Solution to Homework #7: ECE 461/661

Error Constants, Routh Criteria, Sketching a Root Locus

Error Constants:

1) Fill in the following table\

Open-Loop System G(s)	Type 0 - 1 - 2	Kp	Kv	Steady-State Error for a Step Input
$\left(\frac{20}{(s+2)(s+5)}\right)$	0	2	0	1/3
$\left(\frac{20}{(s-2)(s+5)}\right)$	0	-2	0	-1
$\left(\frac{20}{s(s+2)(s+5)}\right)$	1	inf	2	0
$\left(\frac{20(s+1)}{s^2(s+2)(s+5)}\right)$	2	inf	inf	0

Routh Criteria

Using a Routh table, determine the range of K which results in a negative-definate polynominal (i.e. a stable closed-loop system)

2) $$(s+2)(s+5)(s+10)+2k=0$$

Multiply out

```
>> poly([-2,-5,-10])

    1    17    80   100
```

$$s^3+17s^2+80s+100+2k=0$$

1	80	0	
17	100+2k	0	
$\frac{-\begin{vmatrix} 1 & 80 \\ 17 & 100+2k \end{vmatrix}}{17} = \frac{1260-2k}{17}$	0		k > 630
100+2k	0		k > -50
0	0		

answer: -50 < k < 630

3) $$(s-1)(s+5)(s+9)(s+10)+2k(s+1)=0$$

$$(s^4+23s^3+161s^2+265s-450)+2k(s+1)=0$$

$$s^4+23s^3+161s^2+(265+2k)s+(2k-450)=0$$

```
>> poly([1,-5,-9,-10])
     1    23   161   265  -450
```

Set up a Routh table

1	161	2k-450	
23	265+2k	0	
$149.478-0.0870k$	2k-450	0	k < 1719
$\frac{0.1740k^2-368.011k+39,622,020}{149.478-0.0870k}$	0	0	-190 < k < 1511
2k-450	0	0	k > 225

Result: 225 < k < 1511

Element (3,1)

$$\frac{-\begin{vmatrix} 1 & 161 \\ 23 & 265-2k \end{vmatrix}}{23}=\frac{3438-2k}{23}$$

Element (4,1)

$$\frac{-\begin{vmatrix} 23 & 265+2k \\ 149.478-0.0870k & 2k-450 \end{vmatrix}}{149.478-0.0870k}=\frac{-0.1740k^2+229.91k+49,961}{149.478-0.0870k}$$

```
>>  roots([-0.1740,229.91,49961])

ans =

    1511.3
   -190.0

-190.0 < k < 1511.3
```

4) $s^2(s+2)(s+5)+2k(s+1)=0$

$$s^4+7s^2+10s^2+2ks+2k=0$$

Place in a Routh Table

1	10	2k	
7	2k	0	
10 - 0.2857k	2k	0	k < 35
$\frac{-0.5714k^2+6k}{10-0.2857k}$			0 < k < 10.5
2k			k > 0

Result: **0 < k < 10.5**

Element 4,1

$$\frac{-\begin{vmatrix} 7 & 2k \\ 10-0.2857k & 2k \end{vmatrix}}{10-0.2857k} = \frac{-0.5714k^2+6k}{10-0.2857k}$$

Sketching a Root Locus

Draw the root locus plot for the following systems (it's OK to use Matlab). Calculate and show on your plots

5) $G(s) = \left(\frac{2}{(s+2)(s+5)(s+10)}\right)$

The real-axis loci,

(0, -2), (-10, -infinity)

