ECE 461/661 - Homework Set #10

z-Transform, Converting G(s) to G(z). Due November 14th.

Assume a sampling rate of 10ms.

- Determine a digital filter, G(z), which has approximately the same step response as G(s).
- Verify your design by plotting the step response of G(s) and G(z)

1) $G(s) = \left(\frac{10}{(s+1)(s+2)}\right)$

```
-->s = [-1; -2];
-->T = 0.01;
-->z = exp(s*T)

    0.9900498
    0.9801987


-->DC = 5.

-->k = DC * (1 - z(1)) * (1 - z(2))

    0.0009851
```

$$G(z) = \left(\frac{0.0009851}{(z-0.9900)(z-0.9801)}\right)$$

Now, add zeros at z = 0 to get the delay 'correct'. At s = j1

$$\left(\frac{10}{(s+1)(s+2)}\right)_{s=j1} = 3.1623\angle -71.56^0$$

$$\left(\frac{0.0009851}{(z-0.9900)(z-0.9801)}\right)_{z=e^{j1\cdot T}} = 3.1418\angle -71.88^0$$

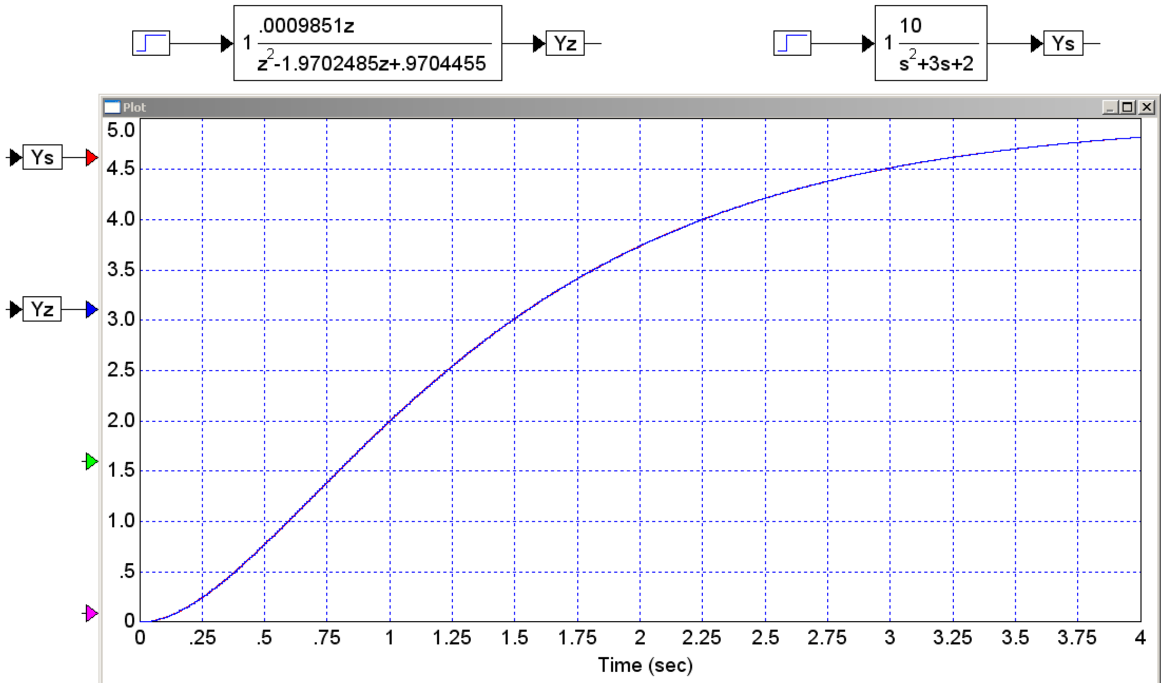
The phase is off by 0.32 degrees. Each zero at z = 0 adds 0.573 degrees

$$z = e^{j1\cdot T} = 1\angle 0.573^0$$

So, we need to add 0.55 zeros. Round to one

$$G(z) = \left(\frac{0.0009851z}{(z-0.9900)(z-0.9801)}\right)$$

Checking in VisSim



.0009851z
z^2-1.9702485z+.9704455
Yz
10
s^2+3s+2
Ys
Plot
Ys
Yz
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
0
.25
.5
.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
3.25
3.5
3.75
4
Time (sec)

