Homework #9: ECE 461

Gain, Lead, PID Compensation. Due Monday, October 30th

Problem 1: Assume

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right)$$

a) Design a compensator, K(s), which results in

- No error for a step input
- A 2% settling time of 4 seconds, and
- 20% overshoot for a step input.

Pick K(s) as

$$K(s) = k\left(\frac{(s+0.195)(s+1.074)}{s(s+a)}\right)$$

This

- Adds a pole at s = 0 making the system type-1
- Cancels the two slow poles
- Adds a second pole so that s = -1 + j2 is on the root locus

To find 'a'

$$GK = \left(\frac{0.2796}{s(s+2.753)(s+a)}\right)$$

At s = -1 + j2

$$\left(\frac{0.2796}{s(s+2.753)(s+a)}\right)_{s=-1+j2} = 1\angle 180^0$$

Note that

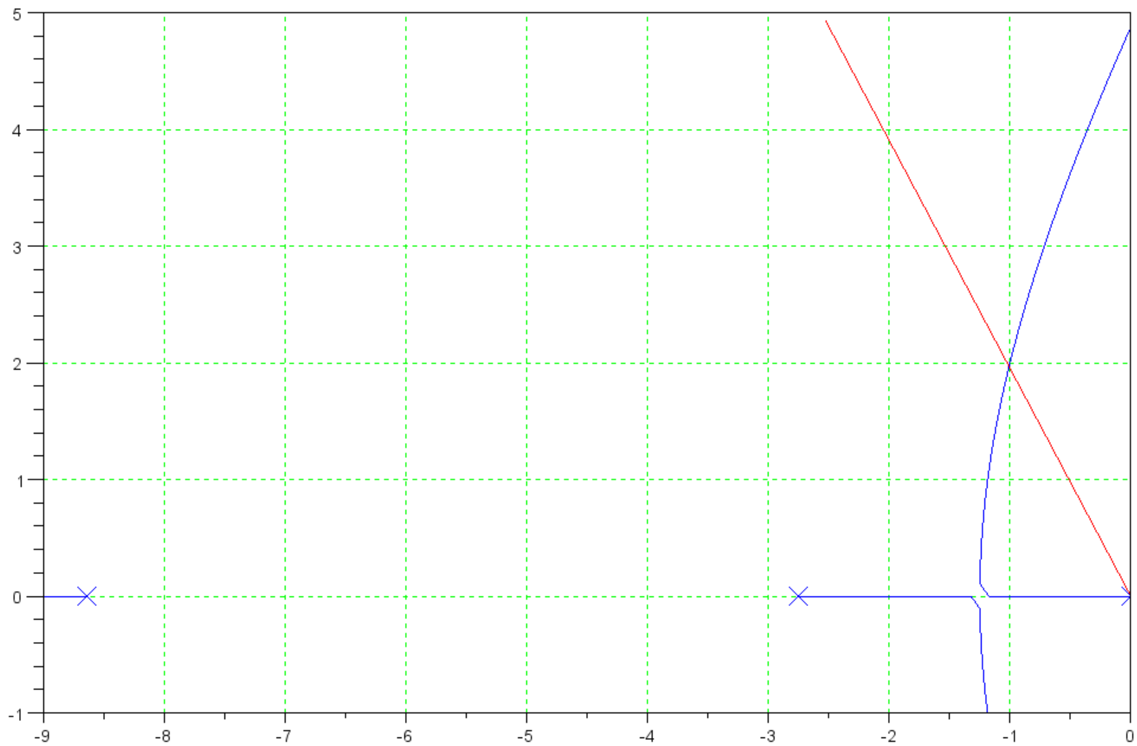
$$\left(\frac{0.2796}{s(s+2.753)}\right)_{s=-1+j2} = 0.0470\angle -165.33^0$$

To make the angles add up to 180 degrees

$$\angle(s+a) = 14.67^0$$

$$a = \frac{2}{ran(14.67^0)} + 1 = 8.64$$

This gives the following root locus (in case you're interested). Note that s = -1 + j2 is on the root locus.



To find k

$$GK = \left(\frac{0.2796}{s(s+2.753)(s+8.64)} \right)_{s=-1+j2} = 0.0060\angle 180^0$$

