Homework #11: ECE 461

Digital Control. Due Monday, November 20th

1) For the following system:

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right)$$

design a digital controller, K(z), so that

- The sampling rate is 0.1 seconds (T = 0.1)
- The resulting closed-loop system has no error for a step input,
- With a 2% settling time of 4 seconds, and
- 20% overshoot for a step input.

a) Specify K(z)

- Add a pole at z = 1 to make it type 1
- Add a zero at z = 0.9807 to cancel the pole at s = -0.195
- Add a zero at z = 0.8982 to cancel the pole at s = -1.074
- Add a zero at z = 0.7593 to cancel the pole at -2.753
- Add two pole to make it causal and to place s = -1 + j2 on the root locus

$$K(z) = k\left(\frac{(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)(z-a)^2}\right)$$

This results in (including a 1/2 sample delay to model the sample-and-hold)

$$GK = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot (e^{-0.05s}) \cdot \left(\frac{k(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)(z-a)^2}\right)$$

Evaluating what we know at

- s = -1 + j2
- z = esT = 0.8868 + j0.1798

$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot (e^{-0.05s}) \cdot \left(\frac{k(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)}\right) = 0.0010\angle -110.6^0$$

This means that the two pole (z-a) term must contribute 69.36 degrees (34.68 degrees each)

$$\angle(z-a) = 34.68^0$$

$$a = 0.8868 - \frac{0.1798}{\tan\left(34.68^0\right)} = 0.6270$$

and

$$K(z) = k\left(\frac{(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)(z-0.6270)^2}\right)$$

To find 'k'

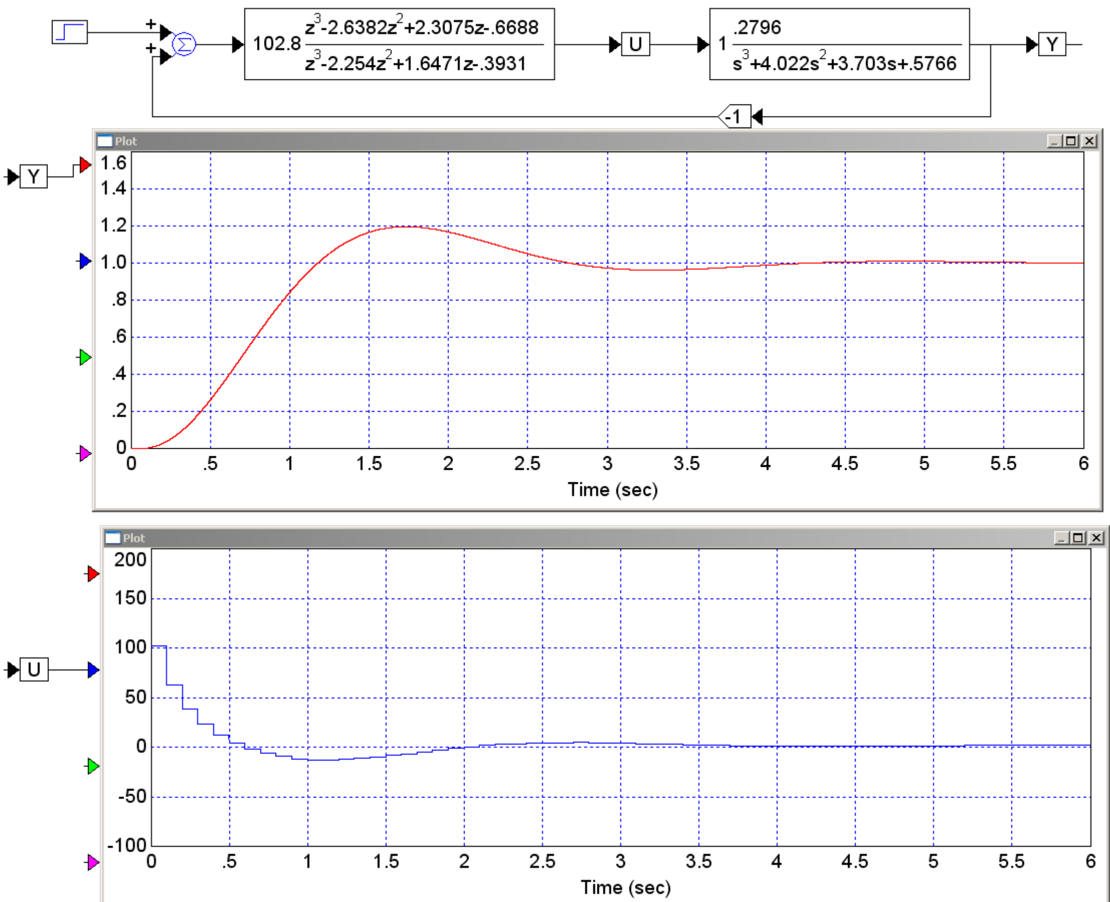
$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot (e^{-0.05s}) \cdot \left(\frac{k(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)(z-0.6270)^2}\right) = 0.0097\angle 180^0$$