2) $G(s) = \left(\frac{10}{s(s+5)}\right)$

s = 0 z = -1

s = -5 z = 0.9512

so

$$G(z) \approx \left(\frac{a}{(z-1)(z-0.9512)}\right)$$

The DC gain is infinity - so avoid DC. Picking another point, like

s = 0.01

$$z = e^{sT} = 1.0001$$

gives

$$\left(\frac{10}{s(s+5)}\right)_{s=0.01} = 199.60$$

$$\left(\frac{a}{(z-1)(z-0.9512)}\right)_{z=1.0001} = 199.60$$

$$a = 0.000976$$

To match the number of zeros at z = 0, find the gain at s = j1

$$\left(\frac{10}{s(s+5)}\right)_{s=j} = 1.9612\angle-101.31^0$$

$$\left(\frac{0.000976}{(z-1)(z-0.9512)}\right)_{z=e^{j0.01}} = 1.9612\angle-101.88^0$$

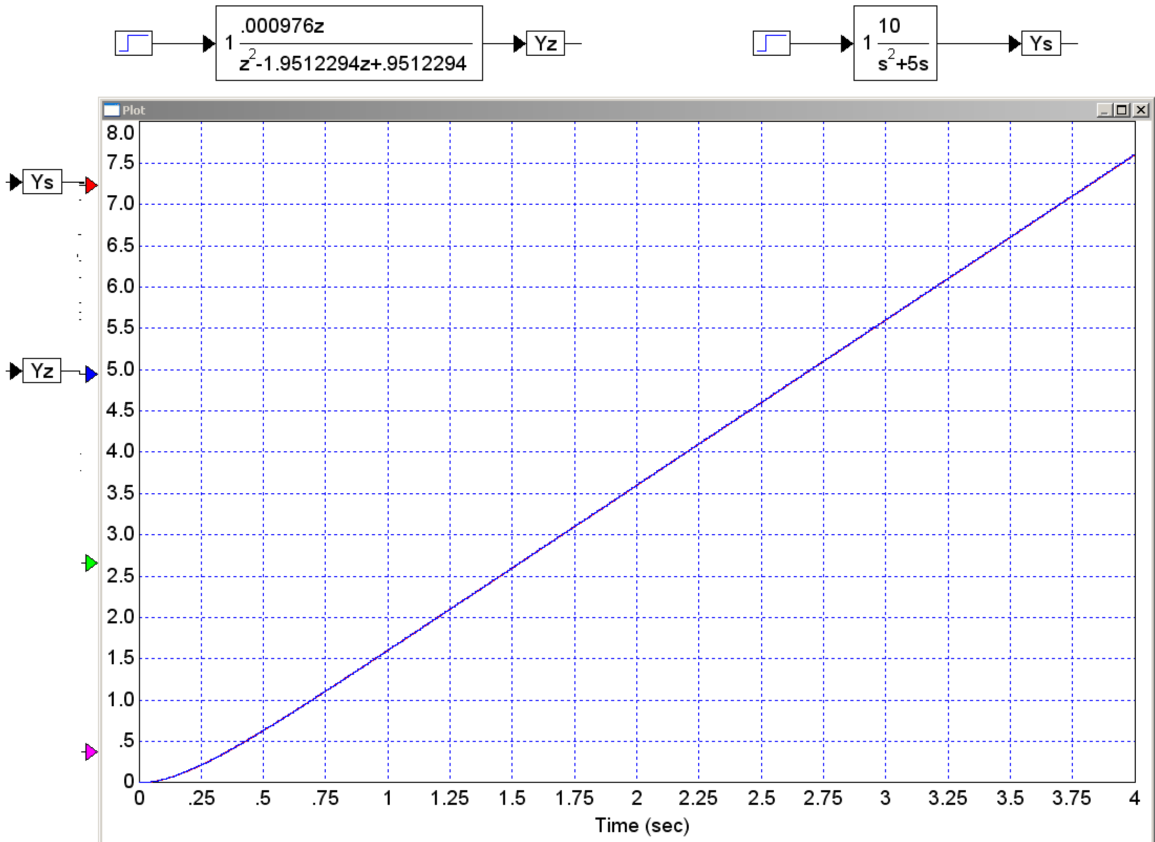
The gain matches, the phase is off by 0.5685 degrees. To match the phase, add zeros at z=0. Each zero adds 0.573 degrees, so the number of zeros you need is

$$n = \left(\frac{0.5685^0}{0.5730^0}\right) = 0.9922$$

Let n = 1

$$G(z) \approx \left(\frac{0.000976z}{(z-1)(z-0.9512)}\right)$$

Checking in VisSim


1 .000976z / z^2-1.9512294z+.9512294
Yz
1 10 / s^2+5s
Ys
Plot
Ys
Yz
Time (sec)

