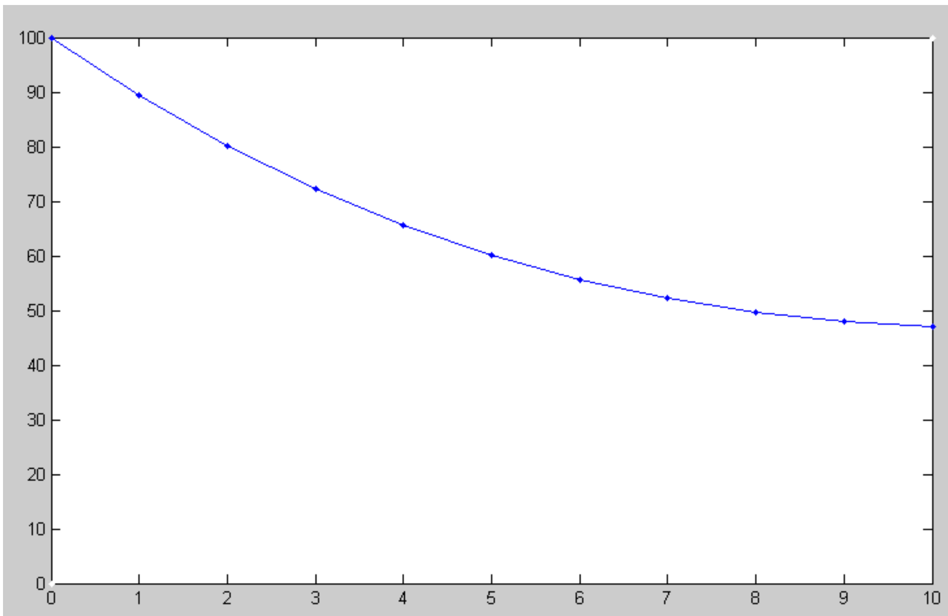
# Homework #8:  ECE 461

Gain, Lead, PID Compensation. Due Monday, October 30th

A 3rd-order approxiamtion for 10-stage heat equation from homework set #5 is

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right)$$



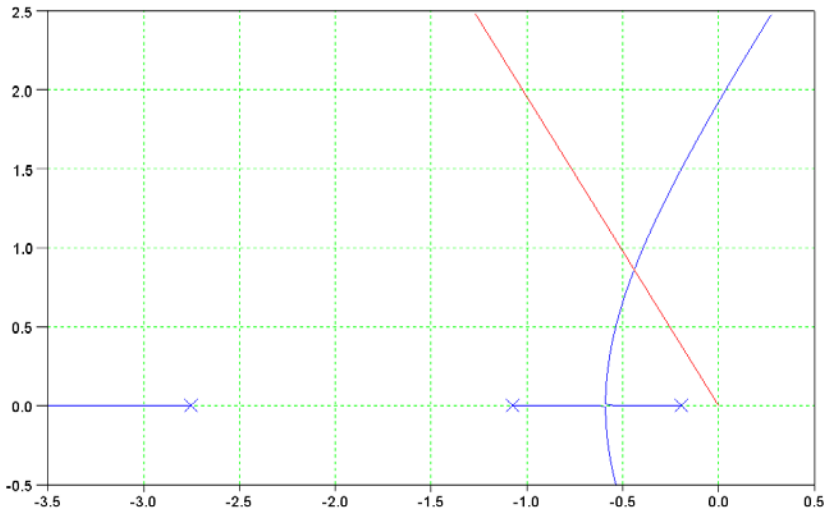
**Problem 1:** Gain Compensation: K(s) = k

a) Design a gain compensator which results in 20% overshoot for a step input.

