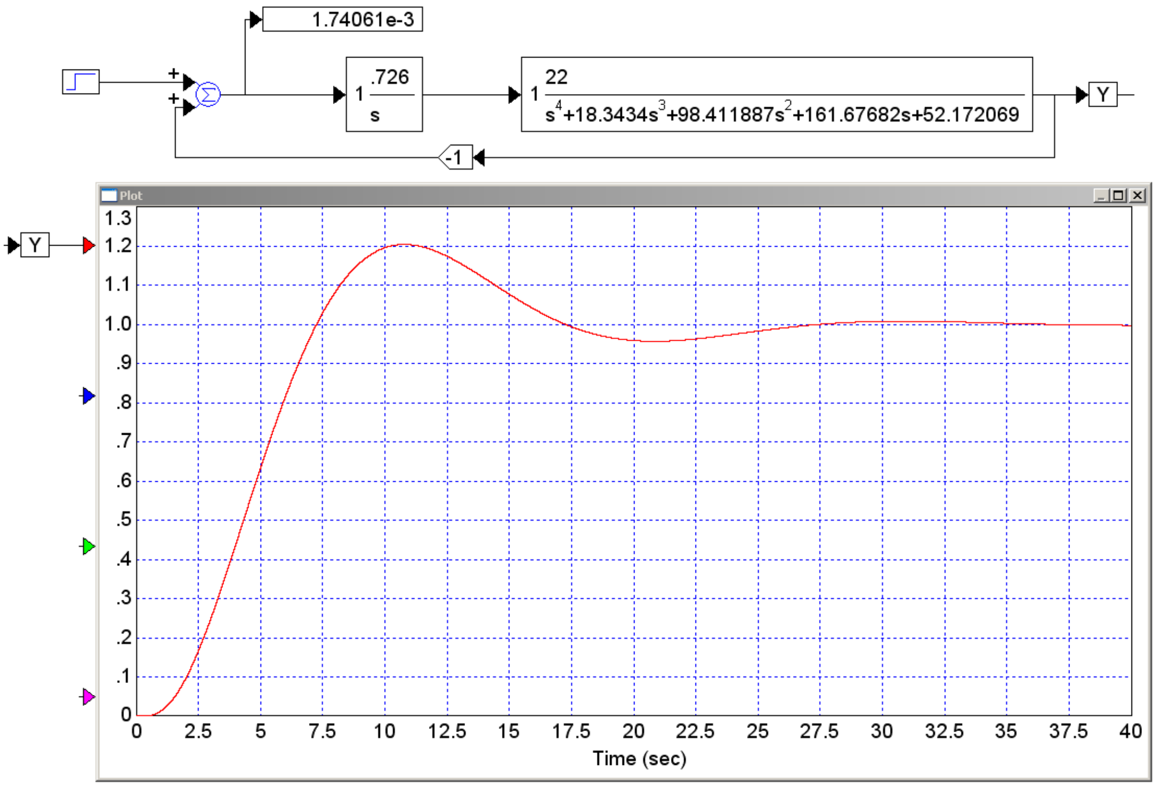
The resulting closed-loop dominant pole(s)

s = -0.1565 + j0.3129

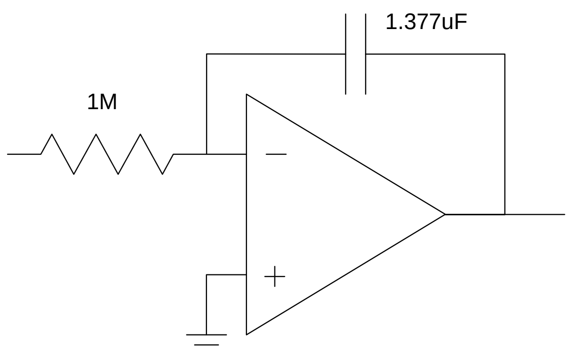
The error constant, Kp

Kp = infinity (type-1 system)

The step response of the closed-loop system, and



A circuit to implement K(s)



