ECE 461/661 - Homework Set #12

Root Locus in the z-plane - Due Wednesday, December 7th

1) Determine the corresponding values (revised)

Dominant Pole	Damping Ratio	Resonance Mm	Phase Margin	0dB Gain Freq
-2 + j9	**0.2169**	**2.3611**	**24.45 deg**	**9 rad/sec**
-3.14 + j10	0.3	**1.74**	**33.4 deg**	10 rad/sec
-3.55 + j10	**0.3348**	+4dB (1.5849)	**36.78 deg**	10 rad/sec
-4.18 + j15	**0.2687**	**1.9319**	30 degrees	15 rad/sec

The 0dB gain frequency is the complex part of the dominant pole

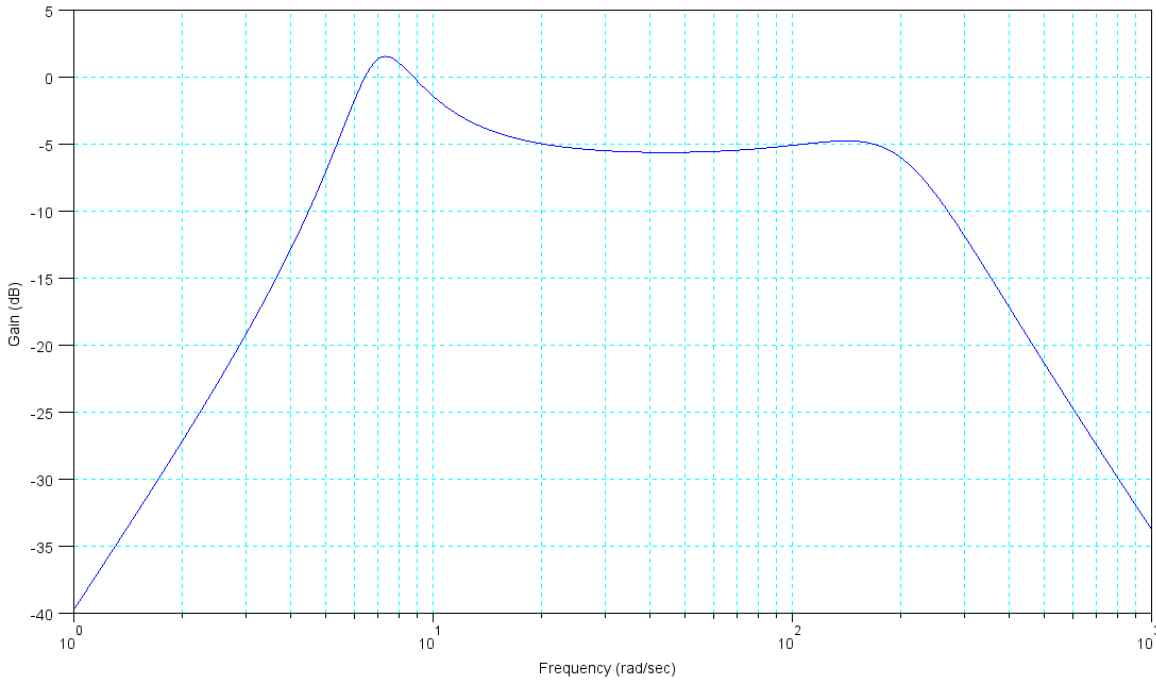
$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$M_m = \left|\frac{1\angle\phi}{1+1\angle\phi}\right|$$

$$\frac{1}{M_m^2} = 2 + 2\cos\phi$$

Phase Margin = $180^0 - |\phi|$

2) Determine the system that has the following gain vs. frequency



Start increasing +40dB / decade

two zeros at s = 0

The gain drops 40bD / decade at 8 rad/sec

two poles at 8 rad/sec

The gain at the corner is +6dB (2)

$$\frac{1}{2\zeta} = 2$$

$$\zeta = 0.25, \qquad \theta = 75.5^0$$

The gain drops another 40dB/decade at 180 rad/sec

Two more poles at 180 rad/sec

The gain at the corder is +0dB (1)

$$\frac{1}{2\zeta} = 1$$

$$\zeta = 0.5, \qquad \theta = 60^0$$

The gain at 20 rad/sec is -6dB

$$G(s) \approx \left(\frac{16{,}200 \cdot s^2}{\left(s+8\angle 75.5^0\right)\left(s+8\angle -75.5^0\right)\left(s+180\angle 60^0\right)\left(s+180\angle -60^0\right)} \right)$$

A 4th-order model for the 10-stage RC filter from homework #6 is

$$G(s) \approx \left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)} \right)$$

3) Gain Compensation: Design a gain compensator so that the closed-loop system has

- A phase margin of 40 degrees.

The phase margin means that at some frequency

$$GK = 1\angle -140^0$$

Search along the line s = jw until the angles add up to -40 degrees.

