ECE 461/661 - Homework Set #11

Root Locus in the z-plane - Due Monday, November 21st

20 points per problem

A 4th-order model for the 10-stage RC filter from homework #6 is

$$G(s) \approx \left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)$$

Assume a sampling rate of 0.1 second (T = 0.1);

1) Gain Compensation: Design a digital compensator

$$K(z) = k$$

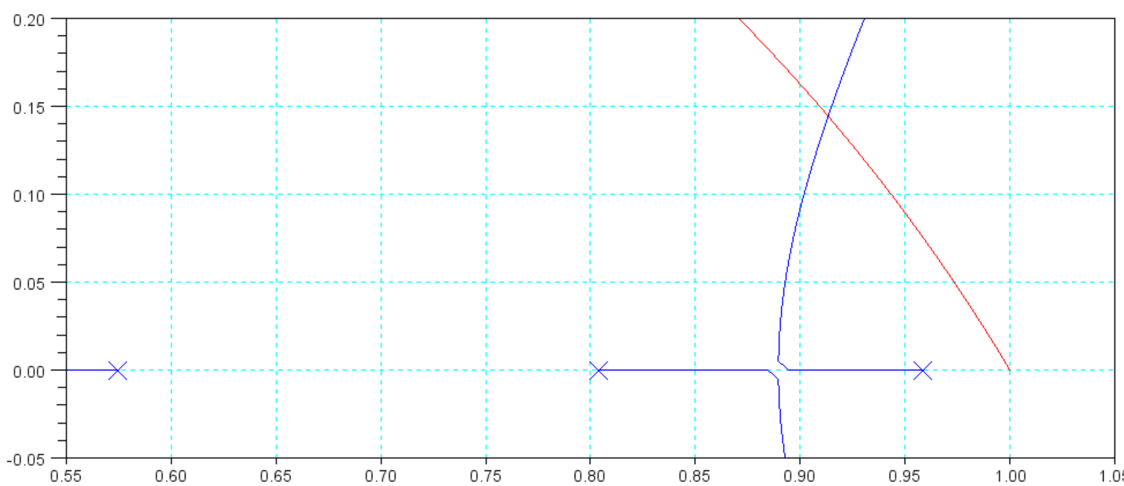
which results in 20% overshoot for a step input. For this value of K(z), give

- The resulting closed-loop dominant pole
- The step response of the closed-loop system.

Method 1) Convert to the z-plane. The DC gain is 0.4217

$$G(z) \approx \left(\frac{0.000933z^2}{(z-0.9585)(z-0.8040)(z-0.5747)(z-0.3606)}\right)$$

Draw the root locus and find the spot that hits the 0.4559 damping line



Find the spot on the root locus that intersects the damping line:

z = 0.9133 + j0.1443

Pick k so that GK = -1 at this point

$$\left(\frac{0.000933z}{(z-0.9585)(z-0.8040)(z-0.5747)(z-03606)}\right)_{z=0.9133+j0.1443} = -0.1386$$

$$k = \frac{1}{0.1386} = 7.215$$

Method #2: Search along the damping line so that

$$angle\left(\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\right)_{s=a(-1+j2)} = 180^0$$

