Homework #7: ECE 461

Error Constants, Routh Criteria, Sketching a Root Locus. Due Monday, October 16th

Error Constants

1) Determine the system type, error constant, and the steady-state error a closed loop system would have for the following systems:

System G(s)	System Type	Kp	Kv	Steady-State Error for a Step Input
$\left(\frac{10}{(s+1)(s+5)}\right)$	**0**	**2**	**0**	**0.33**
$\left(\frac{10}{(s-1)(s+5)}\right)$	**0**	**-2**	**0**	**-1**
$\left(\frac{10}{s(s+1)(s+5)}\right)$	**1**	**inf**	**2**	**0**
$\left(\frac{10}{s^2(s+1)(s+5)}\right)$	**2**	**inf**	**inf**	**0**

Routh Criteria

Determine the range of k for

- The following polynomials to be negative definite
- The closed-loop system $\left(\frac{Gk}{1+Gk}\right)$ to be stable (same thing)

2) $$(s+1)(s+5)(s+10)+2k=0$$

$$G(s)=\left(\frac{2k}{(s+1)(s+5)(s+10)}\right)$$

$$s^3+16s^2+65s+50+2k=0$$

1	65	0	
16	50+2k	0	
$\frac{-\begin{vmatrix} 1 & 65 \\ 16 & 50+2k \end{vmatrix}}{16} = 61.875-0.125k$	0	0	k < 495
50+2k	0	0	k > -25
0	0	0	

result:

-25 < k < 495

3) $(s-1)(s+5)(s+10)(s+20)+2k(s+4)=0$

$$G(s)=\left(\frac{2k(s+4)}{(s-1)(s+5)(s+10)(s+20)}\right)$$

Multiplying it out

$$s^4+34s^3+315s^2+650s-1000+2k(s+4)=0$$

Grouping terms

$$s^4+34s^3+315s^2+(650+2k)s+(8k-1000)=0$$

1	315	8k-1000	
34	650+2k	0	
$\frac{-\begin{vmatrix}1 & 315\\ 34 & 650+2k\end{vmatrix}}{34}=295.88-0.0588k$	8k-1000	0	k < 5030
$\frac{-\begin{vmatrix}34 & 650+2k\\ 295.88-0.0588k & 8k-1000\end{vmatrix}}{295.88-0.0588k}=$ $-0.1176k^2+281.5294k+226323$	0	0	-635 < k < 3029
8k - 1000	0	0	k > 125
0	0	0	

Net result:

125 < k < 3029

4) $s^2(s+1)(s+5)+2k=0$

$$G(s)=\left(\frac{2k}{s^2(s+5)(s+10)}\right)$$

Multiplying it out

$$s^4+6s^3+5s^2+2k=0$$

1	5	2k
6	0	0
$\frac{-\begin{vmatrix}1 & 5\\ 6 & 0\end{vmatrix}}{6}=5$	$\frac{-\begin{vmatrix}1 & 2k\\ 6 & 0\end{vmatrix}}{6}=2k$	
$\frac{-\begin{vmatrix}6 & 0\\ 5 & 2k\end{vmatrix}}{5}=-2.4k$	$\frac{-\begin{vmatrix}6 & 0\\ 5 & 0\end{vmatrix}}{5}=0$	
$\frac{-\begin{vmatrix}5 & 2k\\ -2.4k & 0\end{vmatrix}}{-2.4k}=2k$	$\frac{-\begin{vmatrix}5 & 0\\ -2.4k & 0\end{vmatrix}}{-2.4k}=0$	
$\frac{-\begin{vmatrix}-2.4k & 0\\ 2k & 0\end{vmatrix}}{2k}=0$	$\frac{-\begin{vmatrix}-2.4k & 0\\ 2k & 0\end{vmatrix}}{2k}=0$	

k < 0

k > 0

ans

No Solution

Sketching a Root Locus

5) $G(s) = \left(\frac{2}{(s+1)(s+5)(s+10)}\right)$

The real axis loci

(-1, -5), (-10, -infinity)