$$GK = \left(\frac{0.2796k}{(s+0.195)(s+1.074)(s+2.753)}\right)$$

Sketch the root locus along with the damping line for 20% overshoot

$$\zeta = 0.4554$$




Find the point on the root locus where the damping ratio is 0.4554

s = -0.4366 + j0.8733

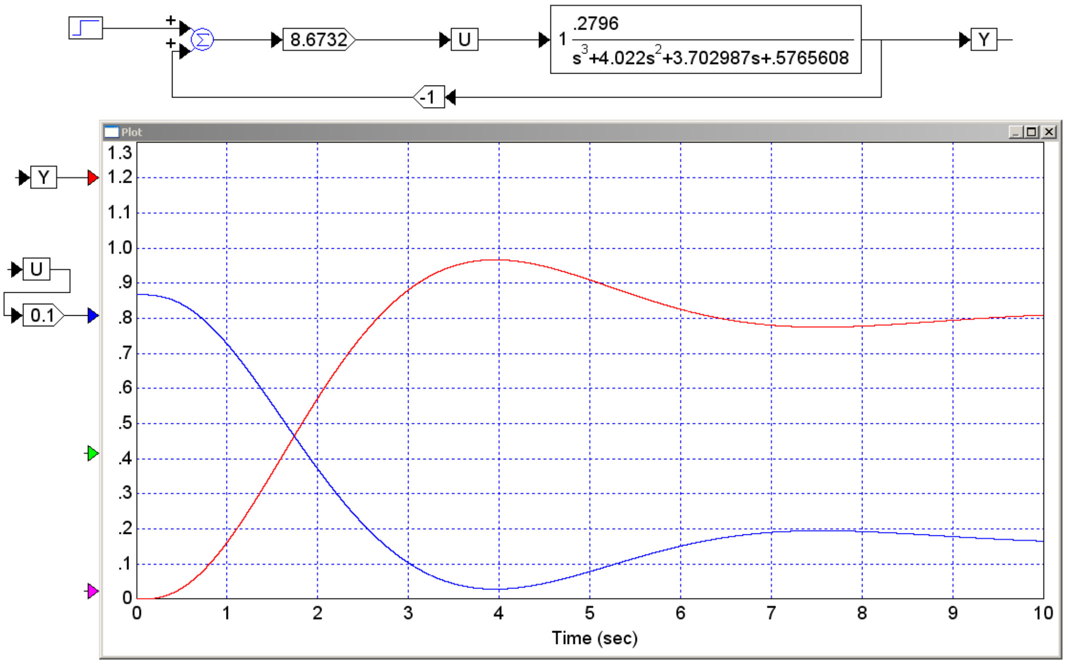
At this point, G*K = -1

$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right)_{s=-0.4366+j0.8733} = 0.1153\angle 180^0$$

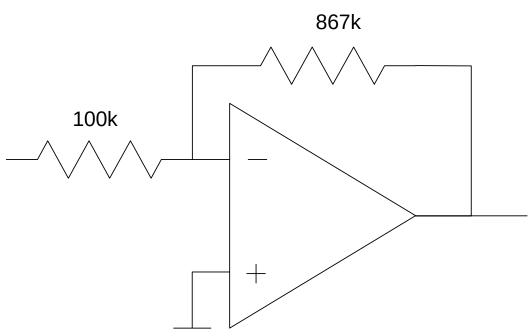
Therefore

$$k = \frac{1}{0.1153} = 8.6732$$

b) Verify your design by taking the step response of the closed-loop system using Matlab or VisSim (or like program)




c) Give a circuit to implement K(s)




d) Write a C program to implement your compensator (modify the program heat.m)

```
Error = REF - V(10);

V0 = 8.67*E
```

