ECE 461/661 - Final Exam: Name ____________________

Fall 2018

1) Assume X and Y are related by the following transfer function

$$Y = \left(\frac{20}{(s+2)(s+5)} \right) X$$

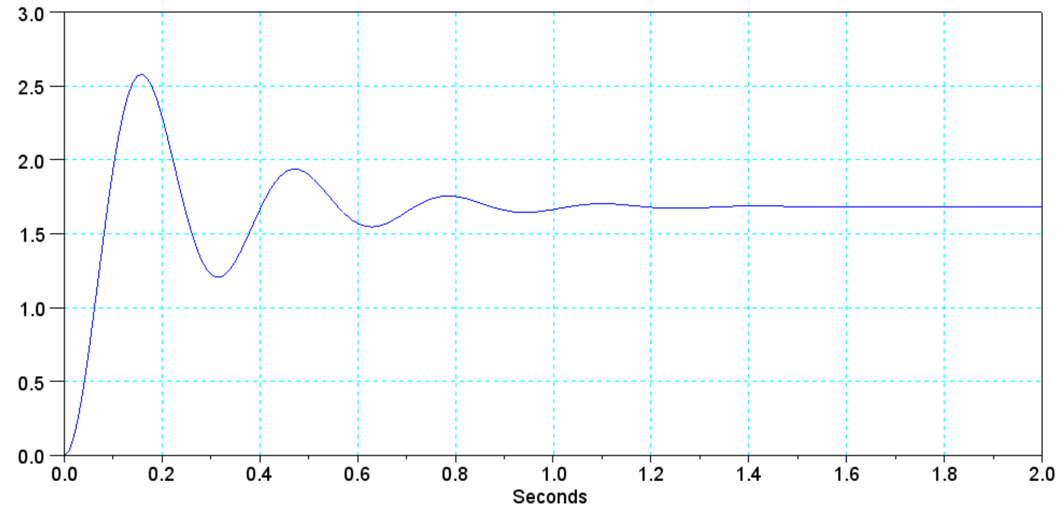
1a) Find y(t) assuming

$$x(t) = 2 + 3\cos(4t)$$

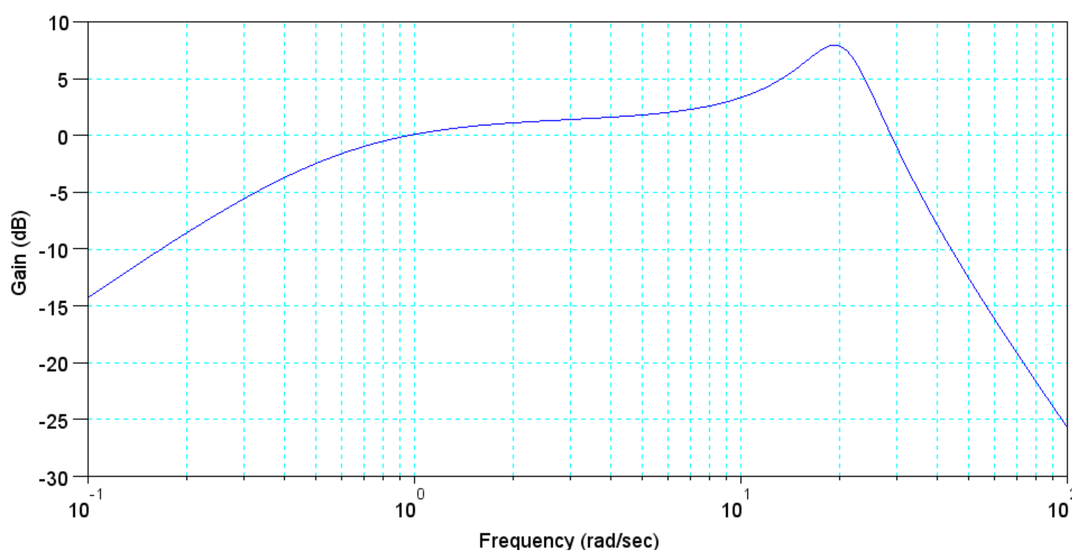
1b) Find y(t) assuming

$$x(t) = 2u(t) = \begin{cases} 2 & t > 0 \\ 0 & otherwise \end{cases}$$

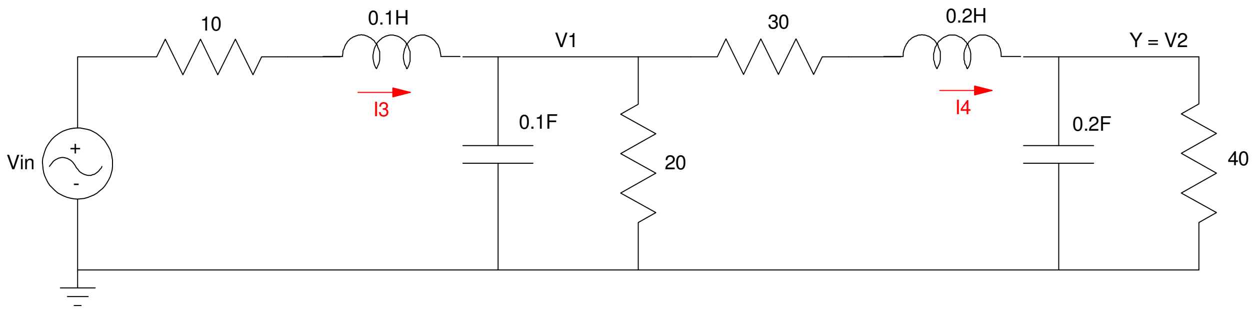
2a) Determine the transfer function for the system with the following step response