so

$$k = \frac{1}{0.0060} = 167.97$$

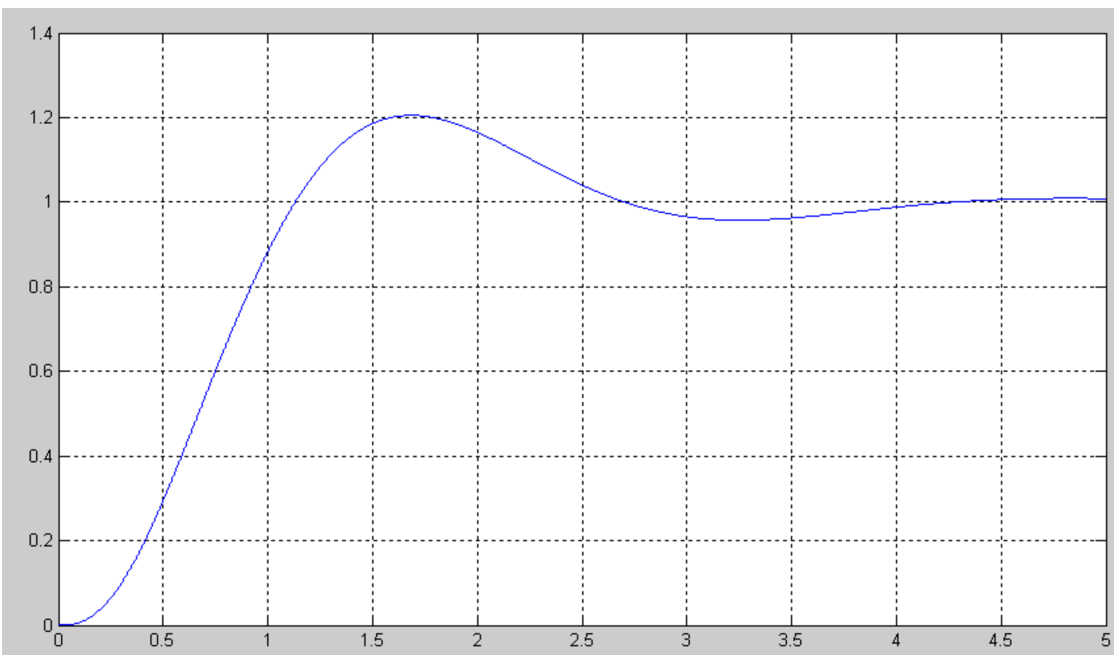
and

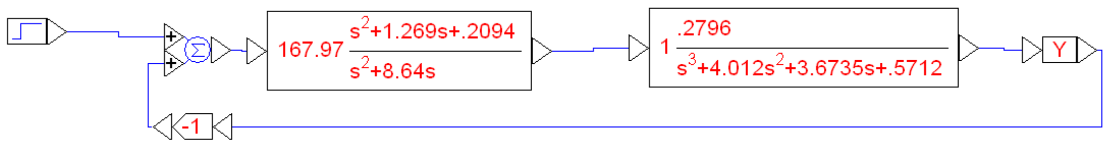
$$\boxed{K(s) = 167.97\left(\frac{(s+0.195)(s+1.074)}{s(s+8.64)} \right)}$$

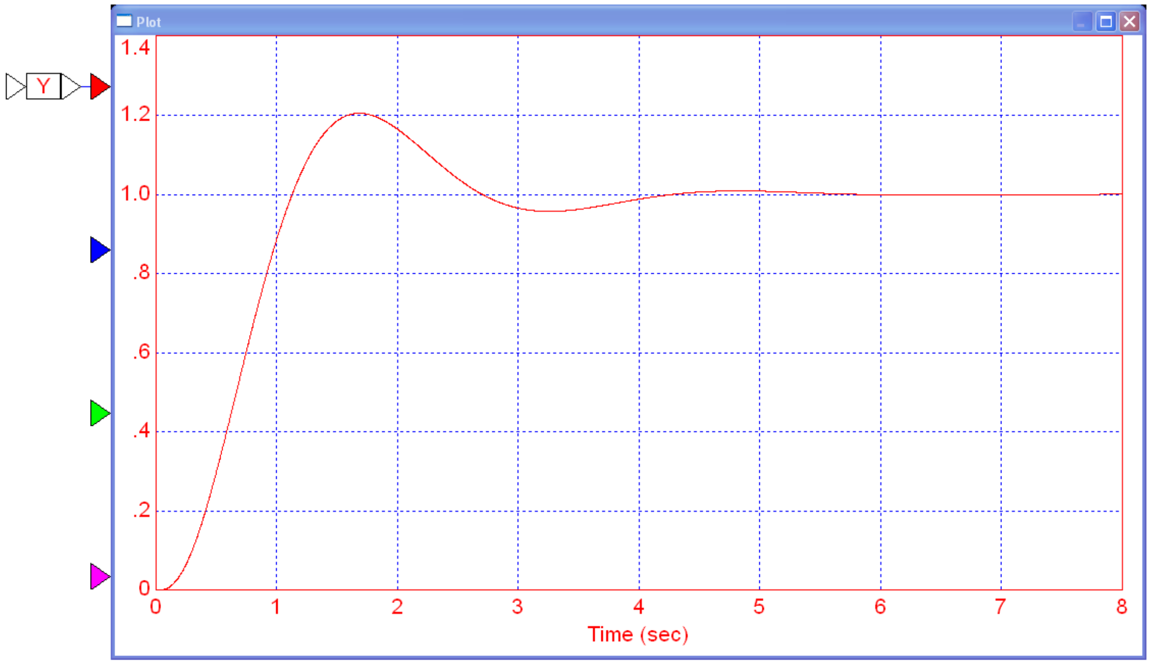
b) Find the step response of the closed-loop system using Matlab or VisSim (or similar program)

Matlab Code:

```
>> G = zpk([],[-0.195,-1.064,-2.753],0.2796)

           0.2796
-----------------------------
(s+0.195) (s+1.064) (s+2.753)

>> K = zpk([-0.195,-1.074],[0,-8.64],167.97)

167.97 (s+0.195) (s+1.074)
--------------------------
        s (s+8.64)

>> Gcl = minreal(G*K / (1+G*K))
>> eig(Gcl)

ans =

  -1.0781
  -0.9933 + 1.9987i
  -0.9933 - 1.9987i
  -9.3922

>> t = [0:0.001:5]';
>> y = step(Gcl,t);
>> plot(t,y);
>> grid on
```





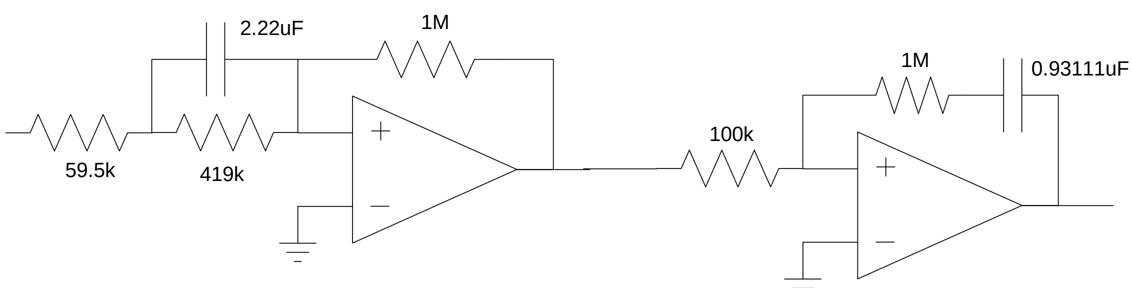


c) Give a circuit to implement K(s).

$$K(s) = 167.97\left(\frac{(s+0.195)(s+1.074)}{s(s+8.64)}\right)$$

Reqrite this as

$$K(s) = 10\left(\frac{(s+0.195)}{s}\right) \cdot 16.797\left(\frac{(s+1.074)}{(s+8.64)}\right)$$



Problem 2: A better model includes a delay to approximate the neglected poles. Assume

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.65s}$$

a) Design a compensator, K(s), which results in

- No error for a step input
- A 2% settling time of 10 seconds, and (change from 4 seconds)
- 20% overshoot for a step input.