**Problem 2:** Lead Compensation: $K(s) = k\left(\frac{s+a}{s+10a}\right)$

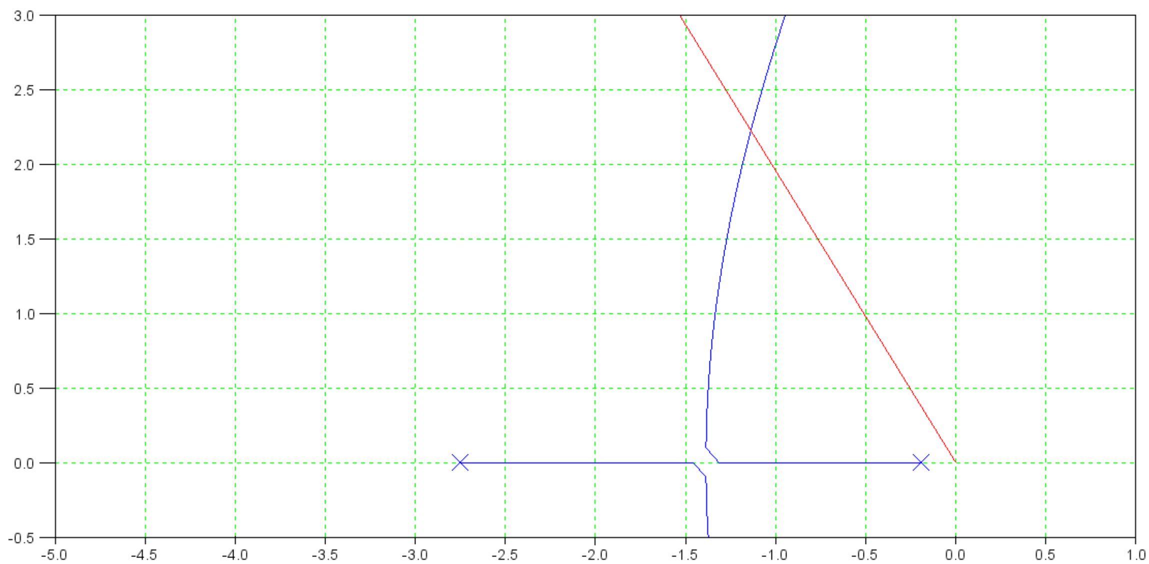
a) Design a lead compensator which results in 20% overshoot for a step input.

- Keep the pole at s = 0.195
- Cancel the pole at s = -1.074.
- Replace it with a pole 10x further out

$$K(s) = k\left(\frac{s+1.074}{s+10.74}\right)$$

$$GK = \left(\frac{0.2796k}{(s+0.195)(s+10.74)(s+2.753)}\right)$$

Sketch the root locus of GK




Find the spot on the root locus where you have 20% overshoot (zeta = 0.4554)

s = -1.1295 + j2.2591

At this point, pick k so that GK = -1

$$GK = \left(\frac{0.2796k}{(s+0.195)(s+10.74)(s+2.753)}\right)_{s=-1.1295+j2.2591} = 0.0042\angle 180^0$$