3) $G(s) = 4\left(\frac{s+2}{s+4}\right)$

s = -2:

z = 0.9802

s - 04

z = 0.9608

so

$$G(z) = a\left(\frac{z-0.9802}{z-0.9608}\right)$$

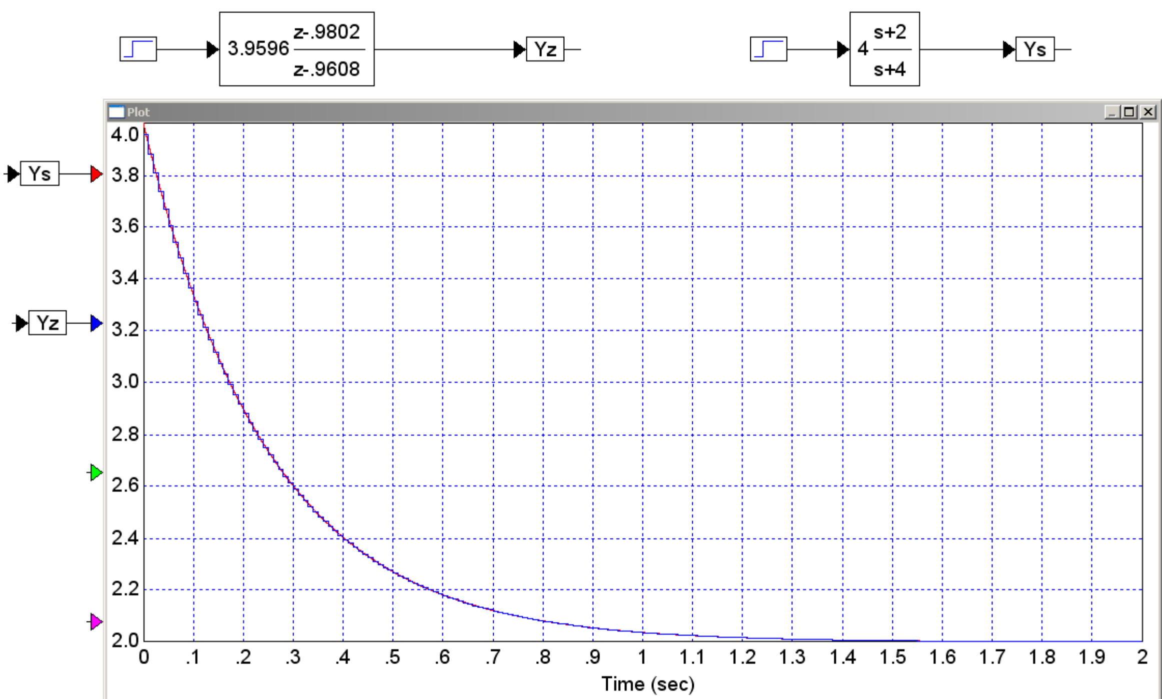
Matching the DC gain

$$4\left(\frac{s+2}{s+4}\right)_{s=0} = 2$$

$$a\left(\frac{z-0.9802}{z-0.9608}\right)_{z=1} = 2$$

$$a = 3.9596$$

$$G(z) = 3.9596\left(\frac{z-0.9802}{z-0.9608}\right)$$



4) $G(s) = \left(\frac{104}{(s+2+j10)(s+2-j10)}\right)$

s = -2 + j10

$$z = e^{sT} = 0.9802\angle 5.7296^0$$

$$G(z) = \left(\frac{a}{\left(z-0.9802\angle 5.7296^0\right)\left(z-0.9802\angle -5.7296^0\right)}\right)$$

Matching the DC gain

$$\left(\frac{104}{(s+2+j10)(s+2-j10)}\right)_{s=0} = 1$$

$$\left(\frac{a}{\left(z-0.9802\angle 5.7296^0\right)\left(z-0.9802\angle -5.7296^0\right)}\right)_{z=1} = 1$$