Let

$$K(s) = k\left(\frac{(s+0.195)(s+1.074)}{s(s+a)}\right)$$

For =0.4 + j0.8 to be on the root locus

$$GK = \left(\left(\frac{0.2796}{s(s+a)(s+2.753)}\right) \cdot e^{-0.65s}\right)_{s=-0.4+j0.8} = 1\angle 180^0$$

Taking the part that we know

$$GK = \left(\left(\frac{0.2796}{s(s+2.753)}\right) \cdot e^{-0.65s}\right)_{s=-0.4+j0.8} = 0.1631\angle -165.14^0$$

meaning

$$\angle(s+a) = 14.86^0$$

$$a = \frac{0.8}{\tan(14.86^0)} + 0.4 = 3.4143$$

To find k

$$GK = \left(\left(\frac{0.2796}{s(s+3.4143)(s+2.753)}\right) \cdot e^{-0.65s}\right)_{s=-0.4+j0.8} = 0.0523\angle 180^0$$

$$k = \frac{1}{0.0523} = 19.1176$$

and

$$\boxed{K(s) = 19.1176\left(\frac{(s+0.195)(s+1.074)}{s(s+3.4143)}\right)}$$

In Matlab

Input the system

```
G = zpk([],[-0.195,-1.074,-2.753],0.2796);
```

Input the delay of 0.65 seconds. Use a 2nd-order Pade approximation

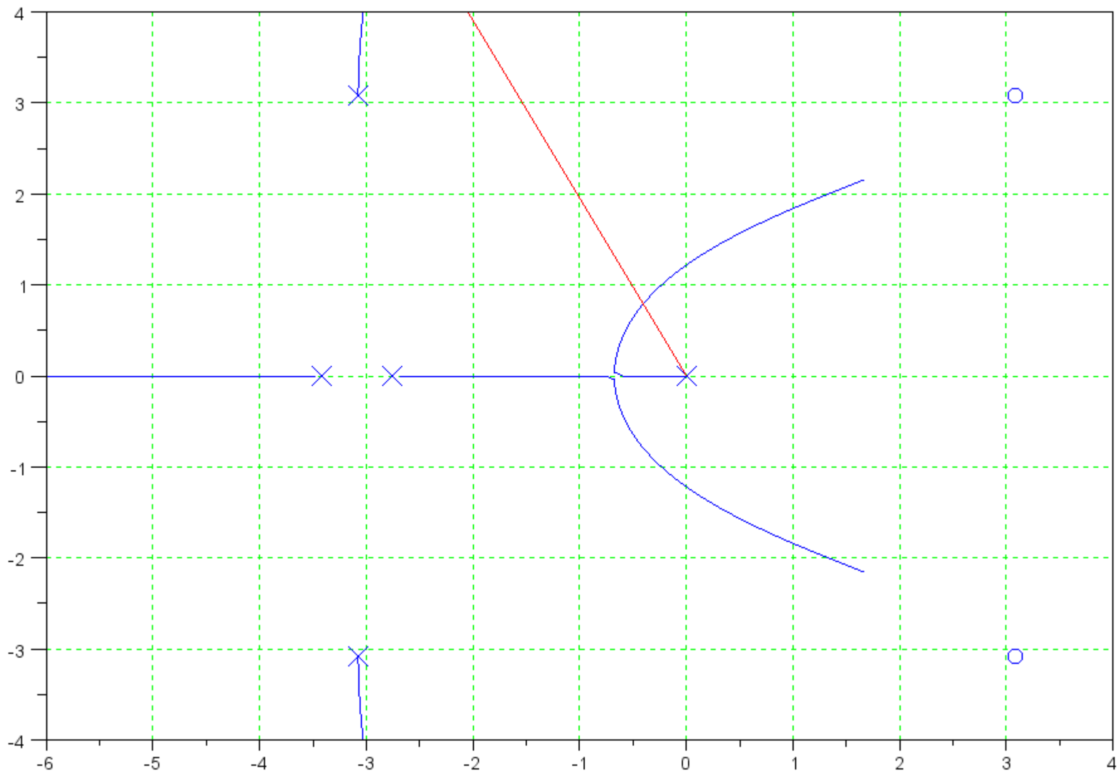
```
[num,den] = pade(0.65, 2);
D = tf(num,den);
```

Input the compensator

```
K = zpk([-0.195,-1.074],[0,-3.4143],1);
```

Draw the root locus just because they're fund to look at

```
k = logspace(-2,2,1000)';
rlocus(G*D*K, k);
```



