Solution to Homework #8: ECE 461

Gain, Lead, PID Compensation

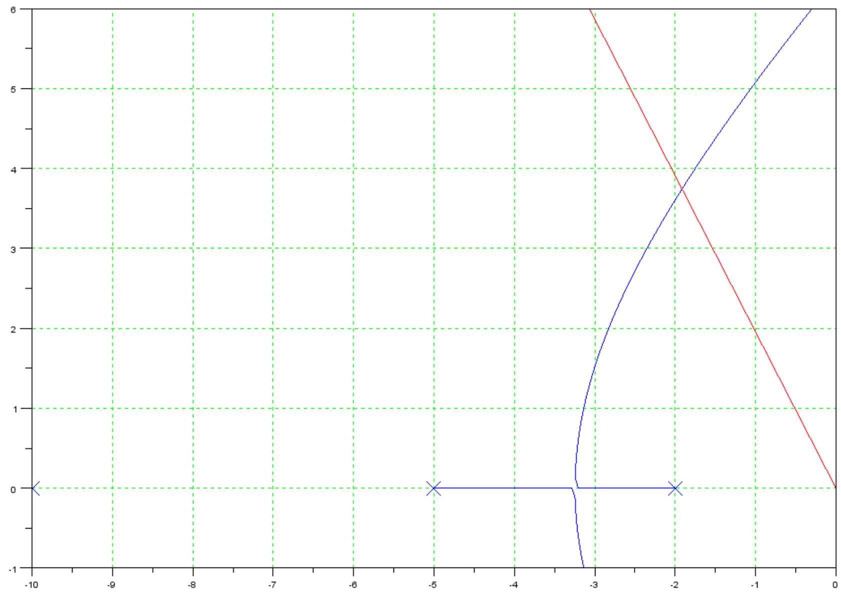
Assume

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)$$

Problem 1: Gain Compensation (K(s) = k)

Design a gain compensator which results in the closed-loop system having 20% overshoot for a step input.

Sketch the root locus. Find the point which intersects the 0.4554 damping line:



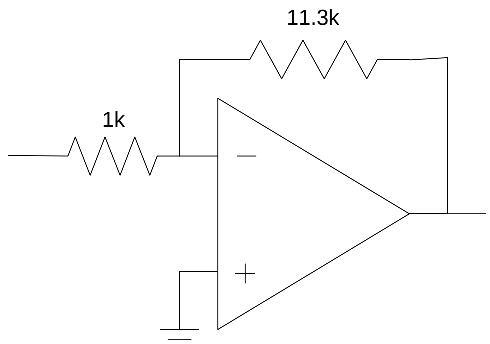
s = -1.8906 + j3.7812

Pick 'k' so that the gain is -1 at this point:

$$\left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)_{s=-1.8906+j3.7812} = 0.0885\angle 180^0$$

$$k = \frac{1}{0.0885} = 11.3038$$

Design an op-amp circuit to implement K(s)



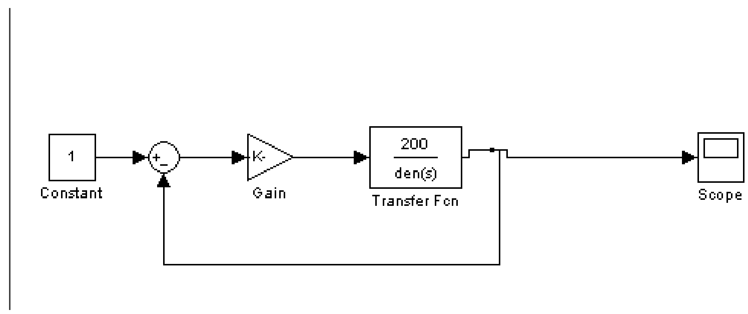
Determine the dominant poles of the closed-loop system