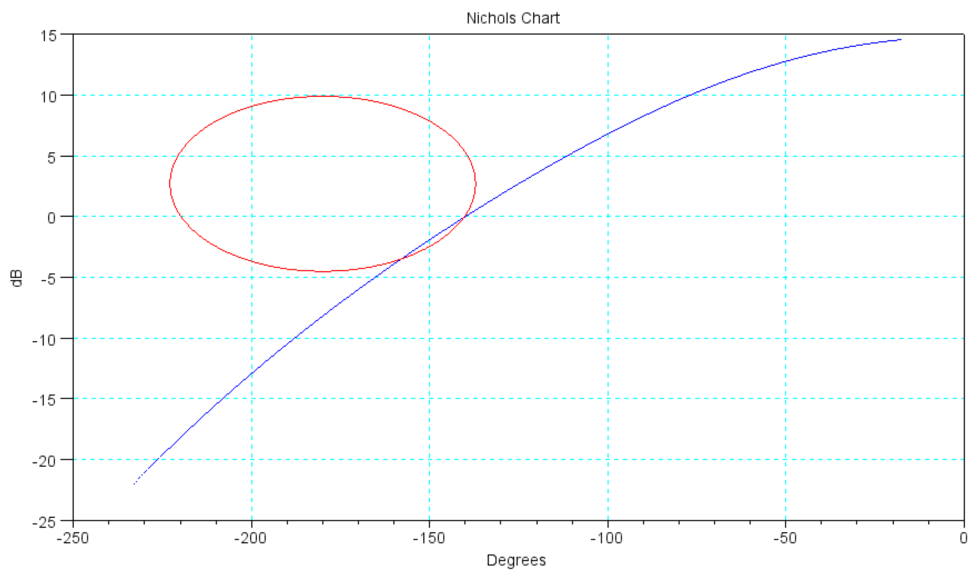
$$\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)} \right)_{s=j1.6878} = 0.0766\angle -140^0$$

Pick 'k' to make the gain one at this frequency

$$k = \frac{1}{0.0766} = 13.058$$

Sidelight: At this point, you're done. To see what's going on, plot Gk on a Nichols chart. The m-circle is

$$M_m = \left| \frac{1\angle -140^0}{1+1\angle -140^0} \right| = 1.4698$$



By designing for a phase margin, you almost got a resonance of Mm = 1.4698 (GK is almost tangent to the M-circle). The gain is a little too large - you intersect the M-circle rather than being tangent to it.

4) (take 1) Design a PI compensator so that the closed-loop system has

- No error for a step input, and
- A phase margin of 40 degrees.
- ~~A 0dB gain frequency of 3 rad/sec~~

Fora PI

- Add a pole at s = 0 (making the system type-1)
- Add a zero at -0.4234 to cancel the slowest stable pole

$$K(s) = k\left(\frac{s+0.4234}{s}\right)$$

So now,

$$GK \approx \left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)s}\right)$$

For a 40 degree phase margin, at some frequency

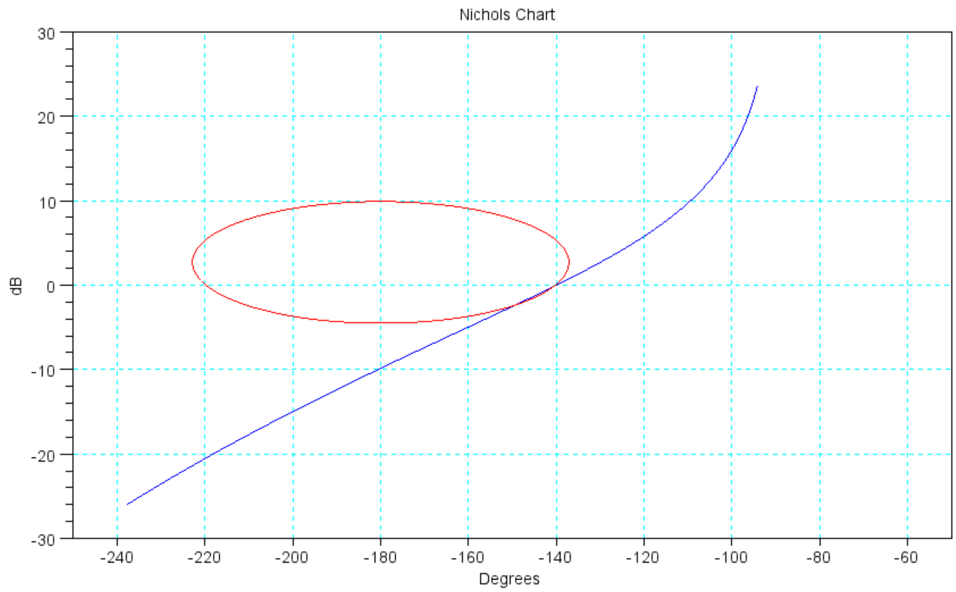
$$GK(j\omega) = 1\angle -140^0$$

$$\left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)s}\right)_{s=j1.2636} = 0.118\angle -140^0$$

$$k = \frac{1}{0.118} = 8.4539$$

$$\boxed{K(s) = 8.4539\left(\frac{s+0.4234}{s}\right)}$$

At this point, you're done. Just for fun, plot GK on a Nichols chart



By designing for a phase margin, you almost got a resonance of Mm = 1.4698 (GK is almost tangent to the M-circle). The gain is a little too large - you intersect the M-circle rather than being tangent to it.