Note that s = -0.4 + j0.8 is on the root locus

To find the gain at this point

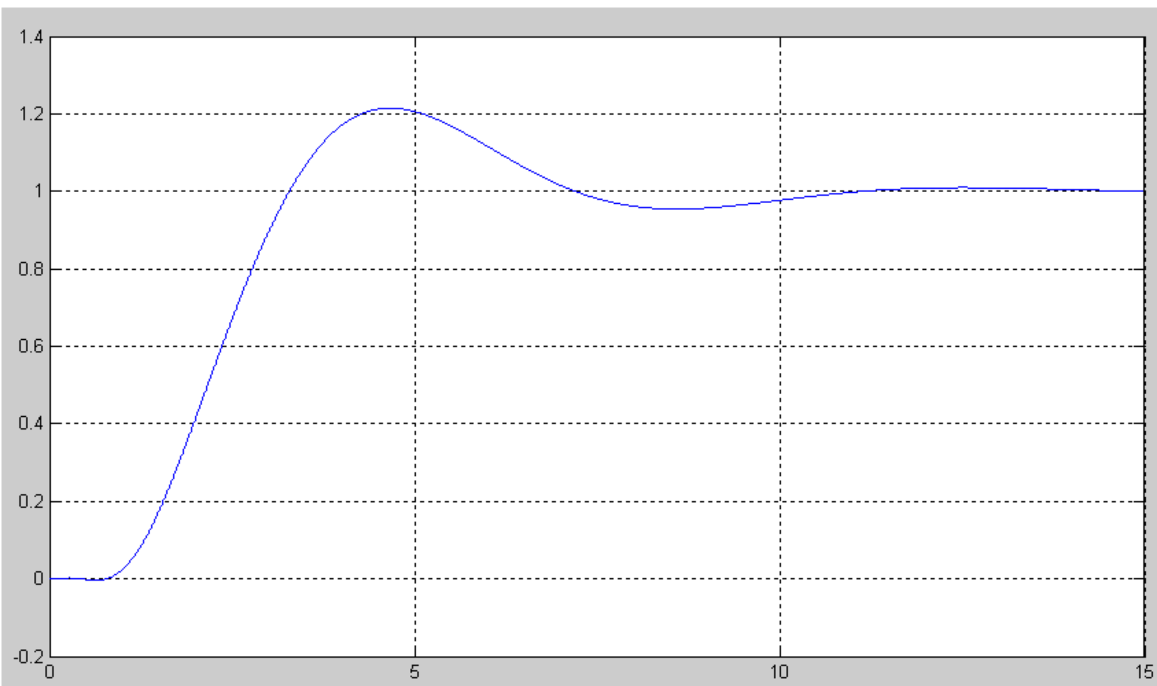
```
-->evalfr(GKD, -0.4 + j*0.8)
 ans  =

  - 0.0527228 + 0.0000851i

-->1/abs(ans)
 ans  =

    18.9671
```

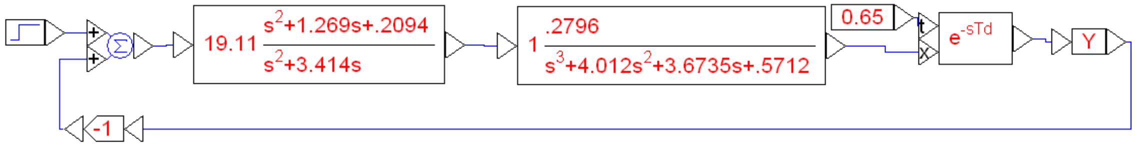
It's a little off due to using a 2nd-order Pade approximation for a delay. If you use a 4th-order or 6th-order, you get closer to the previous number of 19.1176

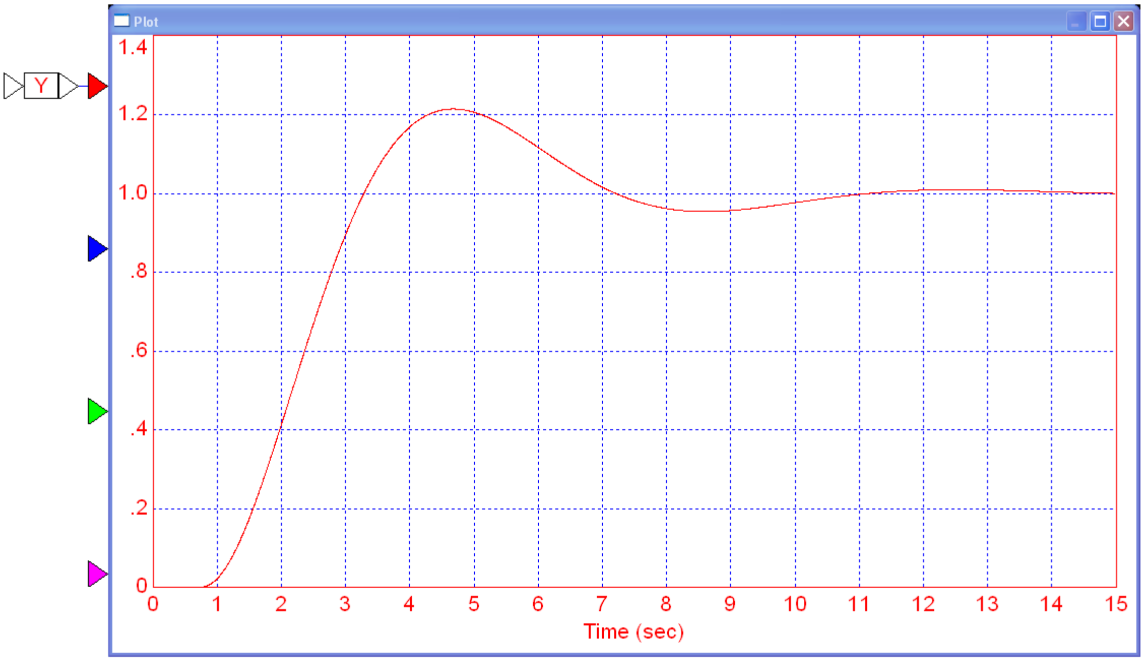
b) Find the step response of the closed-loop system using Matlab or VisSim (or similar program)