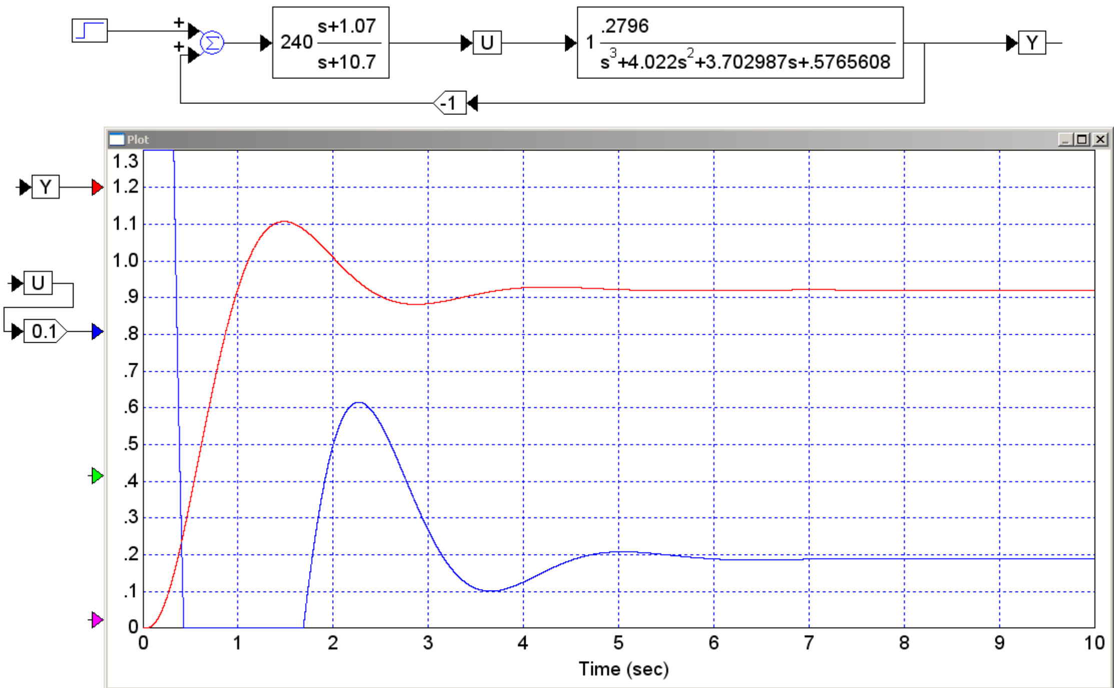
Therefore

$$k = \frac{1}{0.0042} = 240$$

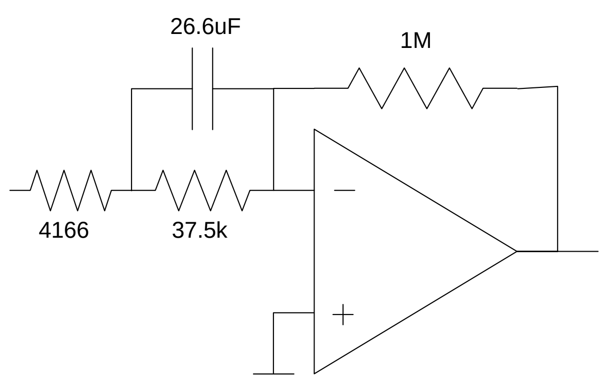
and

$$\boxed{K(s) = 240\left(\frac{s+1.074}{s+10.74}\right)}$$

b) Verify your design by taking the step response of the closed-loop system using Matlab or VisSim (or like program)




c) Give a circuit to implement K(s)




d) Write a C program to implement your compensator (modify the program heat.m)

$$K(s) = 240\left(\frac{s+1.074}{s+10.74}\right) = 20\left(\frac{s+10.74-9.666}{s+10.74}\right) = 20\left(1 - \frac{9.666}{s+10.74}\right)$$

Let

$$X = \left(\frac{9.666}{s+10.74}\right)E$$

sX = 9.666E - 10.74X

Code:

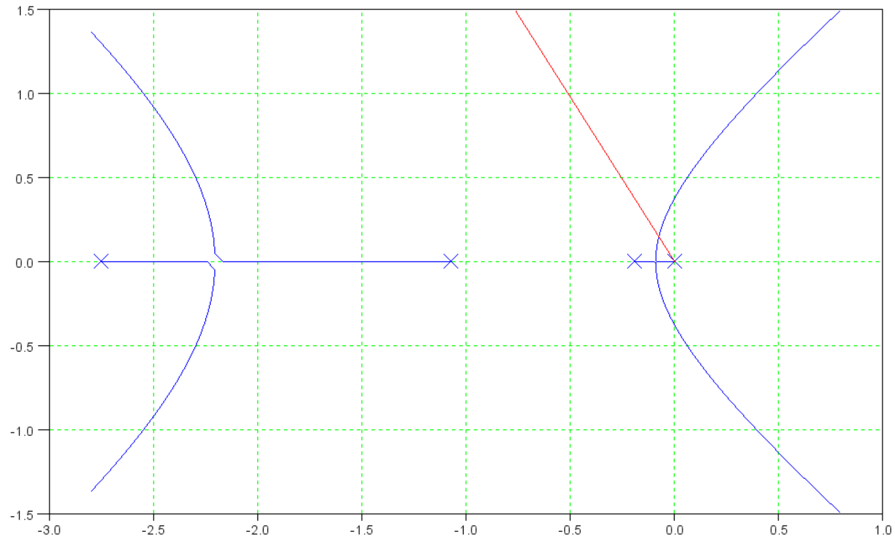
```
E = REF - V(10)
sX = 9.666*E - 10.74*X
X = X + dX * dt
V0 = 20*(1 - X);
```