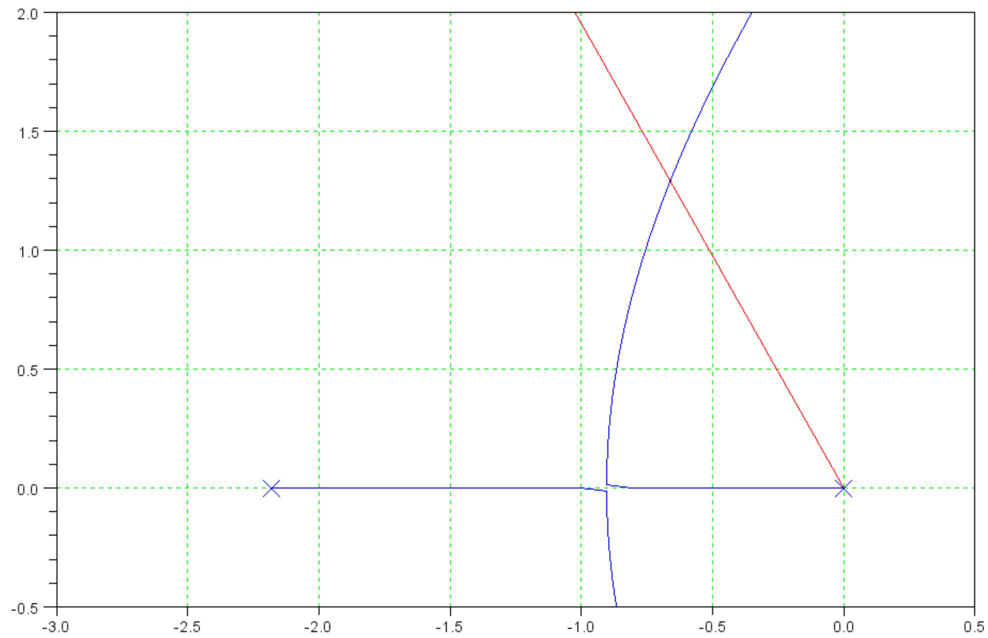
4) Design a PI compensator, $K(s) = k\left(\frac{(s+a)}{s}\right)$, which results in 20% overshoot for a step input.

Cancel the slowest stable pole

$$K(s) = k\left(\frac{s+0.4234}{s}\right)$$

$$GK = \left(\frac{22k}{s(s+2.181)(s+5.539)(s+10.2)}\right)$$

Sketch the root locus



Find the spot on the root locus that intercepts the damping line

s = -0.6546 + j1.3092

Pick 'k' so that GK = -1 at this point

$$\left(\frac{22k}{s(s+2.181)(s+5.539)(s+10.2)}\right)_{s=-0.6546+j1.3092} = 0.1534\angle 180^0$$

$$k = \frac{1}{0.1534} = 6.5187$$

so

$$K(s) = 6.5187\left(\frac{s+0.4234}{s}\right)$$

For this value of K(s), give

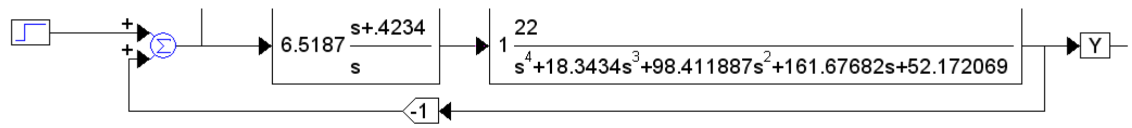
The resulting closed-loop dominant pole(s)