2b) Determine the transfer function for the system with the following frequency response



3a) Write four coupled differential equations to describe the dynamics of this circuit



3b) Express these dynamics in state-space form

$$\begin{bmatrix} sV1 \\ sV2 \\ sI3 \\ sI4 \end{bmatrix} = \begin{bmatrix} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} \; \\ \; \\ \; \\ \; \end{bmatrix} Vin$$

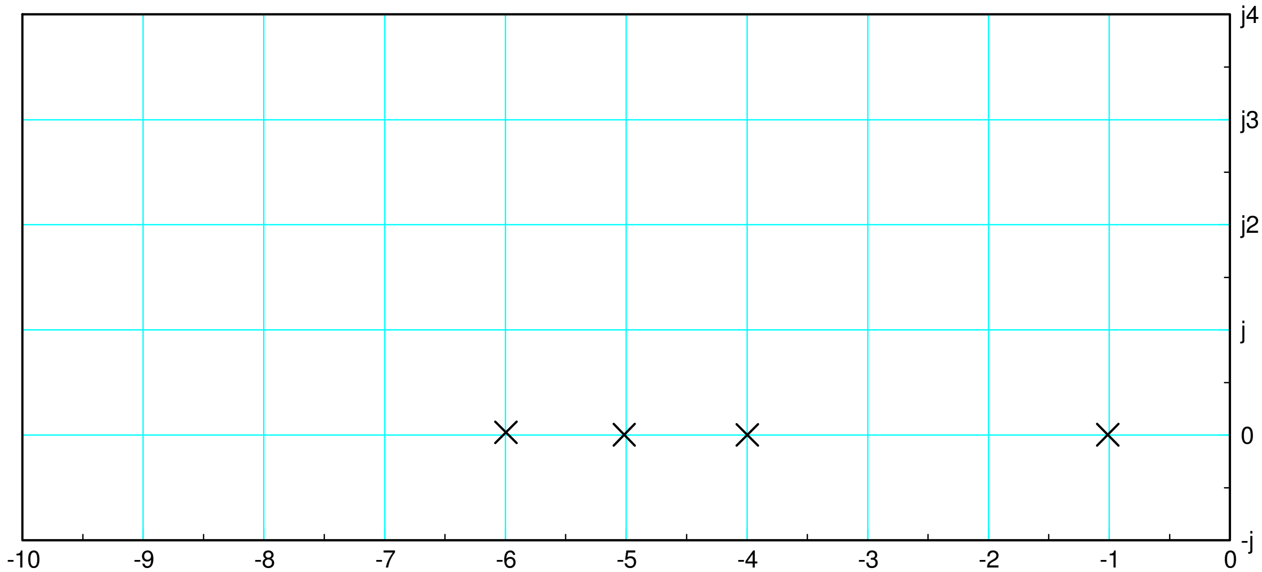
$$Y = \begin{bmatrix} \; & \; & \; & \; \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} \; \end{bmatrix} Vin$$