**Problem 3:** Integral (I) Compensation: $K(s) = \left(\frac{k}{s}\right)$

a) Design an I compensator which results in 20% overshoot for a step input.

$$GK = \left(\frac{0.2796k}{s(s+0.195)(s+1.074)(s+2.753)}\right)$$

Sketch the root locus of GK




Find the spot on the root locus where the damping ratio is 0.4554 (20% overshoot)

$$s = -0.0755 + j0.1509$$

At this point, pick k so that GK = -1

$$\left(\frac{0.2796}{s(s+0.195)(s+1.074)(s+2.753)}\right)_{s=-0.0755+j0.1509} = 3.178\angle 180^0$$

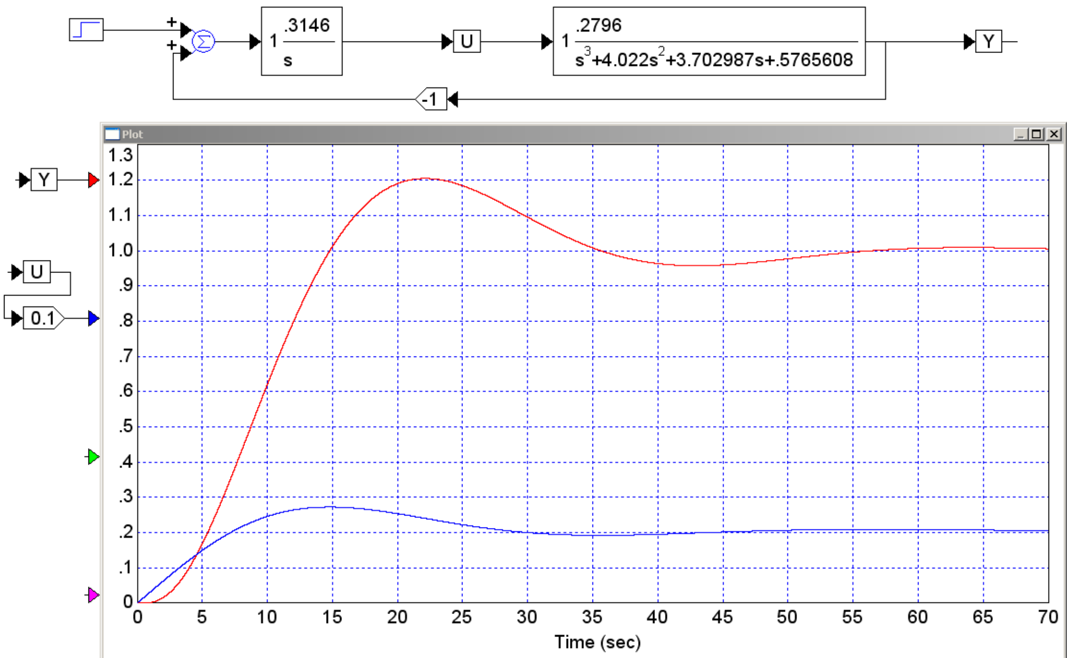
So

$$k = \frac{1}{3.178} = 0.3146$$

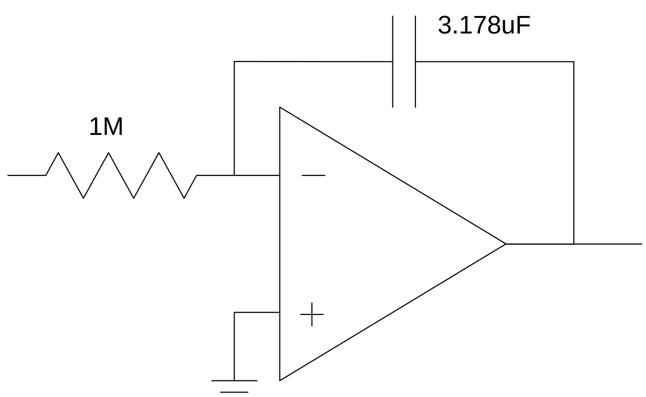
and

$$\boxed{K(s) = \left(\frac{0.3146}{s}\right)}$$

b) Verify your design by taking the step response of the closed-loop system using Matlab or VisSim (or like program)




c) Give a circuit to implement K(s)




d) Write a C program to implement your compensator (modify the program heat.m)

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E = REF - V(10);

dX = E;
X = X + dX * dt;