In Matlab

```
>> G = zpk([],[-0.195,-1.064,-2.753],0.2796);
>> [num,den] = pade(0.65,2);
>> D = tf(num,den);
>> K = zpk([-0.195,-1.074],[0,-3.4143],19.1176)

19.1176 (s+0.195) (s+1.074)
---------------------------
        s (s+3.414)

>> Gcl = minreal( G*D*K / (1 + G*D*K) );
>> eig(Gcl)

  -0.3936 + 0.8005i
  -1.0804
  -0.3936 - 0.8005i
  -4.1211 + 3.5877i
  -6.3523
  -4.1211 - 3.5877i

>> t = [0:0.001:15]';
>> y = step(Gcl,t);
>> plot(t,y);
>> grid on
```

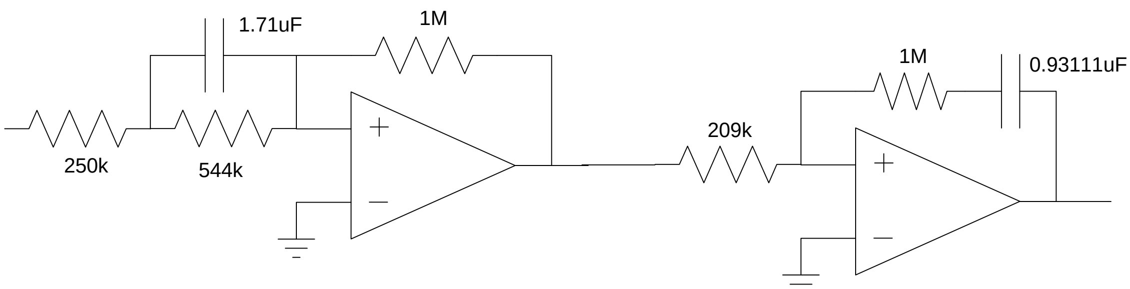






c) Give a circuit to implement K(s).

$$K(s) = 19.1176\left(\frac{(s+0.195)(s+1.074)}{s(s+3.4143)}\right) = 4\left(\frac{(s+1.074)}{(s+3.4143)}\right) \cdot 4.7794\left(\frac{(s+0.195)}{s}\right)$$



Problem 3: A gantry system has the following dynamics:

$$\theta = \left(\frac{0.5}{s^2+14.7}\right) X$$

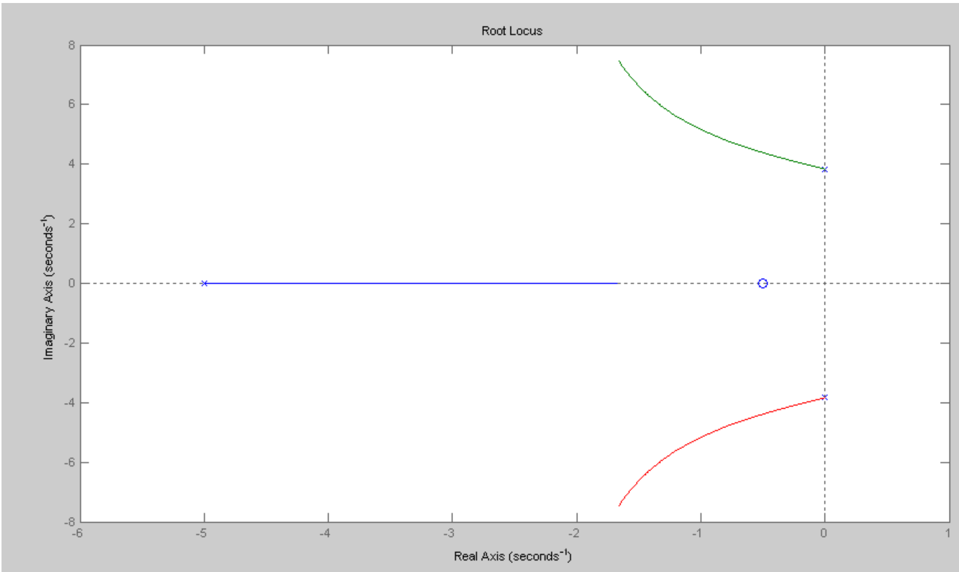
a) Design a compensator, K(s), which results in

- No error for a step input, and
- A 2% settling time less than 20 seconds

Step 1: Stabilize the system

$$K_1(s) = k\left(\frac{s+0.5}{s+5}\right)$$

```
>> G = tf(0.5,[1,0,14.7])

   0.5
----------
s^2 + 14.7

>> K1 = zpk(-0.5,-5,1);
>> k = logspace(-2,2,1000)';
>> rlocus(G*K1,k);
```



Pick a spot that's stable and placed the closed-loop poles there

```
>> s = -1 + j*5.16;
>> evalfr(G*K,s)

  -0.0264 + 0.0000i

>> 1/abs(ans)

   37.8544
```

so

$$\boxed{K_1(s) = 37.85\left(\frac{s+0.5}{s+5}\right)}$$

```
>> K1 = zpk(-0.5,-5,37.8544);
>> G2 = minreal(G*K1 / (1+G*K1));
>> zpk(G2)

        18.9272 (s+0.5)
--------------------------------
(s+3.003) (s^2 + 1.997s + 27.63)
```