The breakaway point(s),

-3.3333 *found numerically: angle(G(x + j0.01)) = 180 degrees*

The jw crossing(s),

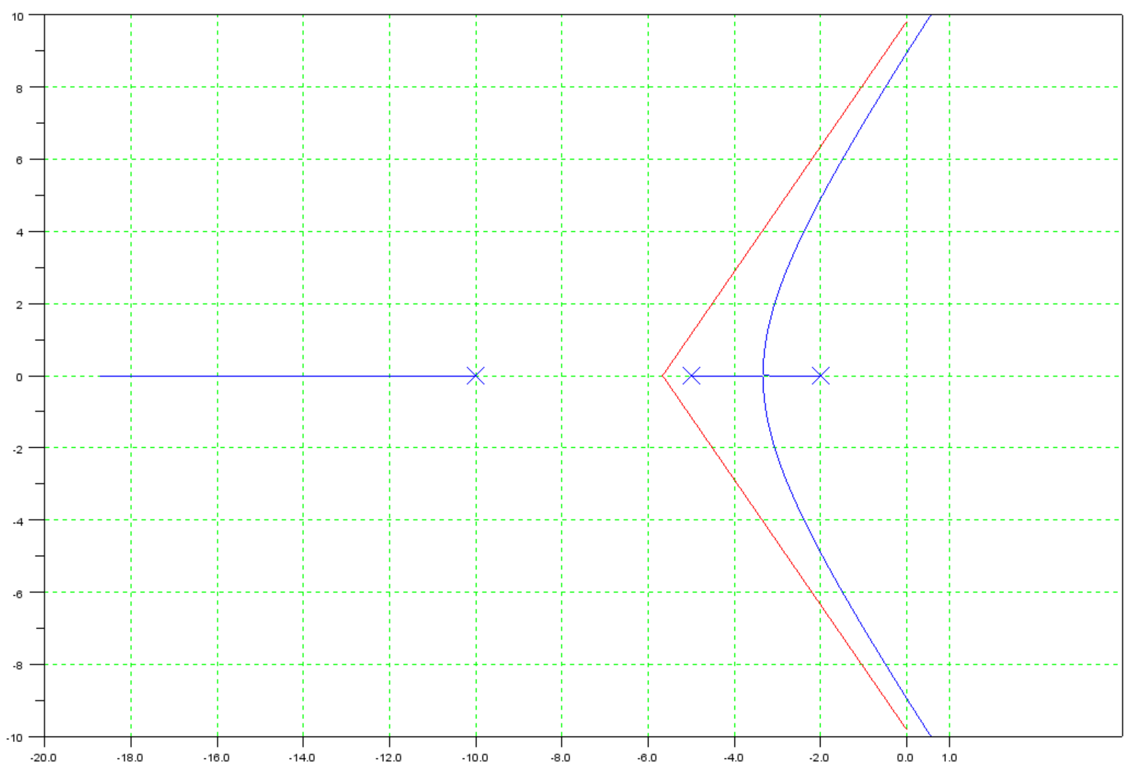
j8.9443 *found numerically: angle(G(jx)) = 180 degrees*

The asymptotes, and

3 asymptotes

angle = $\{\pm 60^0, 180^0\}$

Intersect = $\left(\frac{-2-5-10}{3}\right) = -5.666$



6) $$G(s) = \left(\frac{2(s+1)}{(s-1)(s+5)(s+9)(s+10)}\right)$$

The real-axis loci,

(+1, -1), (-5, -9), {-10, -infinity)

The breakaway point(s),

-6.4026 *found numerically: angle(G(x + j0.01)) = 180 degrees*

The jw crossing(s),

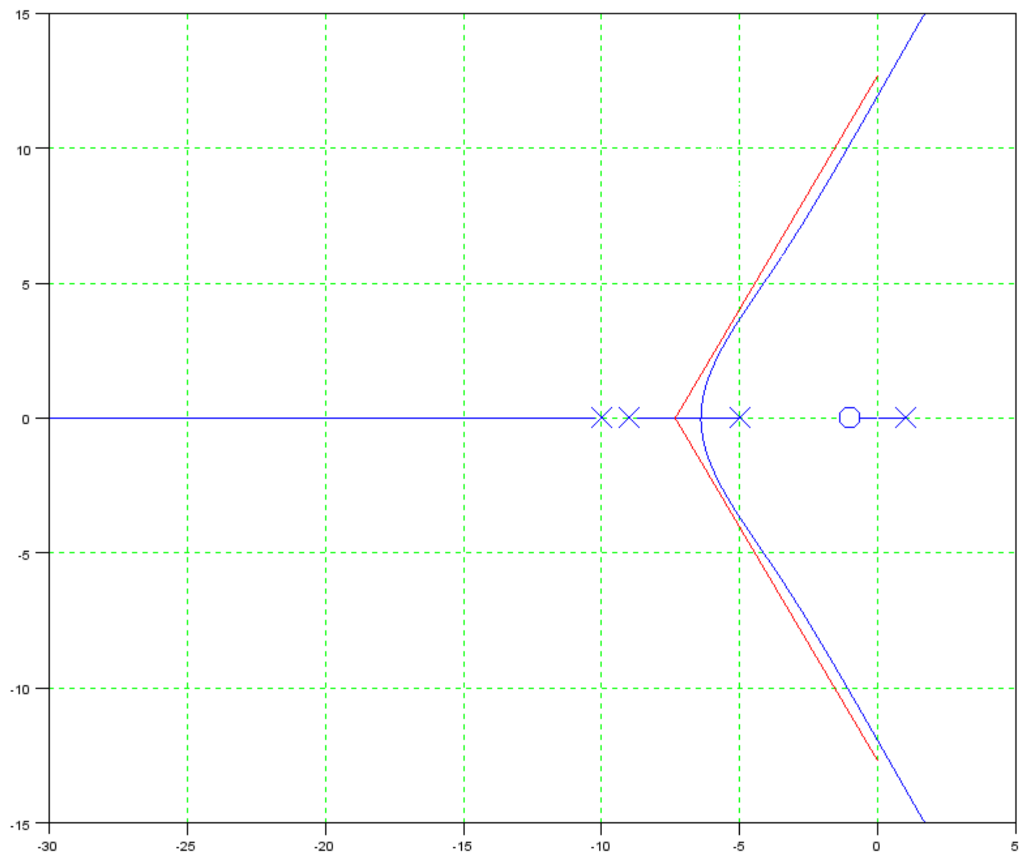
j11.9583 *found numerically: angle(G(jx)) = 180 degrees*

