ECE 461/661 - Homework Set #9

Systems with Delays, Osciallatory Systems, Unstable Systems. Due November 7th. 20pt / problem

1) A better model for the 10-stage RC filter from homework #5 uses a 480ms delay to approximate the effect of the poles that are dropped:

$$G(s) \approx \left(\frac{0.4}{(s+2.181)(s+0.4234)}\right) \cdot e^{-0.48s}$$

Design a gain compensator, K(s), which results in

- No error for a step input,
- 20% overshoot for a step input, and
- Ts = 4 seconds.

Check your design in VisSim / Simulink / or similar program.

Translation:

- Make the system type-1
- Place the closed-loop dominant pole a s = -1 + j2

Let

$$K(s) = k\left(\frac{(s+2.181)(s+0.4234)}{s(s+a)}\right)$$

so that

$$GK = \left(\frac{0.4}{s(s+a)}\right) \cdot e^{-0.48s}$$

Pick 'a' so that s = -1 + j2 is on the root locus. Checking the angles:

$$\left(\left(\frac{0.4}{s}\right) \cdot e^{-0.48s}\right)_{s=-1+j2} = 0.2891\angle -171.56^0$$

For the angles to add up to 180 degrees

$$angle(s+a)_{s=-1+j2} = 8.431^0$$

meaning

$$a = \frac{2}{\tan\left(8.431^0\right)} + 1 = 14.4935$$

To find the gain, k,

$$GK = \left(\left(\frac{0.4}{s(s+14.4935)}\right) \cdot e^{-0.48s}\right)_{s=-1+j2} = 0.0212\angle 180^0$$