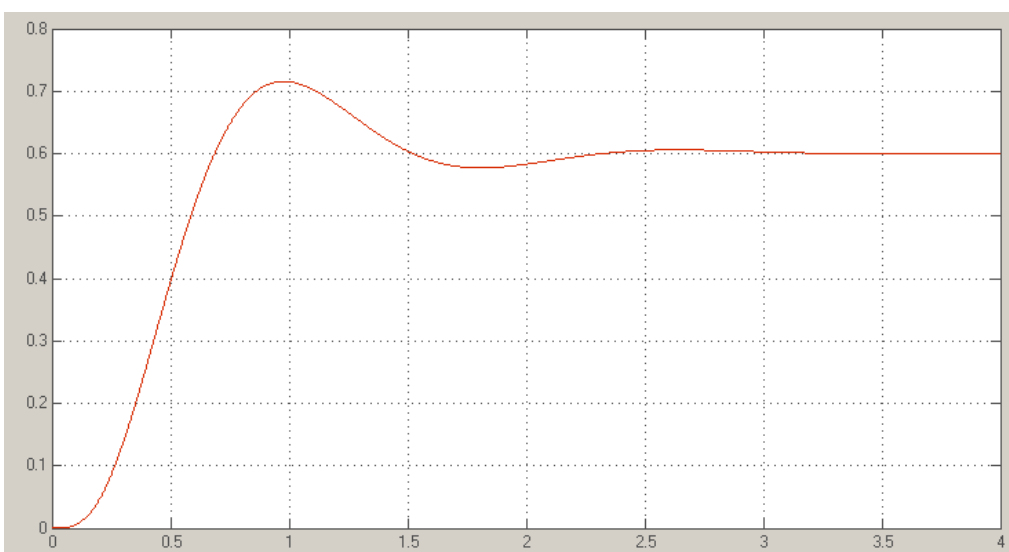
The dominant pole is were we placed it

s = -1.8906 + j3.7812

Plot the step response of the closed-loop system using Matlab (or similar program)

Using SimuLink





Problem 2: Lead Compensation $\left(K(s) = k\left(\frac{s+a}{s+10a}\right)\right)$

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)$$

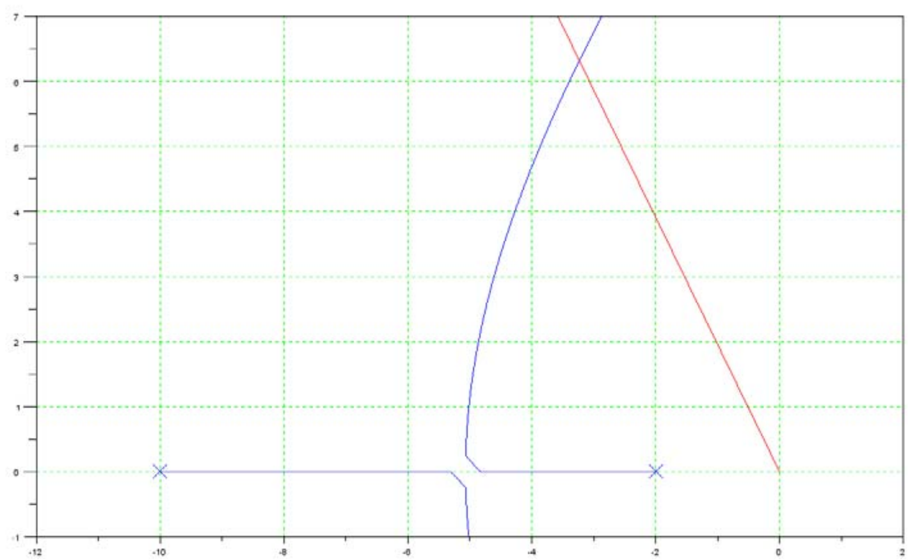
Design a lead compensator which results in the closed-loop system having 20% overshoot for a step input.

Keep the pole at s = -2. Cancel the pole at s = -5

$$K(s) = k\left(\frac{s+5}{s+50}\right)$$

Draw the root locus of GK

$$GK = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)\left(\frac{s+5}{s+50}\right)k$$