4) Sketch the root locus for

$$G(s) = \left(\frac{100}{(s+1)(s+4)(s+5)(s+6)} \right)$$

Determine the following:

Real Axis Loci	
Breakaway Points (approx)	
jw Crossing (approx)	
Asymptotes	show on graph



5) Assume a system has the following dynamics:

$$G(s) = \left(\frac{100}{(s+2)(s+4)(s+10)} \right)$$

Design a compensator, K(s), so that the closed-loop system has

- No error for a step input, and
- Closed-loop dominant poles at **s = -3 + j2**

(note: this implies $GK(-3+j2) = 1\angle 180^0$)

6) Assume a system has the following dynamics:

$$G(s) = \left(\frac{100}{(s+2)(s+4)(s+10)} \right)$$

Design a digital compensator, K(z), so that the closed-loop system has

- No error for a step input,
- Closed-loop dominant poles at **s = -3 + j2**, and

Assume a sampling rate of T = 100ms (T = 0.1)

7) Assume a system has the following dynamics:

$$G(s) = \left(\frac{100}{(s+2)(s+4)(s+10)} \right)$$

Design a compensator, K(s), so that the closed-loop system has

- A DC gain of one (no error for a step input),
- A 0dB gain frequency of 5 rad/sec, and
- A phase margin of 50 degrees

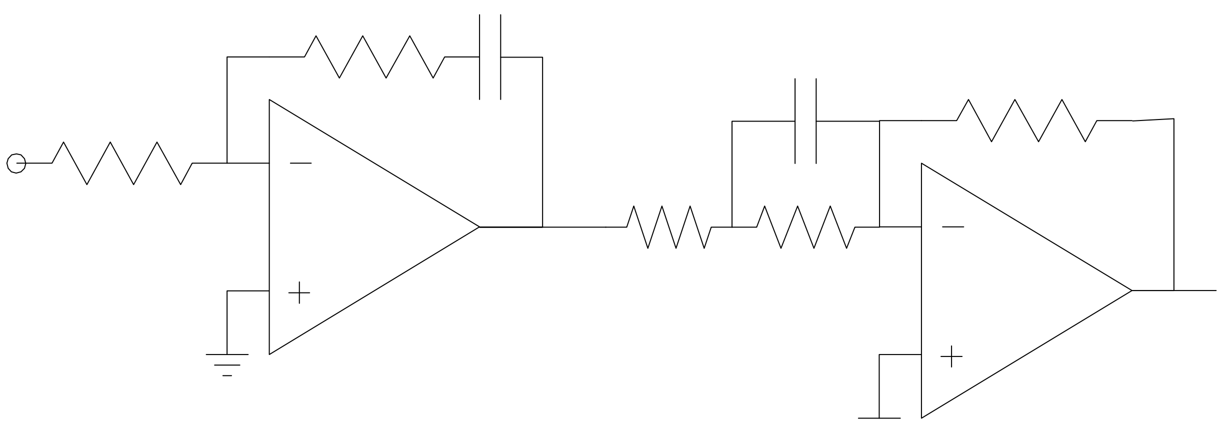
(note: this implies $GK(j5) = 1\angle -130^0$)