$$s = -0.7554 + j1.5108$$

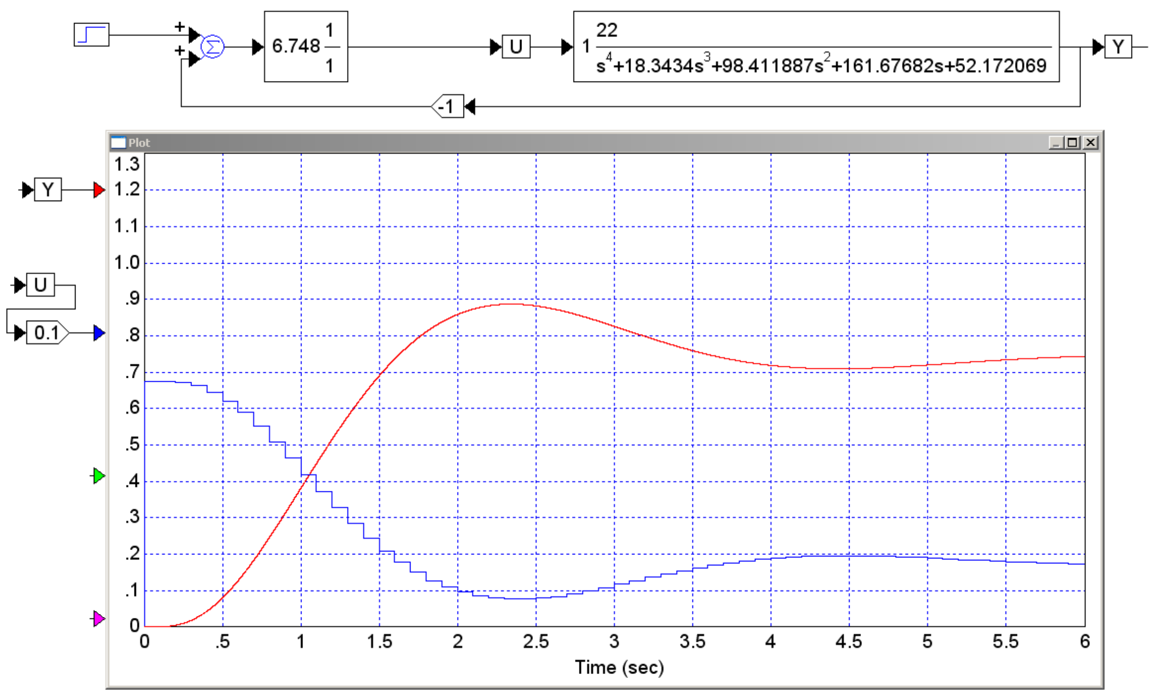
$$z = e^{sT} = 0.9167 + 0.1396$$

At this point

$$\left(\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\right)_{s=-0.7554+j1.5108} = 0.1482\angle 180^0$$

$$k = \frac{1}{0.1482} = 6.7480$$

Check in VisSim:



2) Lead Compensation: Design a digital compensator

$$K(z) = k\left(\frac{z-a}{z-b}\right)$$

which results in 20% overshoot for a step input. For this value of K(z), give

- The resulting closed-loop dominant pole(s)
- The step response of the closed-loop system, and

Method #1) Find G(z)

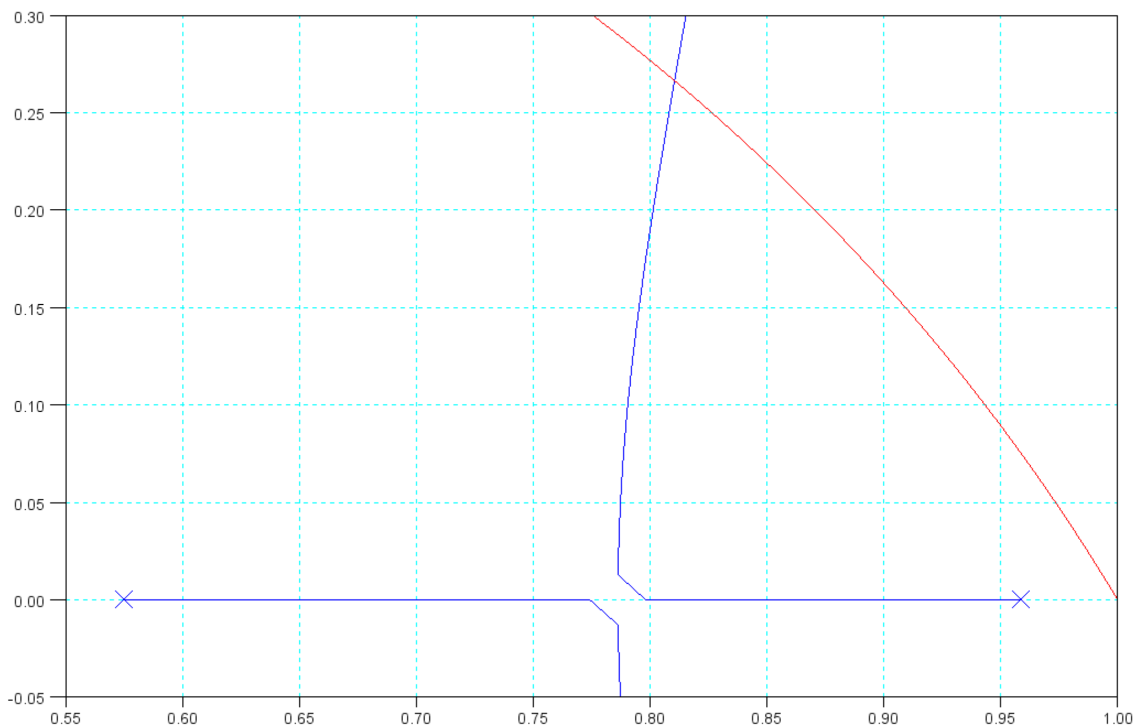
$$G(z) \approx \left(\frac{0.000933z^2}{(z-0.9585)(z-0.8040)(z-0.5747)(z-0.3606)}\right)$$