Find the spot where the root locus intersects with the 0.4554 damping line

s = -3.1953 + j6.3905

At this point, pick 'k' so that GK = -1

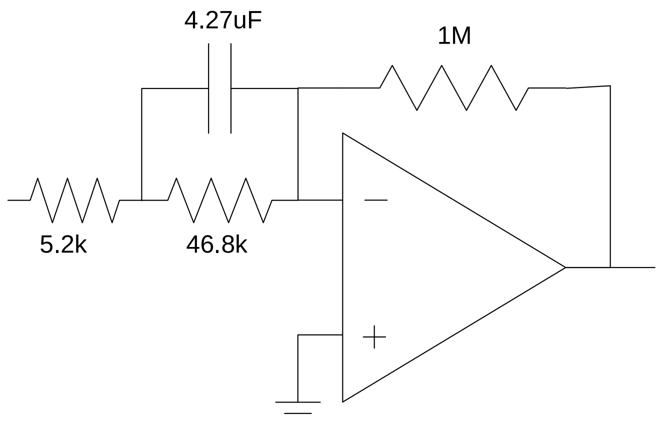
$$\left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)\left(\frac{s+5}{s+50}\right)_{s=-3.1953+j6.3905} = 0.0052\angle 180^0$$

$$k = \frac{1}{0.0052} = 192.42$$

so

$$K(s) = 192.42\left(\frac{s+5}{s+50}\right)$$

Design an op-amp circuit to implement K(s)

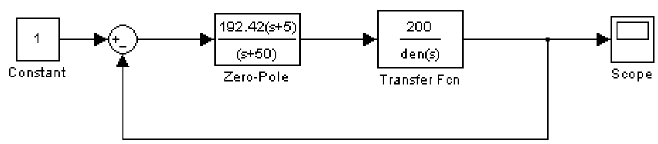


Determine the dominant poles of the closed-loop system

The dominant poles are where we placed them:

$s = -3.1953 + j6.3905$

Plot the step response of the closed-loop system using Matlab (or similar program)



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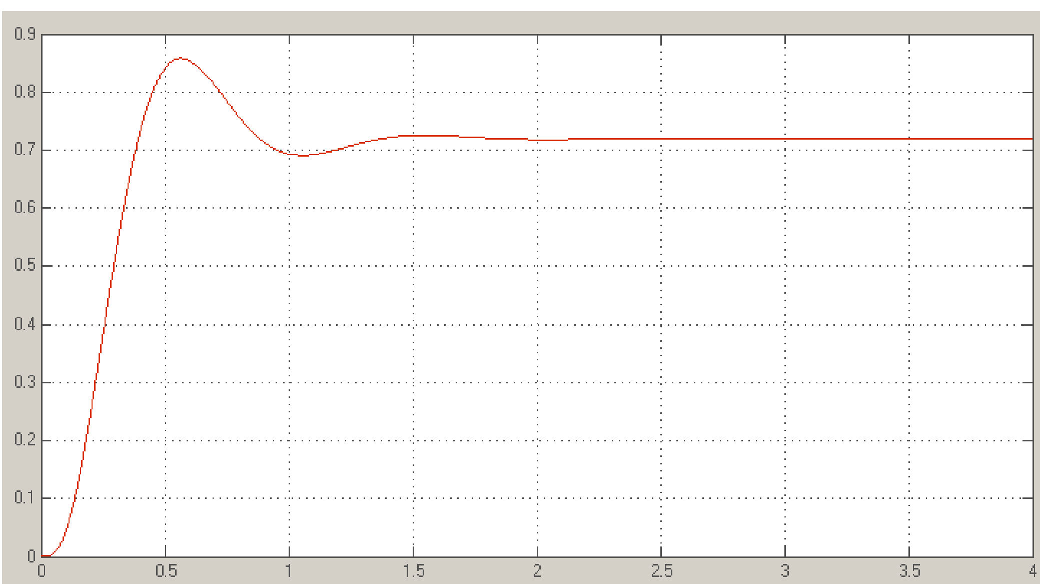