V0 = 0.3146 * X;
```

**Problem 4:** PI Compensation: $K(s) = P + \frac{I}{s} = k(\frac{s+a}{s})$

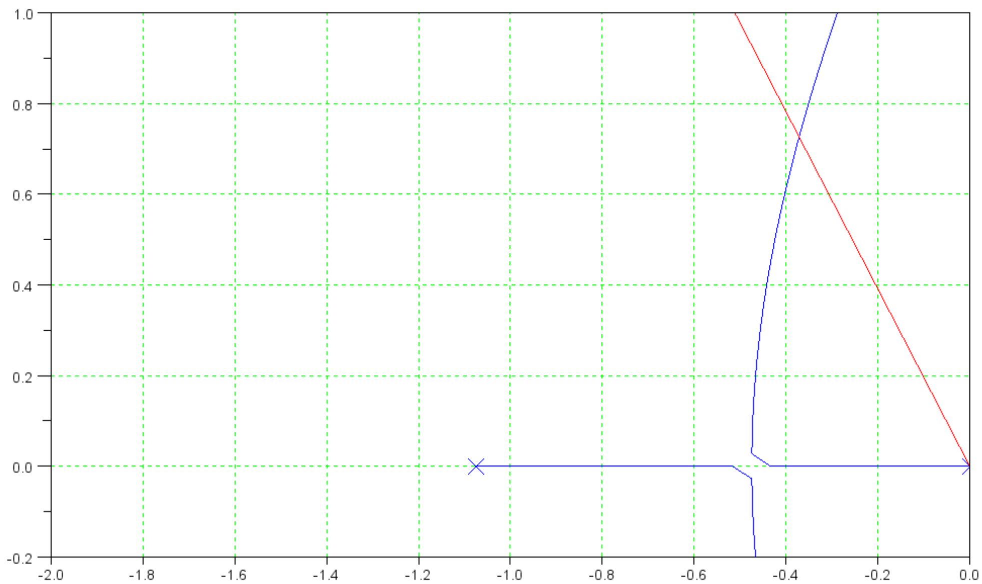
a) Design a PI compensator which results in 20% overshoot for a step input.

- Add a pole at s = 0
- Add a zero at 0.195 to cancel the slowest stable pole

$$K(s) = k\left(\frac{s+0.195}{s}\right)$$

$$GK = \left(\frac{0.2796k}{s(s+1.074)(s+2.753)}\right)$$

Sketch the root locus of GK




Find the spot on the root locus where the damping ratio is 0.4554

s = -0.3686 + j0.7371

Pick k so that GK = -1 at this point

$$\left(\frac{0.2796}{s(s+1.074)(s+2.753)}\right)_{s=-0.3686+j0.7371} = 0.1332\angle 180^0$$