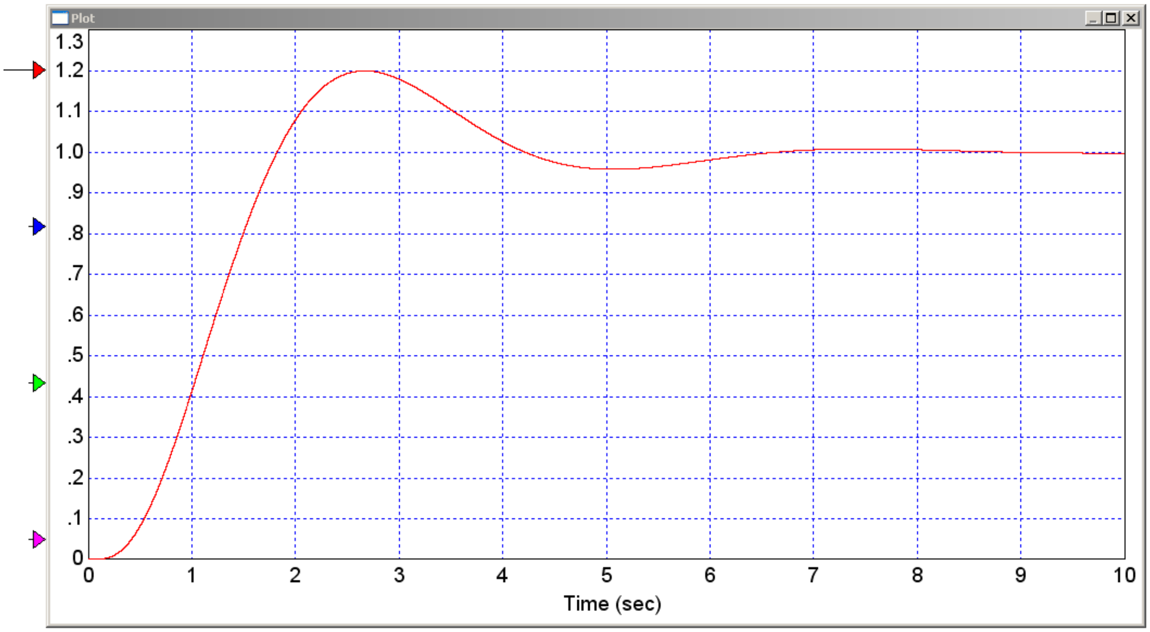
s = -0.6546 + j1.3092

The error constant, Kp

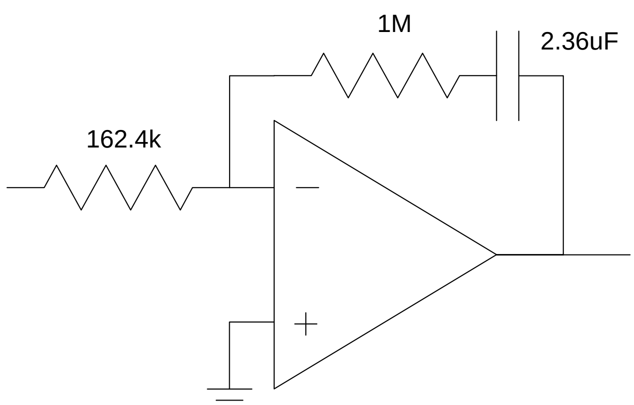
Kp = infinity (type-1 system)

The step response of the closed-loop system, and





A circuit to implement K(s)



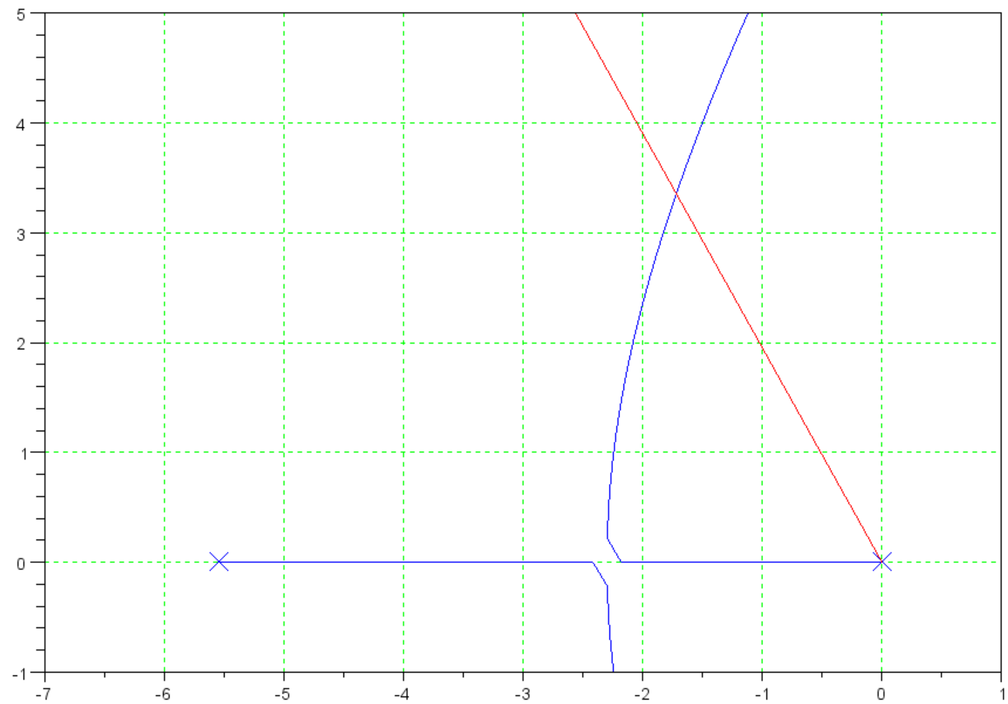
5) Design a PID compensator, $K(s) = k\left(\frac{(s+a)(s+b)}{s}\right)$, which results in 20% overshoot for a step input.