$$a = 0.0101859$$

To find the number of zeros at z = 0

$$\left(\frac{104}{(s+2+j10)(s+2-j10)}\right)_{s=j} = 1.0089\angle -2.2240^0$$

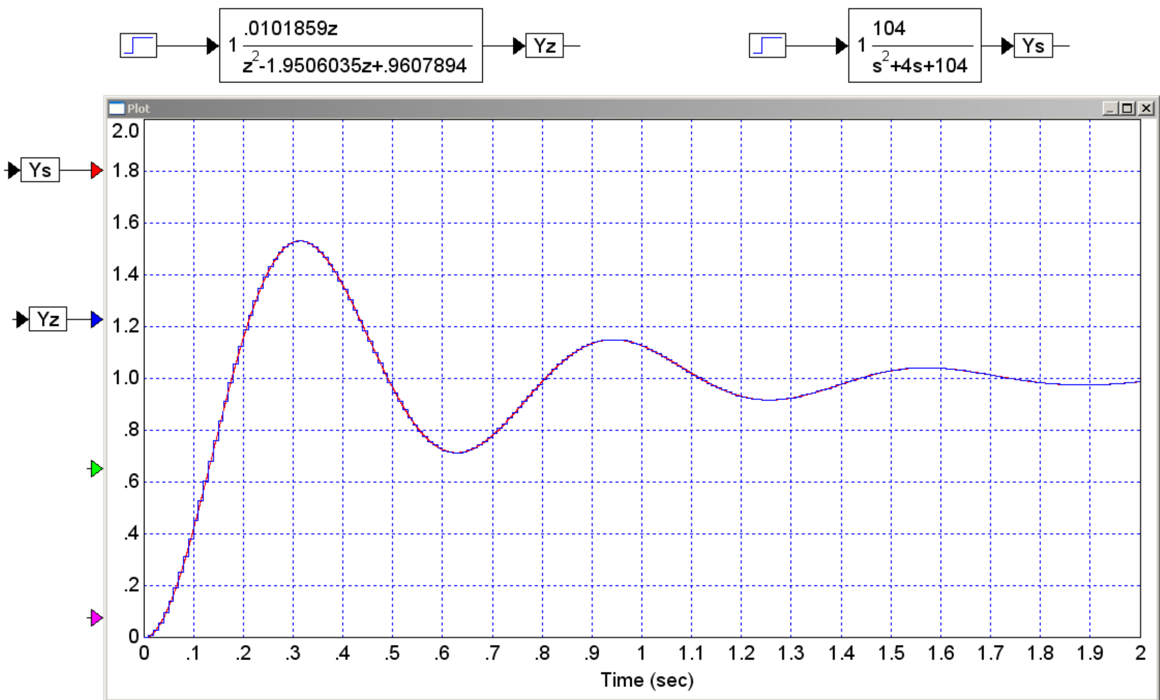
$$\left(\frac{0.0101859}{\left(z-0.9802\angle 5.7296^0\right)\left(z-0.9802\angle -5.7296^0\right)}\right)_{z=e^{j0.01}} = 1.0090\angle -2.7988^0$$

The phase is off by 0.5749 degrees. The number of zeros to add at z = 0 is then

$$n = \left(\frac{0.5749^0}{0.5730^0/zero}\right) = 1.0034$$

Let n = 1

$$G(z) = \left(\frac{0.0101859z}{\left(z-0.9802\angle 5.7296^0\right)\left(z-0.9802\angle -5.7296^0\right)}\right)$$



1
.0101859z
z2-1.9506035z+.9607894
Yz
1
104
s2+4s+104
Ys
Plot
Ys
Yz
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2
Time (sec)

5) Write a program to implement the following system. Assume a sampling rate of 10ms.

$$Y = \left(\frac{0.01z}{(z-1)(z-0.9)(z-0.5)}\right)X$$

Multiply out. In Matlab

```
>> poly([1,0.9,0.5])

ans =

    1.0000   -2.4000    1.8500   -0.4500

>>
```

$$Y = \left(\frac{0.01z}{z^3-2.4z^2+1.85z-0.45}\right)X$$

$$(z^3 - 2.4z^2 + 1.85z - 0.45)Y = (0.01z)X$$

y(k+3) - 2.4y(k+2) + 1.85y(k+1) - 0.45y(k) = 0.01x(k+1)

y(k+3) = 2.4y(k+2) - 1.85y(k+1) + 0.45y(k) + 0.01x(k+1)

Time shift by 3

y(k) = 2.4y(k-1) - 1.85y(k-2) + 0.45y(k-3) + 0.01x(k-2)

Write the C code

```
 while(1) {

   x3 = x2;               // x(k-3)
   x2 = x1;               // x(k-2)
   x1 = x0;               // x(k-1)
   x0 = A2D_Read(0);      // x(k)

   y3 = y2;               // y(k-3)
   y2 = y1;               // y(k-2)
   y1 = y0;               // y(k-1)

   y0 = 2.4*y1 - 1.85*y2 + 0.45*y3 + 0.01*x2;

   D2A(y0);

   Wait_10ms();

   }
       y(k+3) =
```