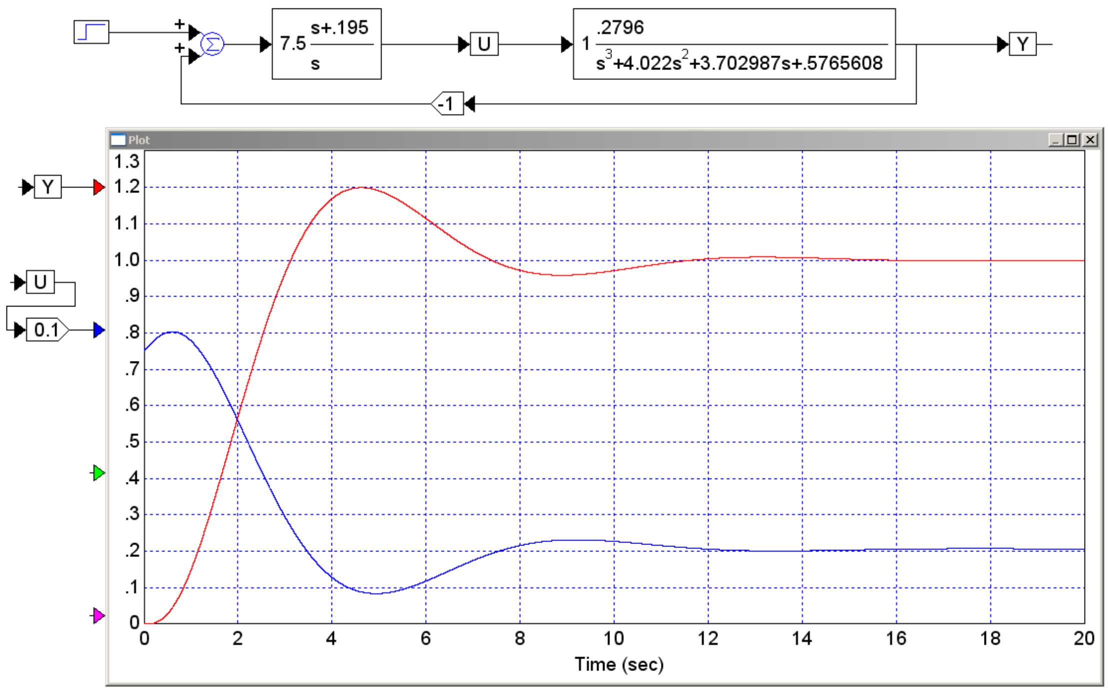
so

$$k = \frac{1}{0.1332} = 7.5054$$

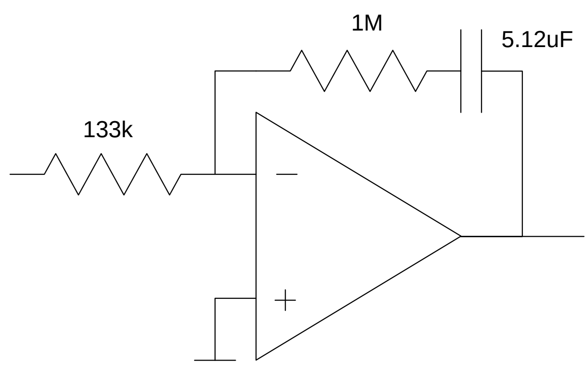
and

$$\boxed{K(s) = 7.5054\left(\frac{s+0.195}{s}\right)}$$

b) Verify your design by taking the step response of the closed-loop system using Matlab or VisSim (or like program)




c) Give a circuit to implement K(s)




d) Write a C program to implement your compensator (modify the program heat.m)

$$K(s) = 7.5\left(\frac{s+0.195}{s}\right) = 7.5 + \frac{1.46}{s}$$

```
E = REF - V(10);

dX = E;
X = X + dX * dt;

V0 = 7.5*E + 1.46*X;
```