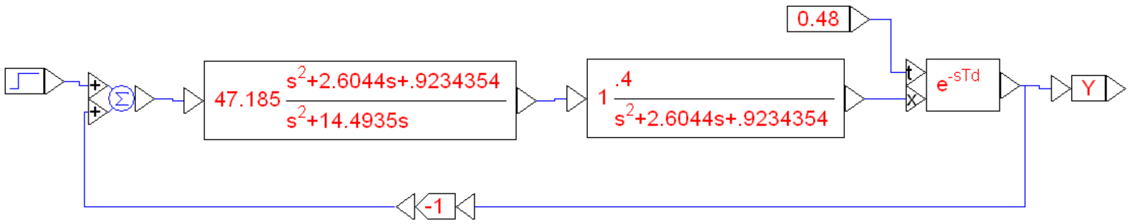
Then

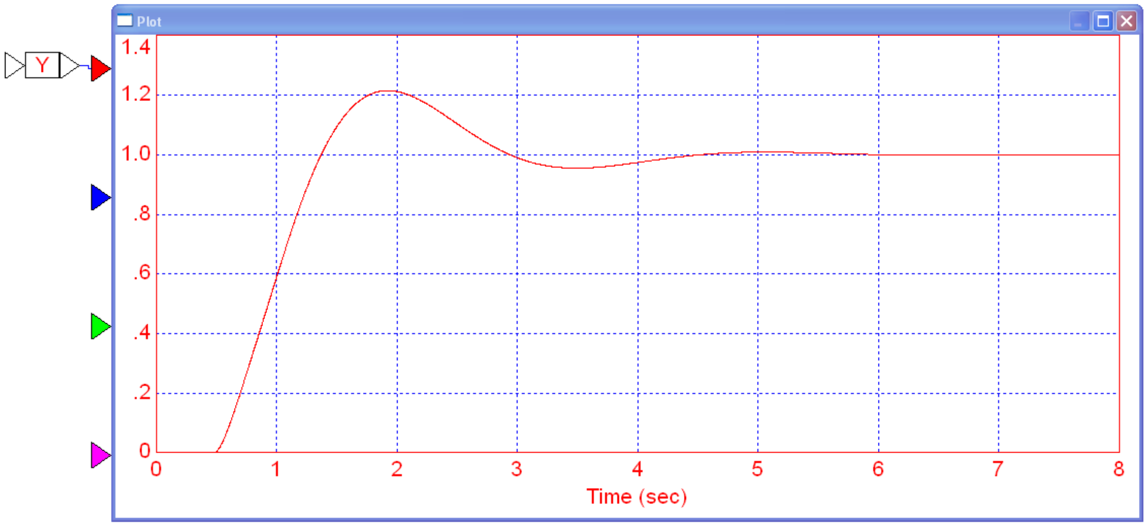
$$k = \frac{1}{0.0212} = 47.185$$

and

$$\boxed{K(s) = 47.185\left(\frac{(s+2.181)(s+0.4234)}{s(s+14.4935)}\right)}$$

Checking in VisSim


0.48
47.185 $\frac{s^2+2.6044s+.9234354}{s^2+14.4935s}$
$1\frac{.4}{s^2+2.6044s+.9234354}$
e^{-sTd}
t
x
Y
-1


Y
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0 1 2 3 4 5 6 7 8
Time (sec)

2) The transfer function for an oscillator system is:

$$G(s) = \left(\frac{10}{s(s+j2)(s-j2)}\right)$$

Design a gain compensator, K(s), which results in

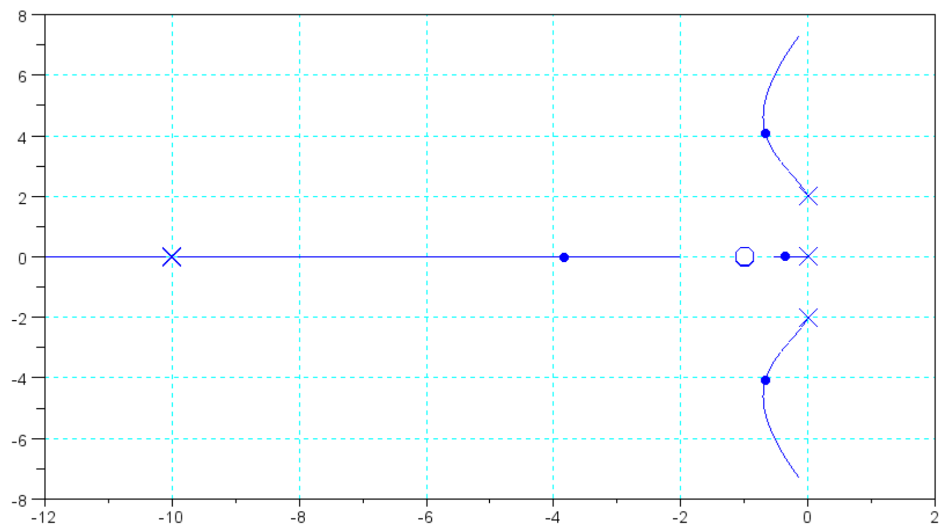
- No error for a step input,
- 20% overshoot for a step input, and
- Ts = 4 seconds.

Check your design in VisSim / Simulink / or similar program.

First, stabilize the system. Let

$$K_1 = k\left(\frac{(s+1)^2}{(s+10)^2}\right)$$

This results in the following root locus



Pick a spot that's stable (shown in the blue dots). Let

s = -1 + j4

Find 'k' so that GK = -1 at this point

$$GK_1 = \left(\left(\frac{10}{s(s+j2)(s-j2)}\right)\left(\frac{(s+1)^2}{(s+10)^2}\right)\right)_{s=-1+j4} = -0.0291275 + 0.0040877i$$