The breakaway point(s)

$$\frac{d}{ds}((s+1)(s+5)(s+10)) = 0$$

s = **-2.7293**

The jw crossing(s)

$$\left(\frac{2}{(s+1)(s+5)(s+10)}\right)_{s=j\omega} = X\angle 180^0$$

$$G(j8.0623) = 0.0020\angle 180^0$$

$$j\omega = j8.0623$$

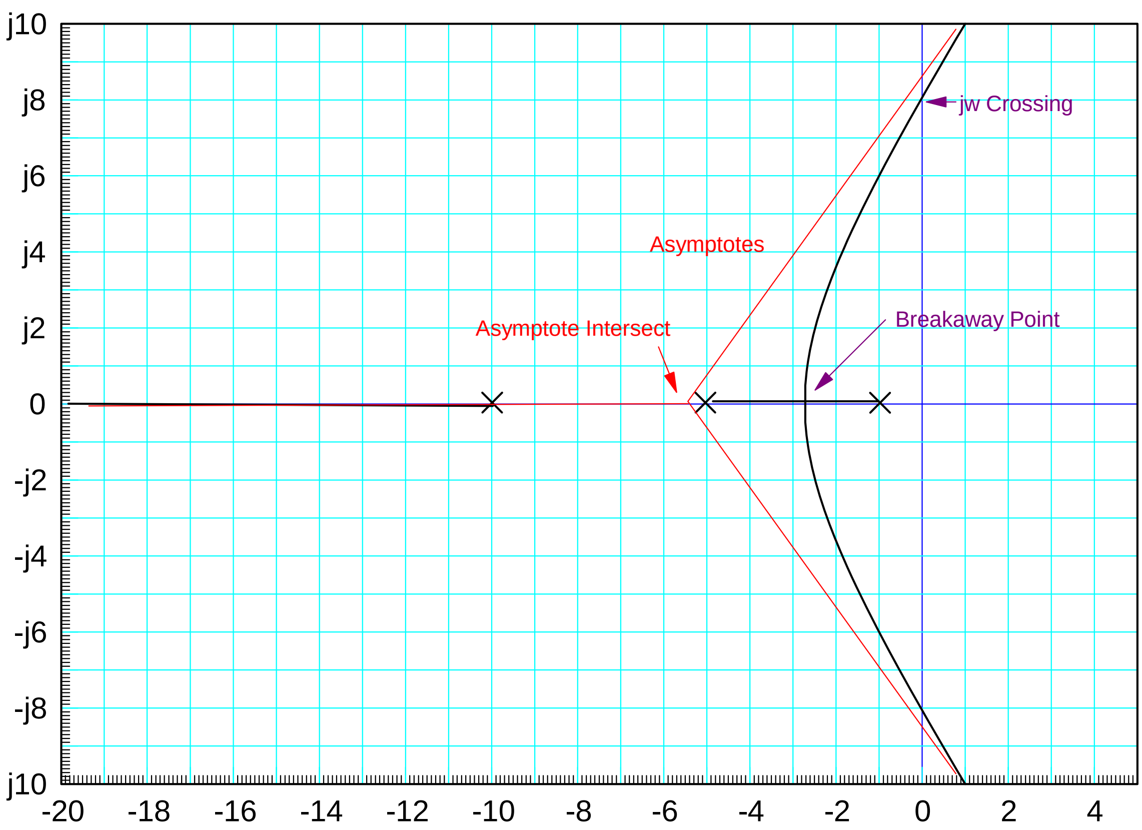
The departure / approach angle from complex poles / zeros

none

$$G(s) = \left(\frac{2}{(s+1)(s+5)(s+10)}\right)$$

Asymptiotes

- **3 (3 poles - 0 zeros)**
- **Angle: {+60 degrees, -60 degrees, 180 degrees)**
- **Intersect = -5.3333**