Problem 3: PI Compensation

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)$$

Design a PI compensator which results in the closed-loop system having

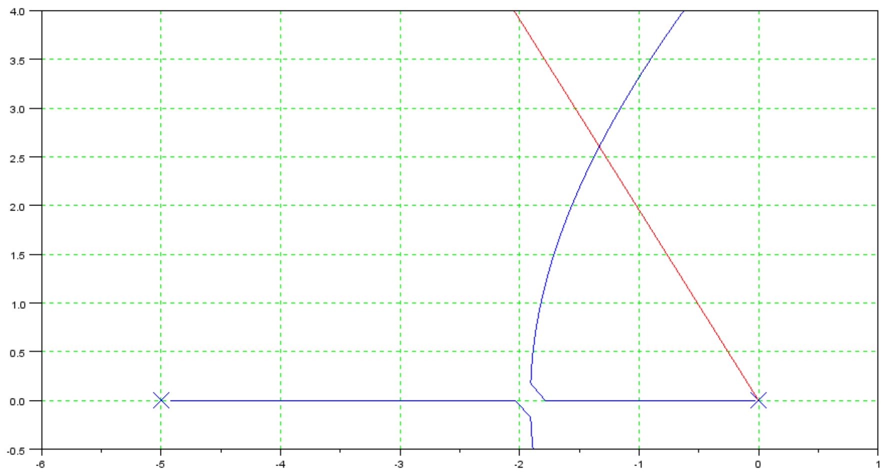
- 20% overshoot for a step input.
- No error for a step input

Cancel the slowest pole (s+2). Add a pole at s = 0 to make it type-1

$$K(s) = k\left(\frac{s+2}{s}\right)$$

Sketch the root locus of GK

$$GK = \left(\frac{200}{s(s+5)(s+10)(s+15)}\right)$$



Find where the root locus intersects the damping line (0.4554 = 20% overshoot)

s = -1.3188 + j2.6376

At this point, pick 'k' so that the gain is -1

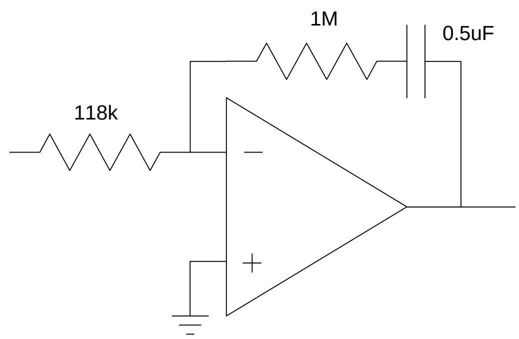
$$\left(\frac{200}{s(s+5)(s+10)(s+15)}\right)_{s=-1.3188+j2.6376} = 0.1185\angle 180^0$$

$$k = \frac{1}{0.1185} = 8.4412$$

so

$$K(s) = 8.4412\left(\frac{s+2}{s}\right)$$

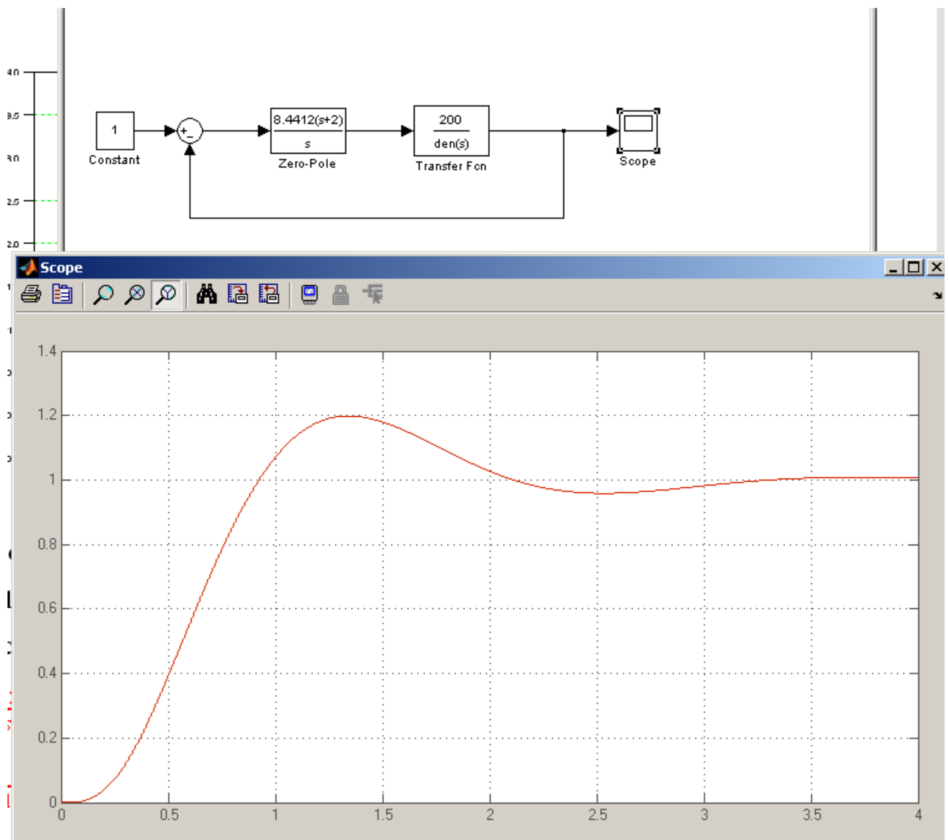
Design an op-amp circuit to implement K(s)