Step 2: Now that the system's stable, add another feedback loop to stabilize

$$G_2 = \left(\frac{18.9272(s+0.5)}{(s+3.003)(s^2+1.997s+27.63)}\right)$$

Let

$$K_2 = k\left(\frac{(s^2+1.997s+27.63)(s+3.003)}{s(s+0.5)(s+a)}\right)$$

Then

$$G_2K_2 = \left(\frac{18.7292k}{s(s+a)}\right)$$

For s = -1 + j2 to be on the root locus

a = 2

$$G_2K_2 = \left(\frac{18.7292k}{s(s+2)}\right)_{s=-1+j2} = 3.745k\angle 180^0$$

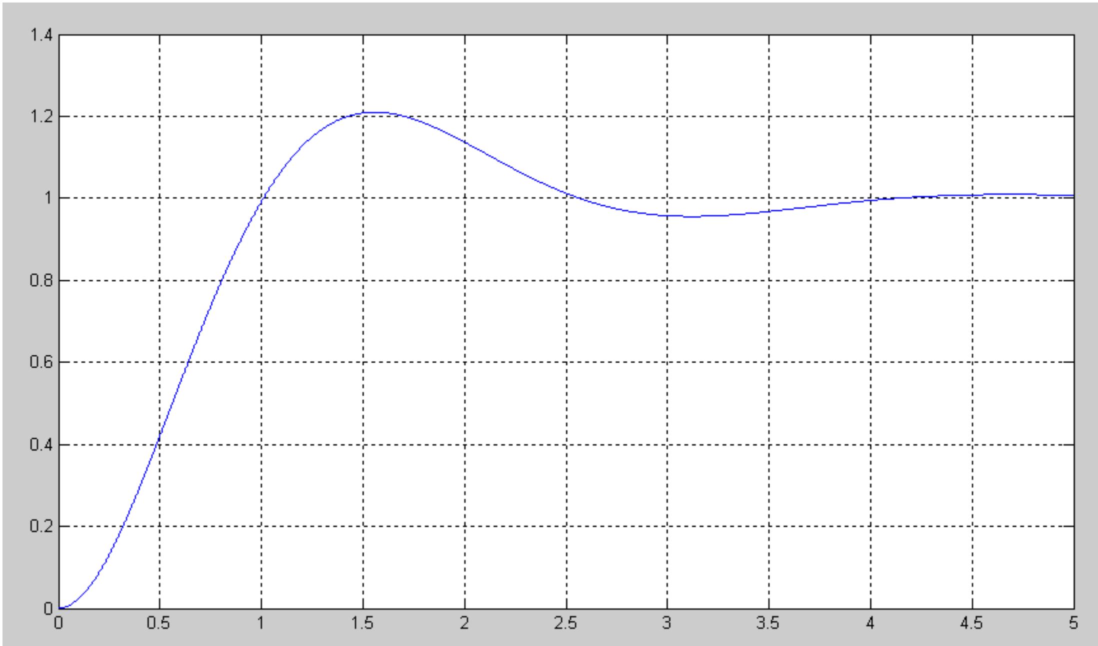
$$k = \frac{1}{3.745} = 0.2670$$

$$\boxed{K_2 = 0.2670\left(\frac{(s^2+1.997s+27.63)(s+3.003)}{s(s+0.5)(s+2)}\right)}$$

b) Find the step response of the closed-loop system using Matlab or VisSim (or similar program)

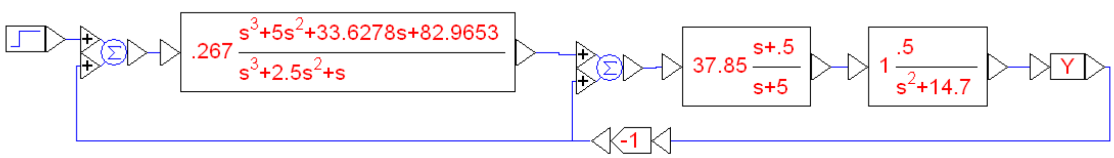
In Matlab, contining

```
> K2 = zpk([-0.9987+j*5.1607,-0.9987-j*5.1607,-3.0027],[0,-0.5,-2],0.2670)


0.267 (s+3.003) (s^2 + 1.997s + 27.63)
--------------------------------------
           s (s+0.5) (s+2)

>> G3 = minreal(G2*K2 / (1+G2*K2));
>> eig(G3)

  -0.9987 + 5.1607i
  -0.9987 + 5.1607i
  -1.0000 + 2.0134i
  -3.0027
  -3.0027
  -0.9987 - 5.1607i
  -0.9987 - 5.1607i
  -1.0000 - 2.0134i

>> t = [0:0.001:5]';
>> y = step(G3,t);
>> plot(t,y);
>> grid on
```