Keep the pole at 0.9585. Add K(z) to cancel the next slowest pole

$$K(z) = \left(\frac{z-0.8040}{z}\right)$$

Sketch the root locus of G*K

Find the point which intersects the damping line



z = 0.8015 + j0.2665

Method #2: For the same reasons, let

$$K(s) = \left(\frac{s+2.181}{s+21.81}\right)$$

$$K(z) = \left(\frac{z-0.8040}{z-0.1129}\right)$$

Find the point on the root locus that intersects the damping line

$$ang\left[\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\cdot\left(\frac{z-0.8040}{z-0.1129}\right)\right]_{s=-a+j2a} = 180^0$$

results in

s = -1.4129 + j2.8258

z = 0.8338 + j0.2421

This gives

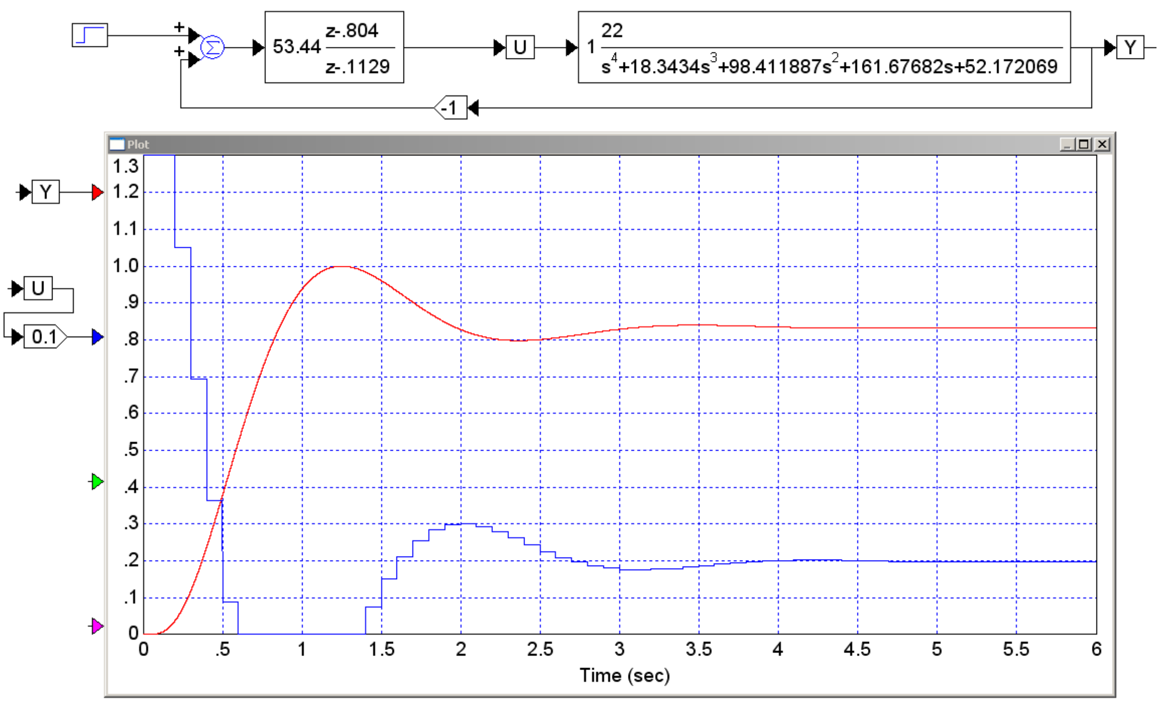
$$\left(\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\cdot\left(\frac{z-0.8040}{z-0.1129}\right)\right)_s = -0.0187$$