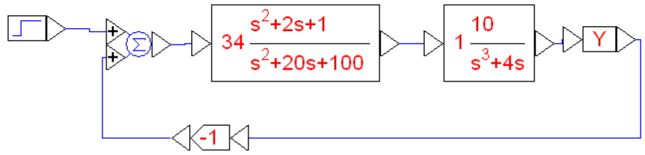
$$k = \frac{1}{|ans|} = 34$$

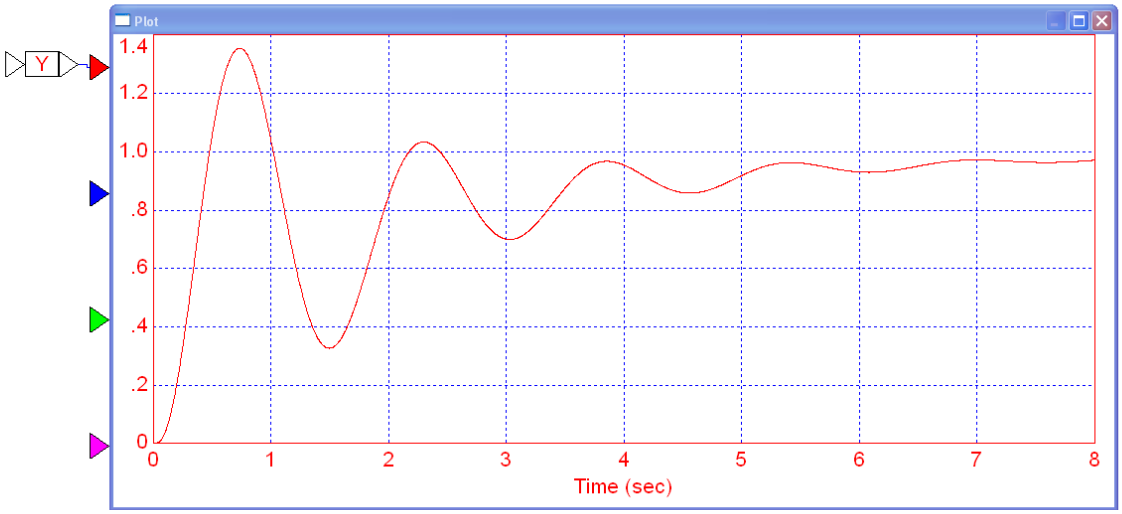
resulting in

$$\boxed{K_1 = 34\left(\frac{(s+1)^2}{(s+10)^2}\right)}$$

The closed-loop system is then

$$G_2 = \left(\frac{GK_1}{1+GK_1} \right) = \left(\frac{340(s+1)^2}{(s+0.3613)(s+3.829)(s+0.67+j4.066)(s+0.67-j4.066)(s+14.47)} \right)$$





Now that the system is stable, meet the design specs. Let

$$K_2 = \left(\frac{(s+0.3613)(s+3.829)(s+0.67+j4.066)(s+0.67-j4.066)}{s(s+1)^2(s+a)} \right)$$

so that

$$G_2K_2 = \left(\frac{340}{s(s+14.47)(s+a)} \right)$$

Pick 'a' so that -1 + j2 is on the root locus

$$\left(\frac{340}{s(s+14.47)} \right)_{s=-1+j2} = 11.16\angle -125.01^0$$

meaning

$$\angle(s+a) = 54.99^0$$

$$a = \frac{2}{\tan(54.99^0)} + 1 = 2.401$$

For the gain to be one:

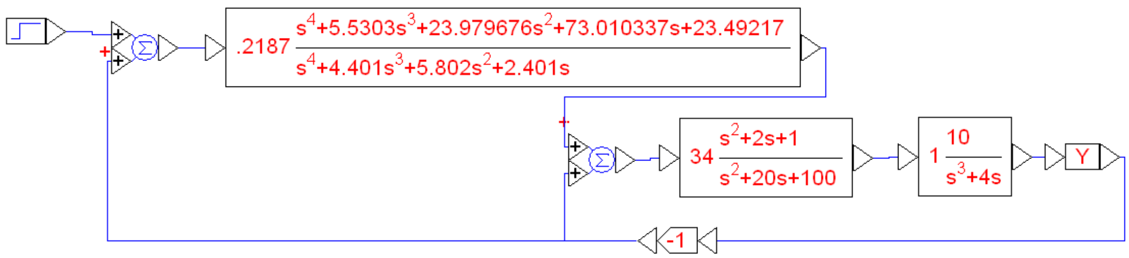
$$G_2K_2 = \left(\frac{340}{s(s+14.47)(s+2.401)} \right)_{s=-1+j2} = 4.5726\angle 180^0$$