8a) Determine a discrete-time equivalent to K(s). Assume a sampling rate of 100ms (T = 0.1)

$$K(s) = \left(\frac{20(s+3)(s+5)}{s(s+10)}\right)$$

8b) Design a circuit to implement K(s)

$$K(s) = \left(\frac{20(s+3)(s+5)}{s(s+10)}\right)$$

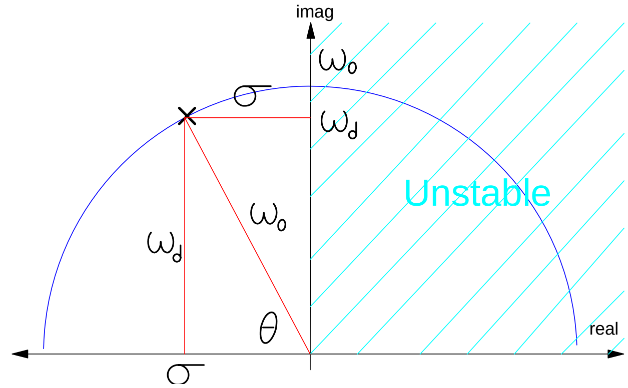


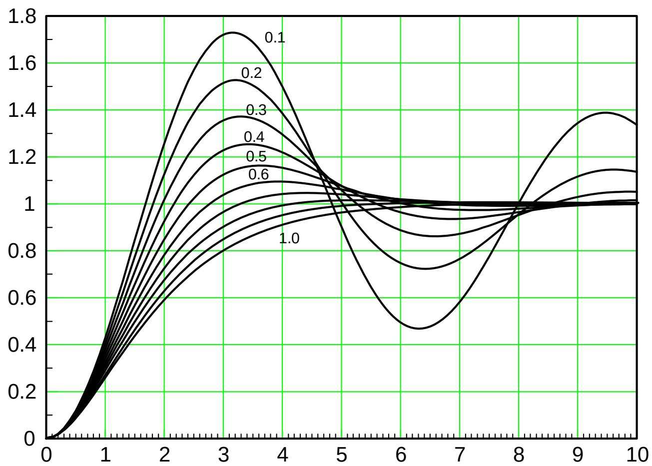
Bonus! What are the names of Santa's Reindeer?

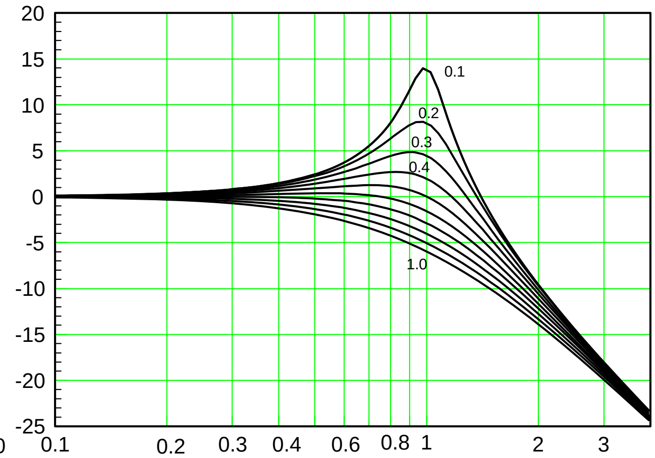
2nd-Order Approximations

$$G(s) = \left(\frac{k\cdot\omega_o^2}{s^2+2\zeta\omega_o s+\omega_o^2}\right) = \left(\frac{k\cdot\left(\sigma^2+\omega_d^2\right)}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

$$s = -\sigma \pm j\omega_d = \omega_o\angle\pm\theta$$







$\zeta = \cos\theta$	damping ratio	$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$	time to peak
$\%OS = \exp\left(-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)\right)$	% Overshoot	$T_s = T_{2\%} = \frac{4}{\sigma}$	2% Settling Time
$\omega_m = \omega_o\sqrt{1-2\zeta^2}$	Max gain frequency	$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$	Max gain
		$\frac{1}{2\zeta}$	Gain at corner freq

ζ	Tp	%OS	ω_m	Mm	Mm (dB)
0.1	3.15	72.81	0.99	5.03	14.02
0.2	3.21	51.97	0.96	2.55	8.14
0.3	3.29	35.5	0.91	1.75	4.85
0.4	3.43	22.4	0.82	1.36	2.7
0.5	3.63	12.31	0.71	1.15	1.25
0.6	3.93	5.26	0.53	1.04	0.35
0.7	4.4	1.34	0.14	1	0
0.8	5.24	0.09	0	1	0
0.9	7.21	0	0	1	0
1.0	-	0	0	1	0