6) $$G(s) = \left(\frac{2(s+4)}{(s-1)(s+5)(s+10)(s+20)}\right)$$

The real axis loci

(+1, -4), (-5, -10), (-20, -infinity)

The breakaway point(s)

$$\frac{d}{ds}\left(\frac{(s-1)(s+5)(s+10)(s+20)}{2(s+4)}\right) = 0$$

s = -6.6077

The jw crossing(s)

G(jw) = X angle 180 degrees

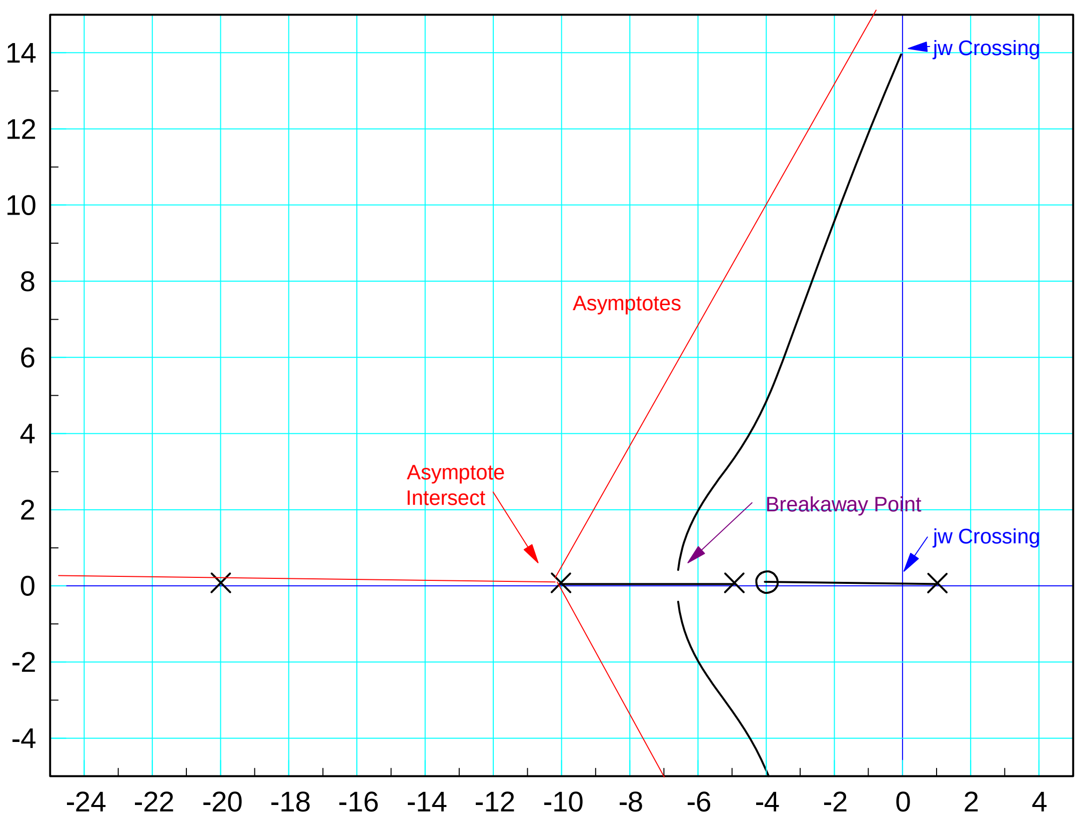
s = j14.0446

The departure / approach angle from complex poles / zeros

none

Asymptiotes

- **3 (4 poles - 1 zero)**
- **Angle: {+60 degrees, -60 degrees, 180 degrees)**
- **Intersect = -10**



7) $G(s) = \left(\frac{2}{s^2(s+5)(s+10)}\right)$

The real axis loci

(0, 0), (-5, -10)