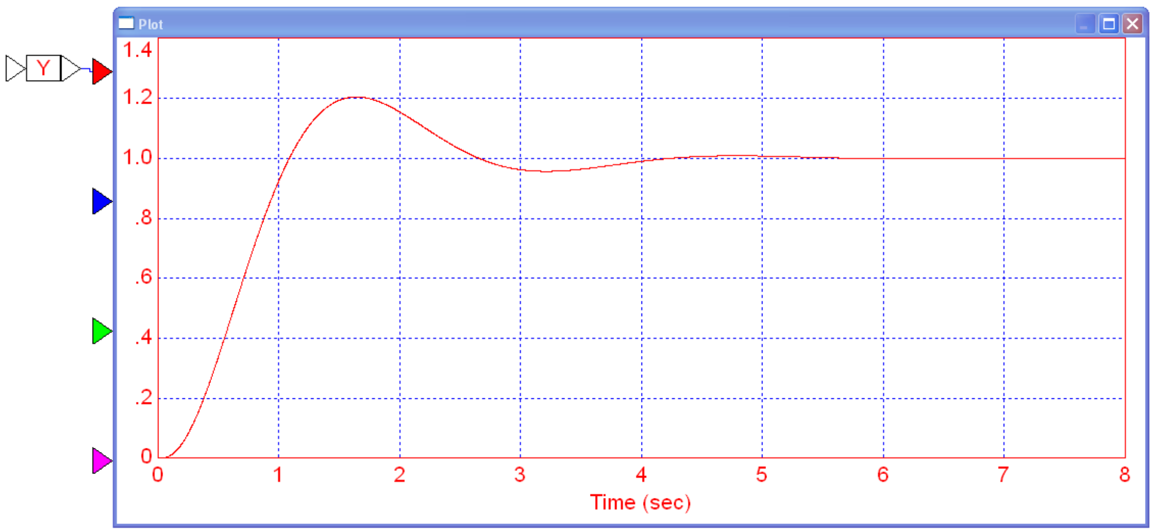
$$k = \frac{1}{4.5726} = 0.2187$$

meaning

$$K_2 = 0.2187\left(\frac{(s+0.3613)(s+3.829)(s+0.67+j4.066)(s+0.67-j4.066)}{s(s+1)^2(s+2.401)}\right)$$

Checking in VisSim





3) The transfer function for an unstable system is:

$$G(s) = \left(\frac{10}{(s-2)(s+2)(s+5)}\right)$$

Design a gain compensator, K(s), which results in

- No error for a step input,
- 20% overshoot for a step input, and
- Ts = 4 seconds.

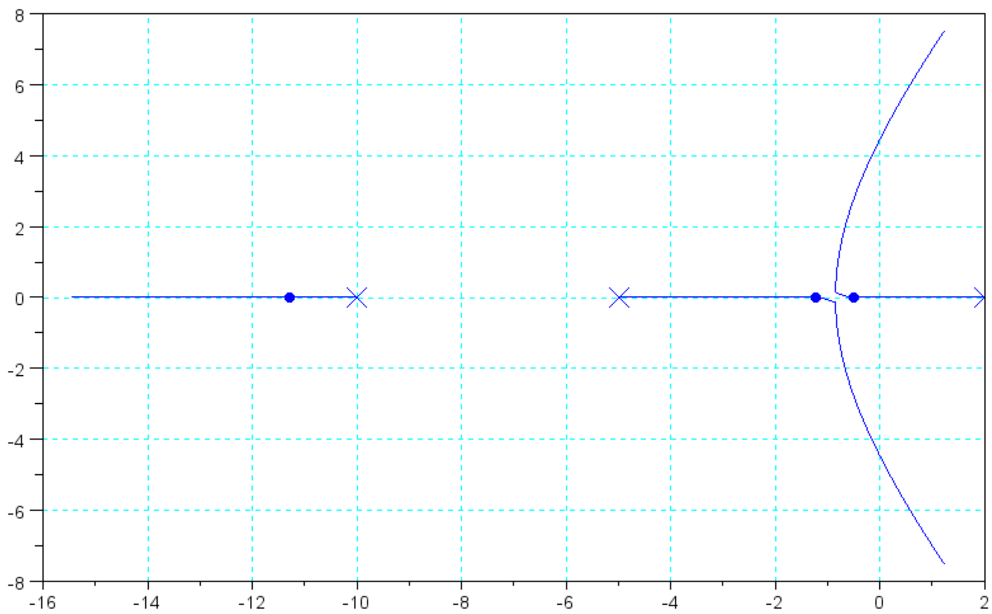
Check your design in VisSim / Simulink / or similar program.