Cancel the two slowest poles

$$K(s) = k\left(\frac{(s+0.4234)(s+2.181)}{s}\right)$$

$$GK = \left(\frac{22k}{s(s+5.539)(s+10.2)}\right)$$

Sketch the root locus



Find the spot on the root locus that intersects the damping line:

s = -1.7027 + j3.4055

Pick 'k' so that GK = -1 at this point

$$\left(\frac{22k}{s(s+5.539)(s+10.2)}\right)_{s=-1.7027+j3.4055} = 0.1230\angle 180^0$$

$$k = \frac{1}{0.1230} = 8.1269$$

$$\boxed{K(s) = 8.1269\left(\frac{(s+0.4234)(s+2.181)}{s}\right)}$$

For this value of K(s), give

The resulting closed-loop dominant pole(s)

s = -1.7027 + j3.4055

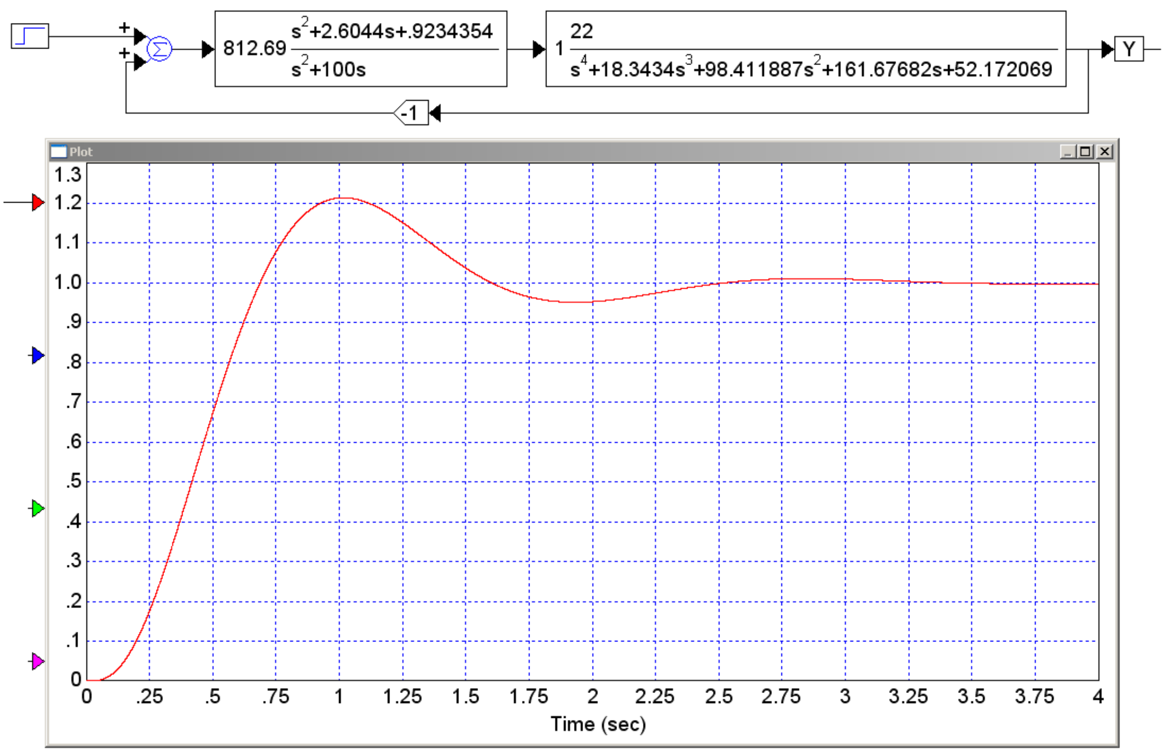
The error constant, Kp

Kp = infinity (type-1 system)

The step response of the closed-loop system,

VisSim and MatLab don't like K(s) since it itn't a proper transfer function (more zeros than poles). Approximate K(s) as

$$K(s) = 8.1269\left(\frac{(s+0.4234)(s+2.181)}{s}\right)\left(\frac{100}{s+100}\right)$$



A circuit to implement K(s)