$$k = \frac{1}{0.0097} = 102.81$$

and

$$K(z) = 102.81\left(\frac{(z-0.9807)(z-0.8982)(z-0.7593)}{(z-1)(z-0.6270)^2}\right)$$

b) Verify your design using Matlab or VisSim (VisSim preferred: it allows you to plot G(s) and K(z)



c) Write a program to implement your K(z).

```
while(1) {

    Y = A2D_Read(0);

    E3 = E2;
    E2 = E1;
    E1 = E0;
    E0 = Ref - Y;

    U3 = U2;
    U2 = U1;
    U1 = U0;
    U0 = 102.8*(E0 - 2.6382*E1  + 2.3075*E2 - 0.6688 *E3 ) +
         2.2540*U1 - 1.6471*U2 + 0.3931*U3;

    D2A(U0);

    Wait_100ms();
    }
```

2) For the following system:

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right)$$

design a digital controller, K(z), so that

- The sampling rate is 0.5 seconds (T = 0.5)
- The resulting closed-loop system has no error for a step input,
- With a 2% settling time of 4 seconds, and
- 20% overshoot for a step input.

a) Specify K(z)

- Add a pole at z = 1 to make it type 1
- Add a zero at z = 0.9071 to cancel the pole at s = -0.195
- Add a zero at z = 0.5848 to cancel the pole at s = -1.074
- Add a zero at z = 0.2525 to cancel the pole at -2.753
- Add two pole to make it causal and to place s = -1 + j2 on the root locus

$$K(z) = k\left(\frac{(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)(z-a)^2}\right)$$

This results in (including a 1/2 sample delay to model the sample-and-hold)

$$GK = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot \left(e^{-0.25s}\right) \cdot \left(\frac{k(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)(z-a)^2}\right)$$

Evaluating what we know at

- s = -1 + j2
- z = esT = 0.3277 + j0.5104

$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot \left(e^{-0.25s}\right) \cdot \left(\frac{(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)}\right) = 0.0084\angle -83^0$$

This means that the two pole (z-a) term must contribute 96.96 degrees (48.48 degrees each)

$$\angle(z-a) = 48.48^0$$

$$a = 0.3277 - \frac{0.5104}{\tan\left(48.48^0\right)} = -0.1242$$

and

$$K(z) = k\left(\frac{(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)(z+0.1242)^2}\right)$$