First, stabilize the system. Let

$$K_1 = \left(\frac{s+2}{s+10}\right)$$

$$GK_1 = \left(\frac{10}{(s-2)(s+5)(s+10)}\right)$$

This results in the following root locus



Pick a spot that's stable. Let s = -0.5

$$\left(\frac{10}{(s-2)(s+5)(s+10)}\right)_{s=-0.5} = -0.0935673$$

$$k = \frac{1}{0.0935} = 10.6875$$

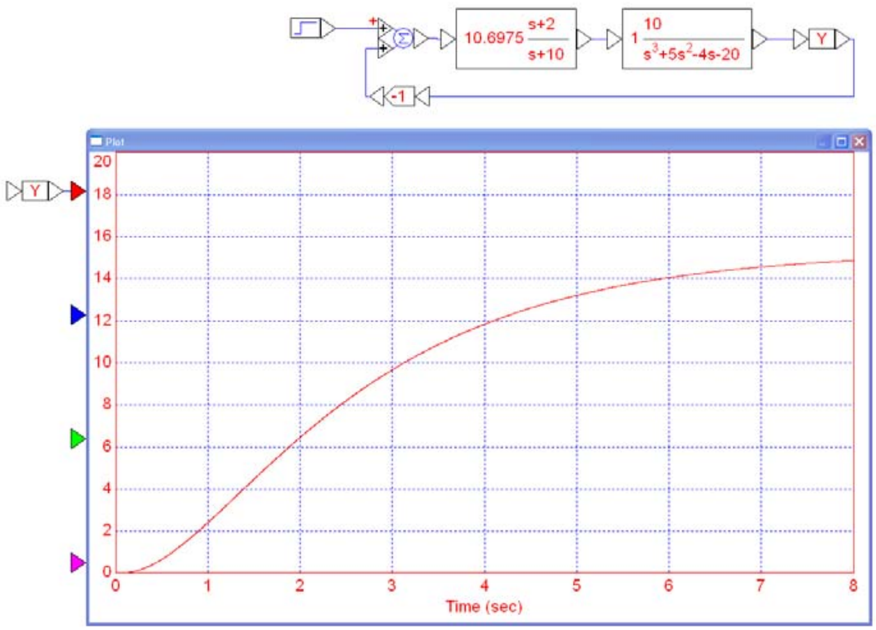
and

$$\boxed{K_1 = 10.6975\left(\frac{s+2}{s+10}\right)}$$

The closed-loop system is then

$$G_2 = \left(\frac{GK_1}{1+GK_1}\right) = \left(\frac{106.875}{(s+0.5)(s+1.2188)(s+11.281)}\right)$$

Checking in VisSim



The step response isn't great, but at least it's stable.

Step 2: Pick a second compensator to meet the design specs

Let

$$K_2 = \left(\frac{(s+0.5)(s+1.2188)}{s(s+a)}\right)$$

$$G_2K_2 = \left(\frac{106.875}{s(s+a)(s+11.281)}\right)$$

Evaluating at s = -1 + j2

$$\left(\frac{106.875}{s(s+11.281)}\right)_{s=-1+j2} = 4.5634\angle -127.57^0$$

For the angles to add up to 180 degrees

$$\angle(s+a) = 52.42^0$$

$$a = \frac{2}{\tan\left(52/42^0\right)} + 1 = 2.5387$$

To find the gain

$$\left(\frac{106.875}{s(s+2.5387)(s+11.281)}\right)_{s=-1+j2} = 1.808\angle 180^0$$

$$k = \frac{1}{1.808} = 0.553$$

and

$$\boxed{K_2 = 0.553\left(\frac{(s+0.5)(s+1.2188)}{s(s+2.5387)}\right)}$$

Checking in VisSim