The breakaway point(s)

s = { 0, -8.2030 }

The jw crossing(s)

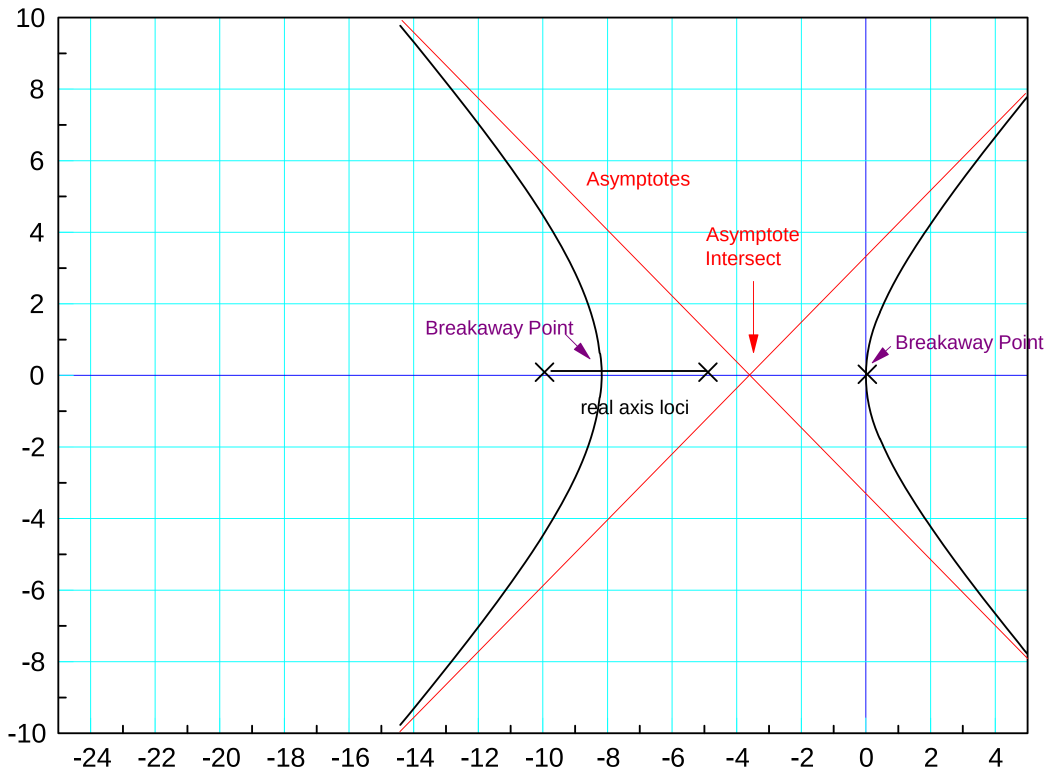
s = j0

The departure / approach angle from complex poles / zeros

none

Asymptiotes

- **4 (4 poles - 0 zeros)**
- **Angle: {±45 degrees, ±135 degrees)**
- **Intersect = -3.75**



8) $G(s) = \left(\frac{(s^2+4)}{s(s+4)(s+5)(s+10)} \right)$

The real axis loci

(0, -4), (-5, -10)

The breakaway point(s)

s = { -0.9342, -7.9345 }

The jw crossing(s)

none

The departure / approach angle from complex poles / zeros

$$\left(\left(\frac{(s+j2)}{s(s+4)(s+5)(s+10)} \right) (s-j2) \right)_{s=j2} = X\angle 180^0$$

$$\left(\frac{(s+j2)}{s(s+4)(s+5)(s+10)} \right)_{s=j2} = 0.0081\angle -59.68^0$$

$$\angle (s-j2) = -120.32^0$$

-120.32 degrees

Asymptiotes

- **2 (4 poles - 2 zeros)**
- **Angle: { ±90 degrees }**
- **Intersect = -9.50**