$$k = \frac{1}{0.0187} = 53.44$$

and

$$K(z) = 53.44\left(\frac{z-0.8040}{z-0.1129}\right)$$

Checking in VisSim



3) I Compensation: Design a digital compensator

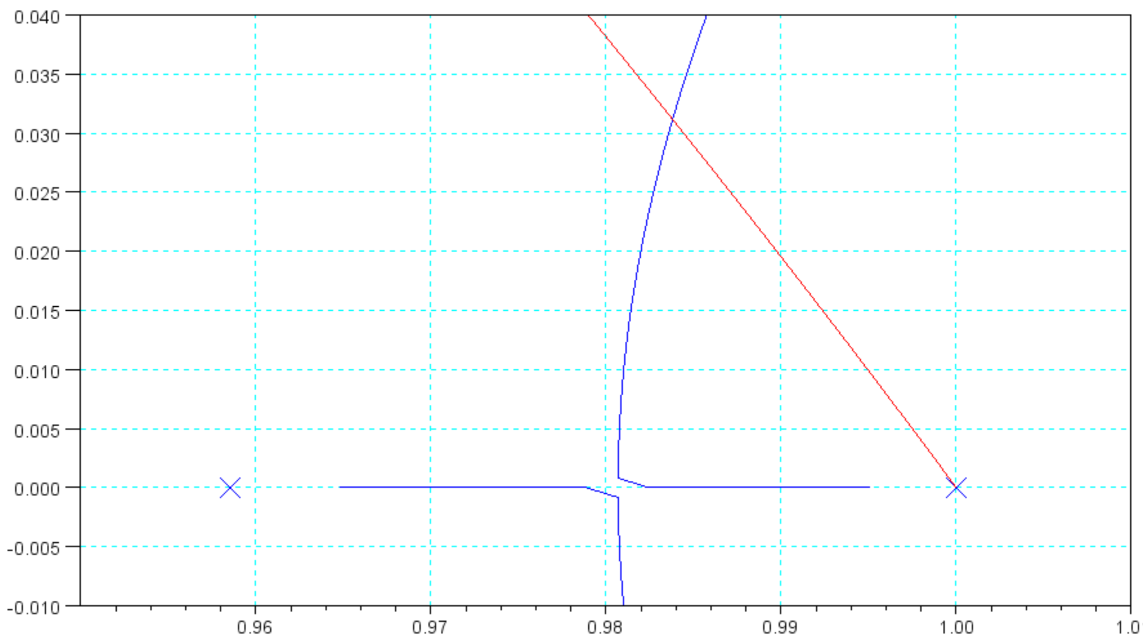
$$K(z) = k\left(\frac{z}{z-1}\right)$$

which results in 20% overshoot for a step input. For this value of K(z), give

- The resulting closed-loop dominant pole(s)
- The step response of the closed-loop system, and

Method #1: Draw the root locus of G(z)*K(z)

$$\left(\frac{0.000933z^2}{(z-0.9585)(z-0.8040)(z-0.5747)(z-0.3606)}\right)\left(\frac{z}{z-1}\right)$$



Find the point that intersects the damping line

At this point, make G*K = -1

Method #2: Solve numerically. Search along the damping line untin the angle is 180 degrees:

$$ang\left[\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\cdot\left(\frac{z}{z-1}\right)\right]_{s=-a+j2a} = 180^0$$

This gives

s = -0.1565 + j0.3129

z = 0.9840 + j0.0308

At this point, GK = -1

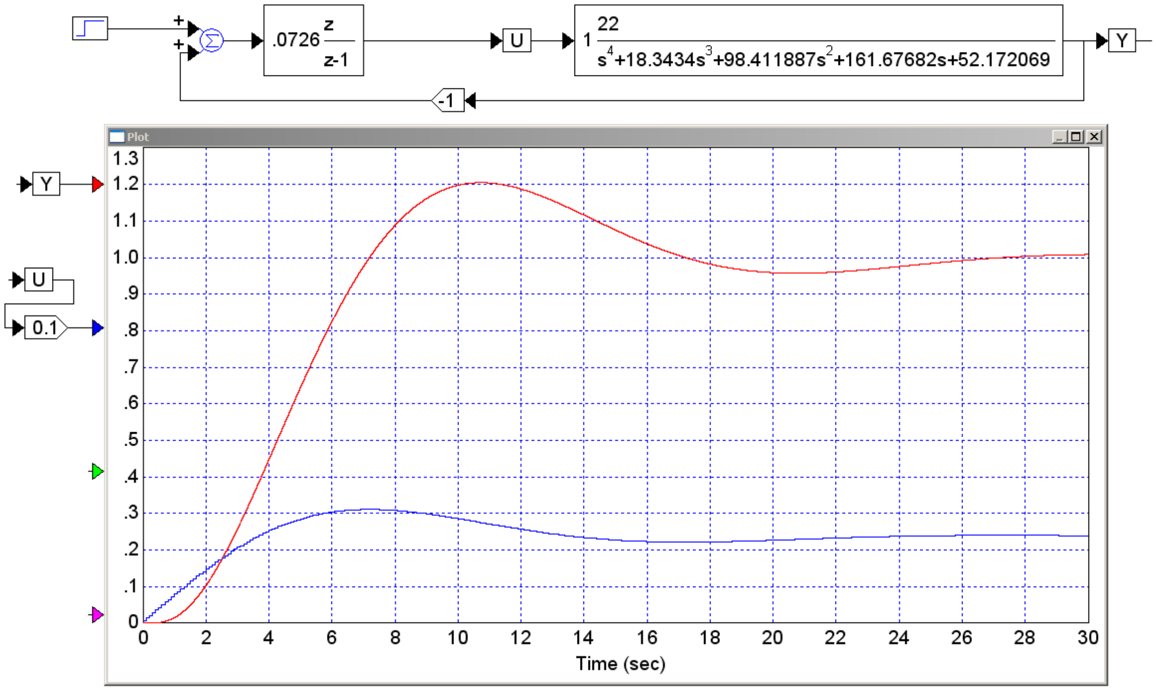
$$\left[\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\cdot\left(\frac{z}{z-1}\right)\right]_{s} = -13.774$$