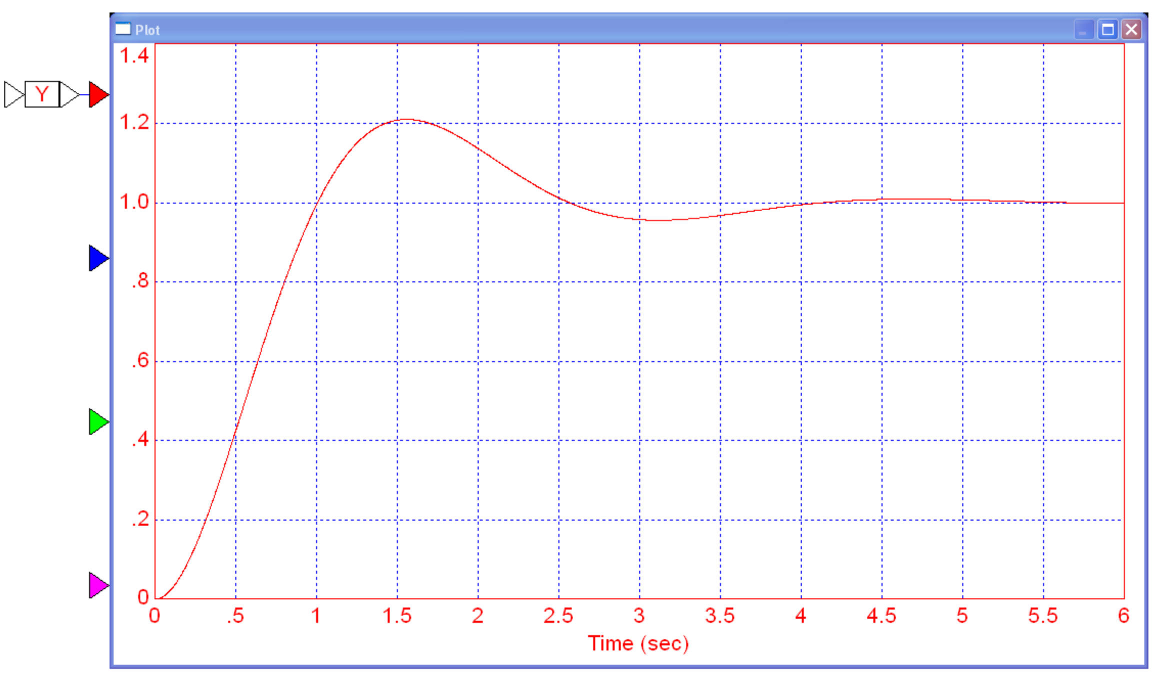


1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

or in VisSim



.267
s3+5s2+33.6278s+82.9653
s3+2.5s2+s
37.85
s+.5
s+5
1
.5
s2+14.7
Y
-1



Y
Plot
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Time (sec)

Problem 4: The dynamics of an inverted pendulum where the input is the position of the base (X) is

$$\theta = \left(\frac{-1}{(s+3.13)(s-3.13)}\right) X$$

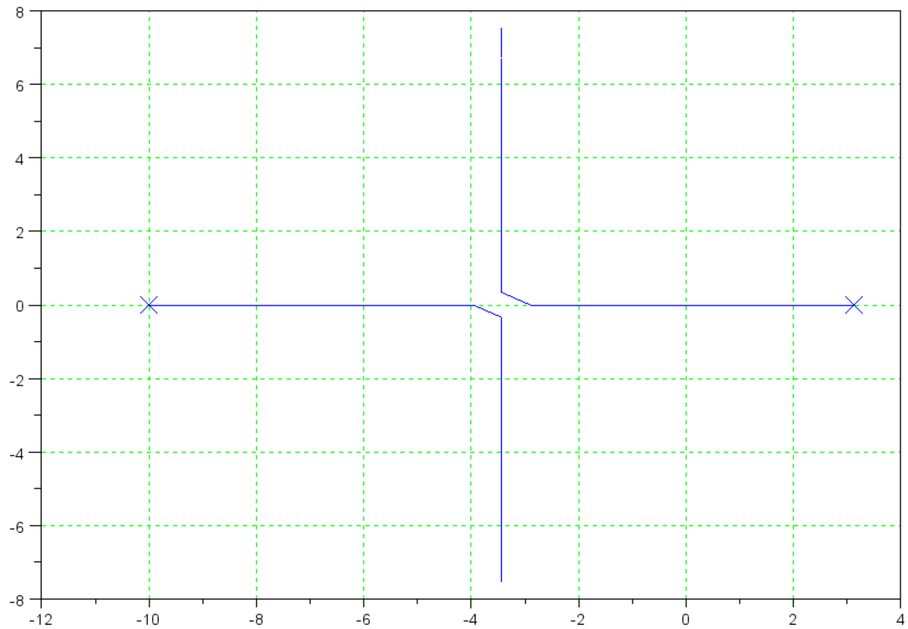
a) Design a compensator, K(s), which results in

- A stable closed-loop system, with
- 20% overshoot for a step input, and
- A 2% settling time less of 4 seconds

Step 1: Stablize the system. Let

$$K_1 = k\left(\frac{s+3.13}{s+10}\right)$$

This gives the following root locus:



Pick a point that's stable. Arbitrarily, let s = -1

$$GK_1 = \left(\frac{-1}{(s+10)(s-3.13)}\right)_{s=-1} = 0.0269\angle 0^0$$

There's a sign error. Let

$$K_1 = -k\left(\frac{s+3.13}{s+10}\right)$$

$$GK_1 = \left(\frac{1}{(s+10)(s-3.13)}\right)_{s=-1} = 0.0269\angle 180^0$$

$$k = \frac{1}{0.0269} = 37.17$$

and

$$K_1 = -37.17\left(\frac{s+3.13}{s+10}\right)$$

The closed-loop system is then

```
>> G = zpk([],[3.13,-3.13],-1)

       -1
-----------------
(s-3.13) (s+3.13)

>> K1 = zpk(-3.13,-10,-37.17)

-37.17 (s+3.13)
---------------
    (s+10)

>> G2 = minreal(G*K1 / (1+G*K1));
>> zpk(G2)

    37.17
--------------
(s+5.87) (s+1)
```