4) (take 2) Design a PI compensator so that the closed-loop system has

- No error for a step input, and
- A phase margin of 40 degrees.
- A 0dB gain frequency of 3 rad/sec

$$G(s) \approx \left(\frac{22}{(s+10.2)(s+5.539)(s+2.181)(s+0.4234)} \right)$$

Add a pole at s = 0. Cancel two poles with two zeros

$$K(s) = \left(\frac{(s+0.4234)(s+2.181)}{s(s+a)} \right)$$

Pick 'a' so that

$$GK(j3) = 1\angle - 140^0$$

Taking the terms you know:

$$GK(j3) = \left(\frac{22}{(s+10.2)(s+5.539)(s)} \right)_{s=j3} = 0.1095\angle - 134.8^0$$

For the angle to add up to -140 degrees

$$\angle(s+a) = -5.1699^0$$

$$a = \frac{3}{\tan(5.1699^0)} = 33.1577$$

and

$$K(s) = \left(\frac{(s+0.4234)(s+2.181)}{s(s+33.1577)} \right)$$

Evaluating GK(j3)

$$GK(j3) = \left(\frac{22}{s(s+5.539)(s+10.2)(s+33.1577)} \right)_{s=j3} = 0.0033\angle - 140^0$$

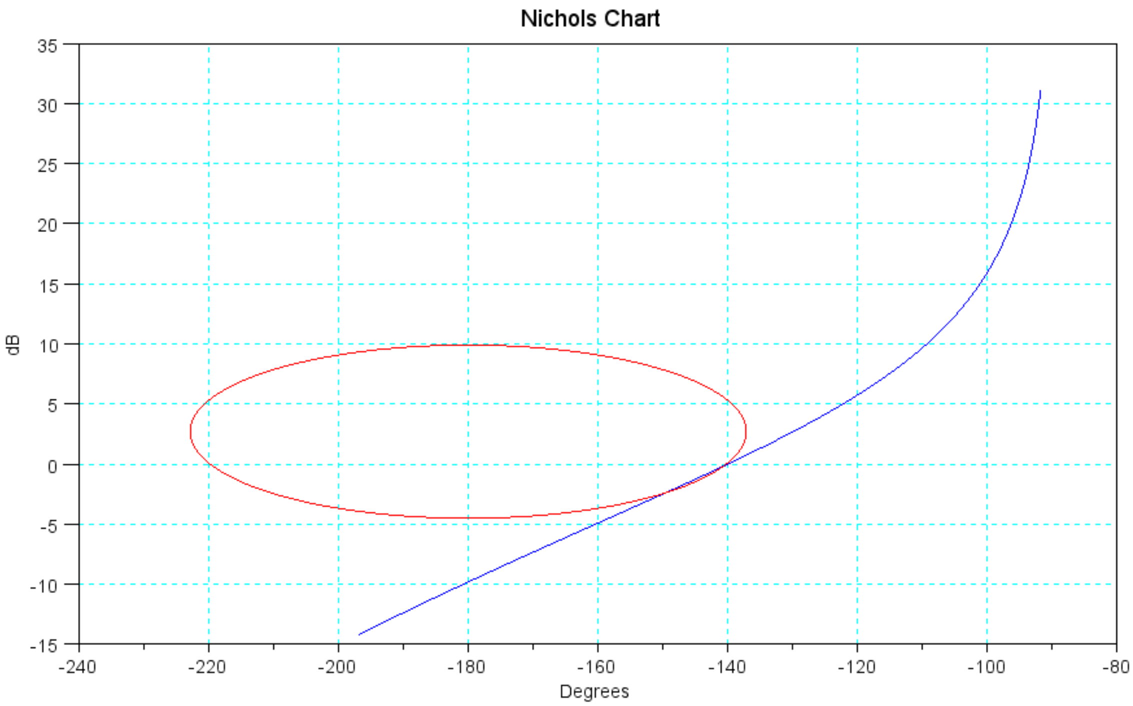
The angle is right (good) but the gain is off.

$$k = \frac{1}{0.0033} = 304.06$$

so

$$K(s) = 304.06\left(\frac{(s+0.4234)(s+2.181)}{s(s+33.1577)} \right)$$

Sidelight: Plot this again on a Nichols chart shows that you are almost tangent to the M-circle (as desired). The resonance is slightly more than it should be since you intersect the M-circle at 0dB / -140 degrees



Nichols Chart
35
30
25
20
15
10
5
0
-5
-10
-15
dB
-240
-220
-200
-180
-160
-140
-120
-100
-80
Degrees