$$k = \frac{1}{13.774} = 0.0726$$

and

$$K(z) = 0.0726\left(\frac{z}{z-1}\right)$$

Checking in VisSim



4) PI Compensator: Designa digital compensator

$$K(z) = k\left(\frac{z-a}{z-1}\right)$$

which results in 20% overshoot for a step input. For this value of K(z), give

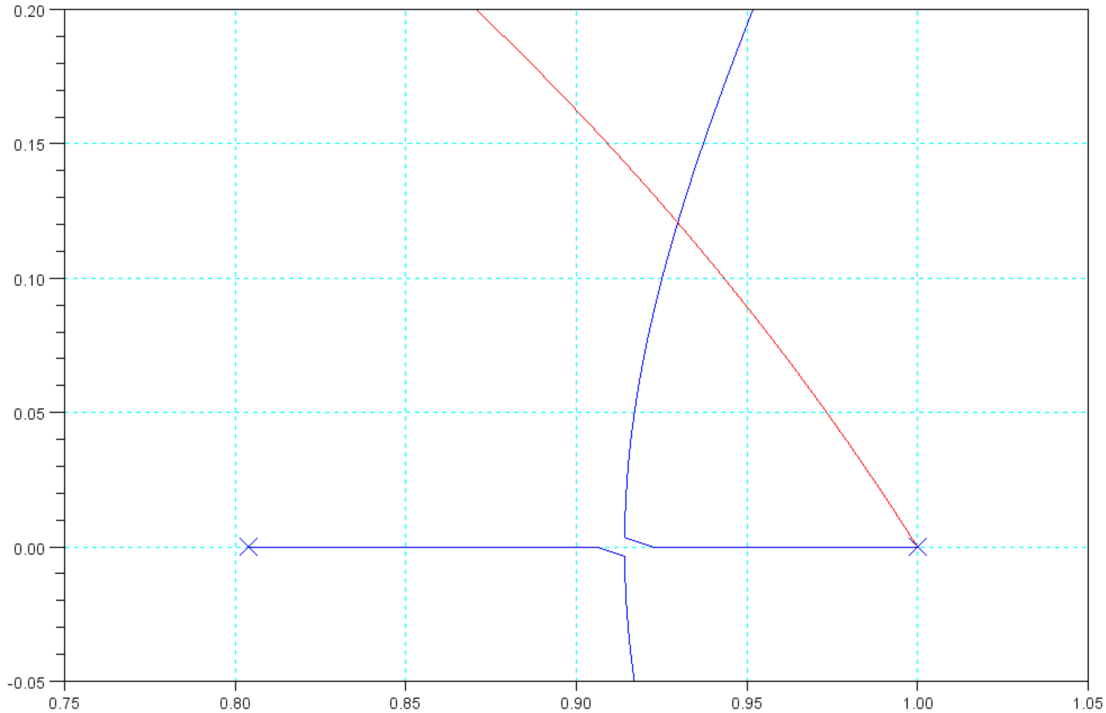
- The resulting closed-loop dominant pole(s)
- The step response of the closed-loop system, and

Let 'a' cancel the slowest stable pole

$$K(z) = k\left(\frac{z-0.9585}{z-1}\right)$$

The root locus of G(z) K(z) is then

$$\left(\frac{0.000933z^2}{(z-0.9585)(z-0.8040)(z-0.5747)(z-0.3606)}\right)\left(\frac{z}{z-1}\right)$$



or... solve numerically

$$ang\left[\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)}\right)\cdot e^{-0.05s}\cdot\left(\frac{z-0.9585}{z-1}\right)\right]_{s=-a+j2a} = 180^0$$

resulting in

s = -0.6191 + j1.2381

z = 0.9328 + j0.1161

At this point

$$GK = -0.1643$$

so

$$k = \frac{1}{0.1643} = 6.0881$$

and

$$K(z) = 6.0881\left(\frac{z-0.9585}{z-1}\right)$$