Step 2: Now meet the design specs. Let

$$K_2(s) = k\left(\frac{(s+1)(s+5.87)}{s(s+2)}\right)$$

so that

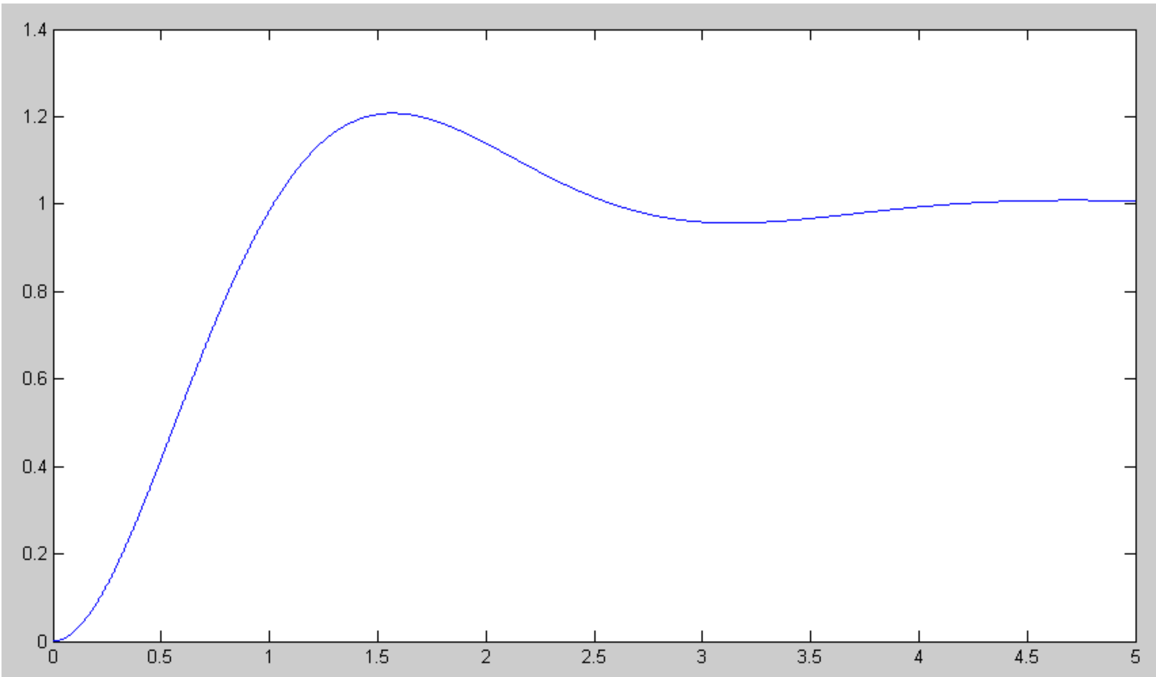
$$G_2K_2 = \left(\frac{37.17}{s(s+2)}\right)_{s=-1+j2} = 7.434\angle 180^0$$

$$k = \frac{1}{7.434} = 0.1345$$

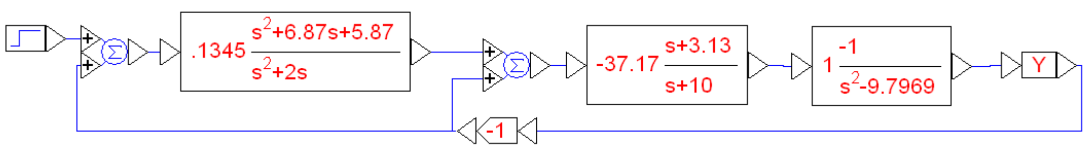
$$K_2(s) = 0.1345\left(\frac{(s+1)(s+5.87)}{s(s+2)}\right)$$

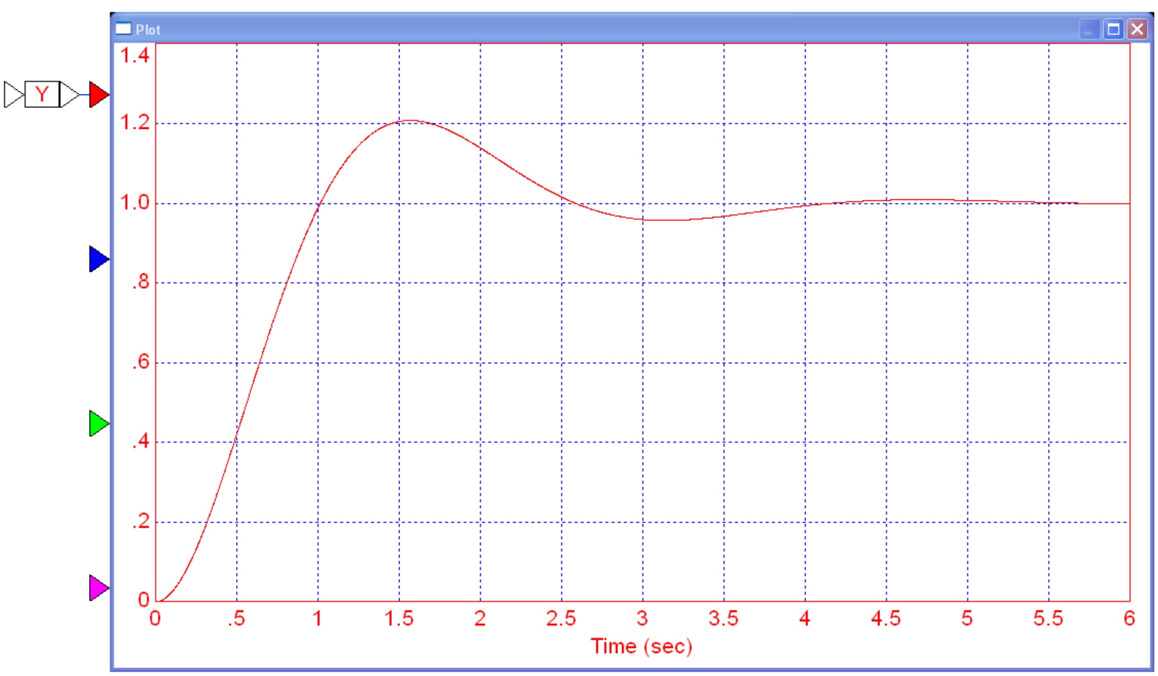
b) Find the step response of the closed-loop system using Matlab or VisSim (or similar program)

```
>> K2 = zpk([-1,-5.87],[0,-2],0.1345);
>> G3 = minreal(G2*K2 / (1 + G2*K2));
>> zpk(G3)

      4.9994
------------------
(s^2 + 2s + 4.999)

>> t = [0:0.001:5]';
>> y = step(G3,t);
>> plot(t,y);
```


1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

or in VisSim


.1345
s2+6.87s+5.87
s2+2s
-37.17
s+3.13
s+10
1
-1
s2-9.7969
Y
-1


Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Time (sec)