The asymptotes, and

3 asymptotes

angle = +60 deg, -60 deg, 180 deg

Intersect = $\left(\frac{(1-5-9-10)-(-1)}{4-1}\right) = -7.3333$



7) $G(s) = \left(\frac{2(s+j2)(s-j2)}{s(s+2)(s+5)(s+10)}\right)$

The real-axis loci,

(0, -2), (-5, -10)

The breakaway point(s),

-0.7024, -7.6930 *found numerically: angle(G(x + j0.01)) = 180 degrees*

The jw crossing(s),

none

The asymptotes, and

2 asymptotes

angle = +90 deg, -90 deg

Intersect = -17 / 2 = -8.5

Approach Angle:

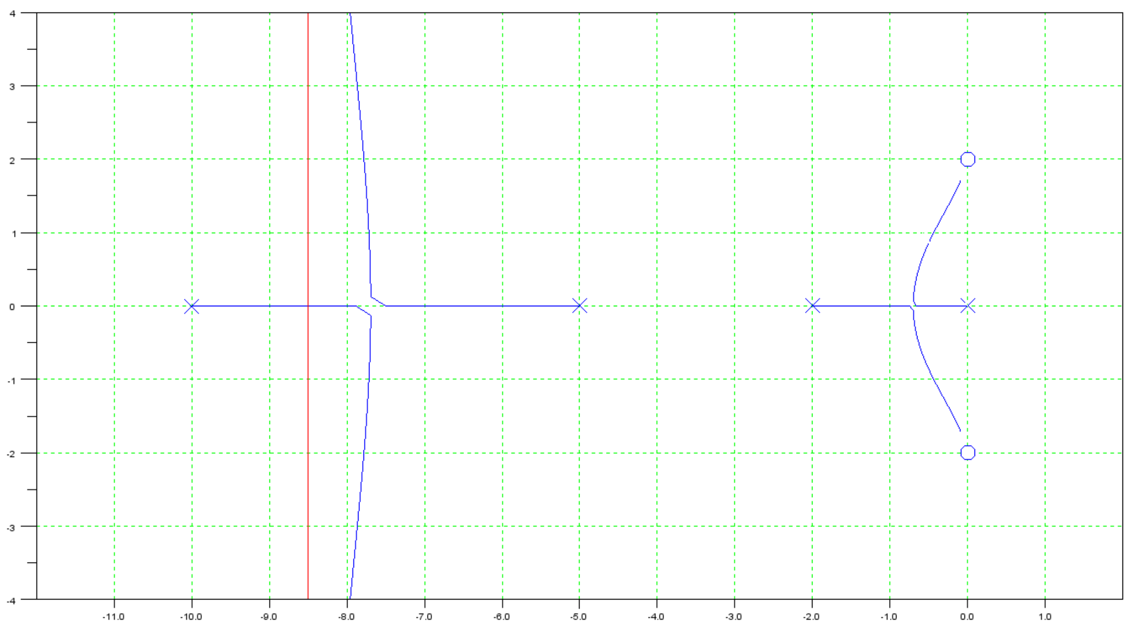
$$\Sigma\,(\text{angles from poles}) - \Sigma\,(\text{angles from zeros}) = 180^0$$

At s= j2

$$\angle s + \angle(s+2) + \angle(s+5) + \angle(s+10) - \angle(s+j2) - \angle(s-j2) = 180^0$$

90 + 45 + 21.8 + 11.3 - 90 - Q = 180

Q = -101.89 degrees



8) $G(s) = \left(\frac{20}{s(s+2)(s+5)(s+10)}\right)$

The real-axis loci,

(0, -2), (-5, -10)

The breakaway point(s),

-0.8305, -8.2874 *found numerically: angle(G(x + j0.01)) = 180 degrees*

The jw crossing(s),

j 2.4254

The asymptotes, and

4 asymptotes

angle = +/- 45 degrees, +/- 135 degrees

Intersect = -17 / 4 = -4.25