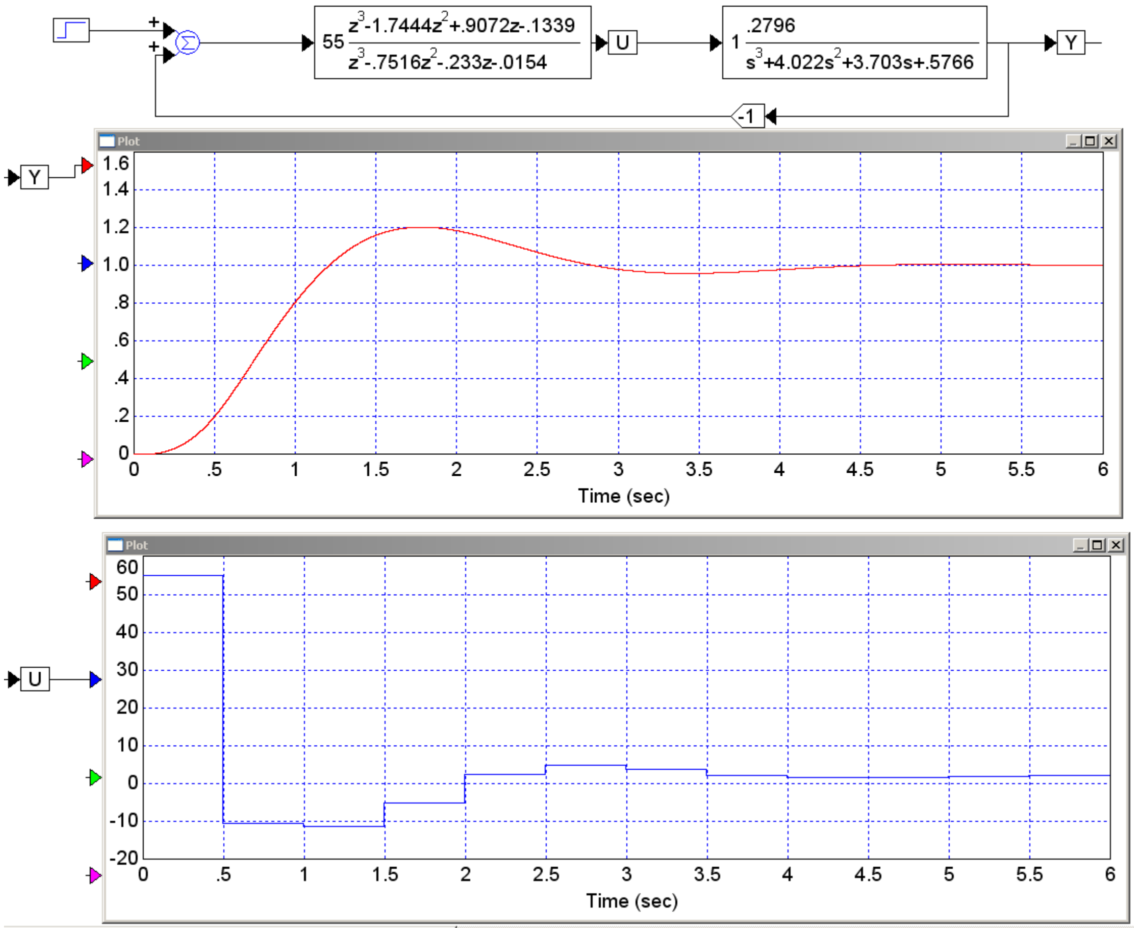
To find 'k'

$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot \left(e^{-0.25s}\right) \cdot \left(\frac{(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)(z+0.1242)^2}\right) = 0.0182\angle 180^0$$

$$k = \frac{1}{0.0182} = 55.077$$

$$K(z) = 55.077\left(\frac{(z-0.9071)(z-0.5848)(z-0.2525)}{(z-1)(z+0.1242)^2}\right)$$

b) Verify your design using Matlab or VisSim (VisSim preferred: it allows you to plot G(s) and K(z)



c) Write a program to implement your K(z).

```
while(1) {

    Y = A2D_Read(0);

    E3 = E2;
    E2 = E1;
    E1 = E0;
    E0 = Ref - Y;

    U3 = U2;
    U2 = U1;
    U1 = U0;
    U0 = 55*(E0 - 1.7444*E1  + 0.9072*E2 - 0.1339 *E3 ) +
         0.7516*U1 + 0.2330*U2 + 0.0154 * U3 +

    D2A(U0);

    Wait_500ms();
    }
```

3) For the following system with a 0.65 second delay:

$$G(s) = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.65s}$$

design a digital controller, K(z), so that

- The sampling rate is 0.5 seconds (T = 0.5)
- The resulting closed-loop system has no error for a step input,
- With a 2% settling time of 10 seconds, and *change: Ts = 10 seconds. Same as HW#9*
- 20% overshoot for a step input.

a) Specify K(z)

- Add a pole at z = 1 to make it type 1
- Add a zero at z = 0.9071 to cancel the pole at s = -0.195
- Add a zero at z = 0.5848 to cancel the pole at s = -1.074
- Add a pole to make it causal and to place s = -0.4 + j0.8 on the root locus

$$K(z) = k\left(\frac{(z-0.9071)(z-0.5848)}{(z-1)(z-a)}\right)$$

This results in (including a 1/2 sample delay to model the sample-and-hold)

$$GK\Delta = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.65s} \cdot e^{-0.25s} \cdot k\left(\frac{(z-0.9071)(z-0.5848)}{(z-1)(z-a)}\right)$$

$$GK\Delta = \left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.9s} \cdot k\left(\frac{(z-0.9071)(z-0.5848)}{(z-1)(z-a)}\right)$$

Evaluating what we know at

- s = -0.4 + j0.8
- z = esT = 0.7541 + j0.3188

$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.9s} \cdot \left(\frac{(z-0.9071)(z-0.5848)}{(z-1)}\right) = 0.2080\angle -85.39^0$$

This means that the pole (z-a) term must contribute 90.31 degrees

$$\angle(z-a) = 94.6^0$$

$$a = 0.7541 - \frac{0.3188}{\tan\left(94.6^0\right)} = 0.7798$$

and

$$K(z) = k\left(\frac{(z-0.9071)(z-0.5848)}{(z-1)(z-0.7798)}\right)$$

To find 'k'