Determine the dominant poles of the closed-loop system

s = -1.3188 + j2.6376

Plot the step response of the closed-loop system using Matlab (or similar program)



Problem 4: General Compensator

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right)$$

Design a compensator which results in the closed-loop system having

- 20% overshoot for a step input.
- No error for a step input, and
- A 2% settling time of 2 seconds.

The closed-loop dominant pole is to be at -2 + j4. Let

$$K(s) = k\left(\frac{(s+2)(s+5)}{s(s+a)}\right)$$

$$GK = \left(\frac{200}{s(s+a)(s+10)(s+15)}\right)$$

At s = -2 + j4

$$\left(\frac{200}{s(s+10)(s+15)}\right)_{s=-2+j4} = 0.3676\angle -160.2328^0$$

For the angles to add up to 180 degrees

$$\angle(s+a)_{s=-2+j4} = 19.7672^0$$

$$a = 2 + \frac{4}{\tan\left(19.7672^0\right)} = 13.1304$$

and

$$K(s) = k\left(\frac{(s+2)(s+5)}{s(s+13.1304)}\right)$$

$$GK = \left(\frac{200}{s(s+13.1304)(s+10)(s+15)}\right)_{s=-2+j4} = 0.0311\angle 180^0$$

This results in

$$k = \frac{1}{0.0311} = 32.1738$$

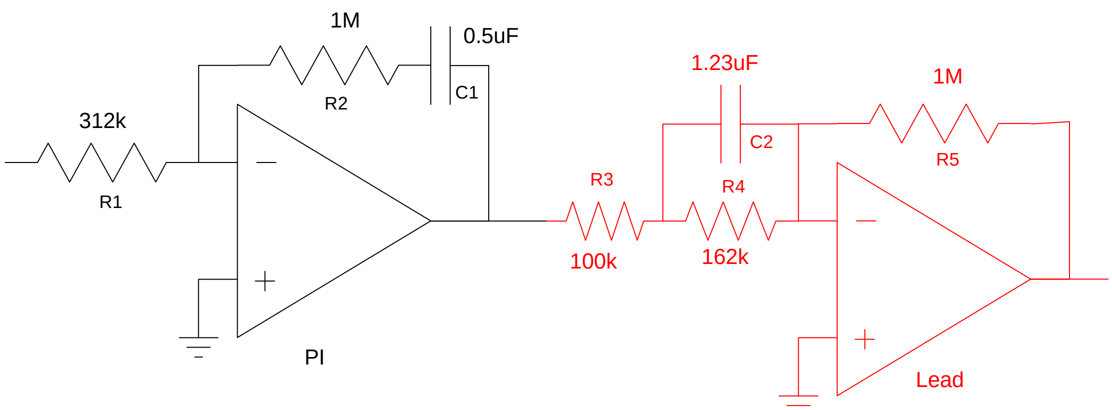
and

$$K(s) = 32.1738\left(\frac{(s+2)(s+5)}{s(s+13.1304)}\right)$$

Design an op-amp circuit to implement K(s)

$$K(s) = 32.1738\left(\frac{(s+2)(s+5)}{s(s+13.1304)}\right)$$

$$K(s) = \left(\frac{3.217(s+2)}{s}\right)\left(\frac{10(s+5)}{(s+13.1304)}\right)$$



Determine the dominant poles of the closed-loop system

s = -2 + j4

Plot the step response of the closed-loop system using Matlab (or similar program)