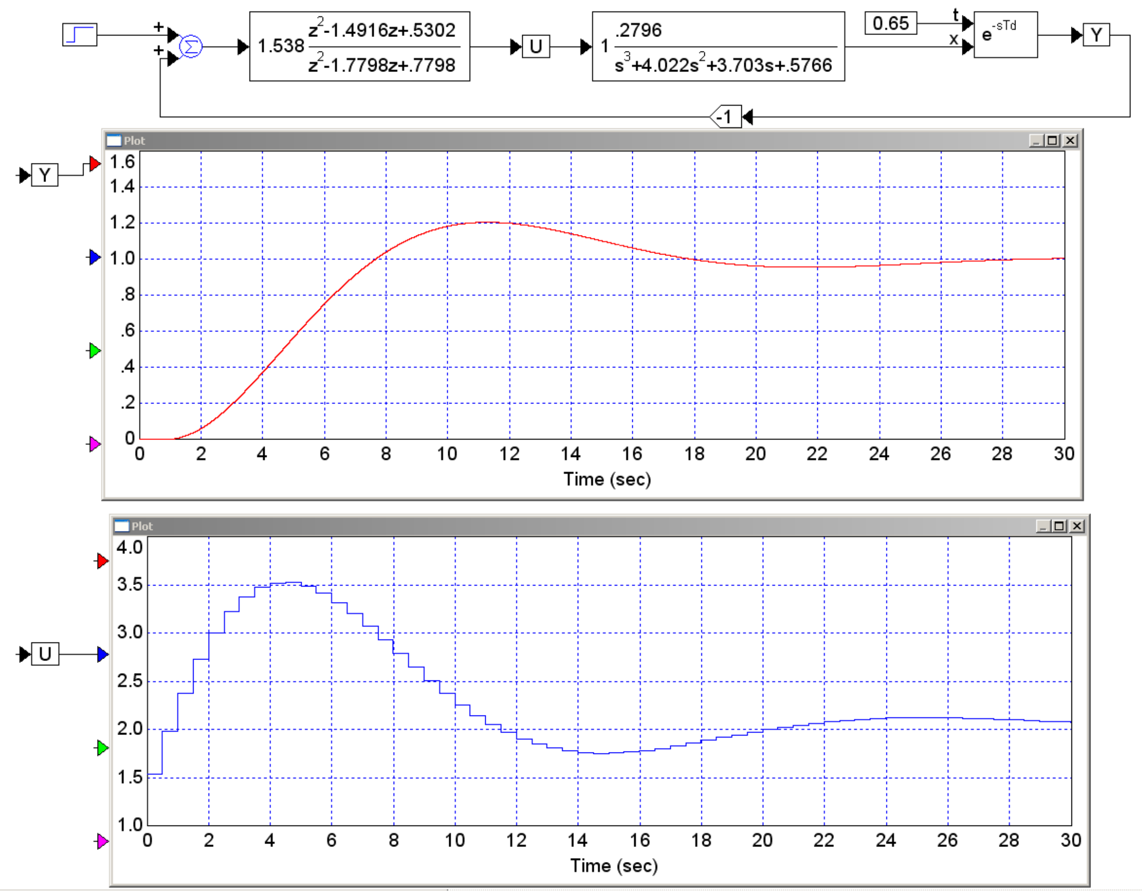
$$\left(\frac{0.2796}{(s+0.195)(s+1.074)(s+2.753)}\right) \cdot e^{-0.9s} \cdot \left(\frac{(z-0.9071)(z-0.5848)}{(z-1)(z-0.7798)}\right) = 0.6502\angle 180^0$$

$$k = \frac{1}{0.6502} = 1.538$$

and

$$\boxed{K(z) = 1.538\left(\frac{(z-0.9071)(z-0.5845)}{(z-1)(z-0.7798)}\right)}$$

b) Verify your design using Matlab or VisSim (VisSim preferred: it allows you to plot G(s) and K(z)



c) Write a program to implement your K(z).

```
while(1) {

    Y = A2D_Read(0);

    E3 = E2;
    E2 = E1;
    E1 = E0;
    E0 = Ref - Y;

    U3 = U2;
    U2 = U1;
    U1 = U0;
    U0 = 1.538*(E0 - 1.4916*E1  + 0.5302*E2 ) +
         1.7798*U1 - 0.7798*U2;

    D2A(U0);

    Wait_500ms();
    }
```