Solution to Homework #9: ECE 461/661

Meeting design specs, Delays, Lightly Damped Systems

Problem 1: Delays. Assume you have a system with a 200ms delay:

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)}\right) \cdot e^{-0.2s}$$

Design a compensator which results in the closed-loop system having

- 20% overshoot for a step input.
- No error for a step input, and
- A 2% settling time of 4 seconds

Let

$$K(s) = k\left(\frac{(s+2)(s+5)}{s(s+a)}\right)$$

At s = -1 + j2

$$GK = \left(\left(\frac{200}{s(s+a)(s+10)(s+15)}\right) \cdot e^{-0.2s}\right)_{s=-1+j2} = 1\angle 180^0$$

$$\left(\left(\frac{200}{s(s+10)(s+15)}\right) \cdot e^{-0.2s}\right)_{s=-1+j2} = 0.8379\angle -160.14^0$$

meaning that the term (s+a) contributes another 19.85 degrees

$$\angle(s+a) = 19.85^0$$

$$a = 1 + \frac{2}{\tan\left(19.85^0\right)} = 6.5377$$

and

$$K(s) = k\left(\frac{(s+2)(s+5)}{s(s+6.5377)}\right)$$

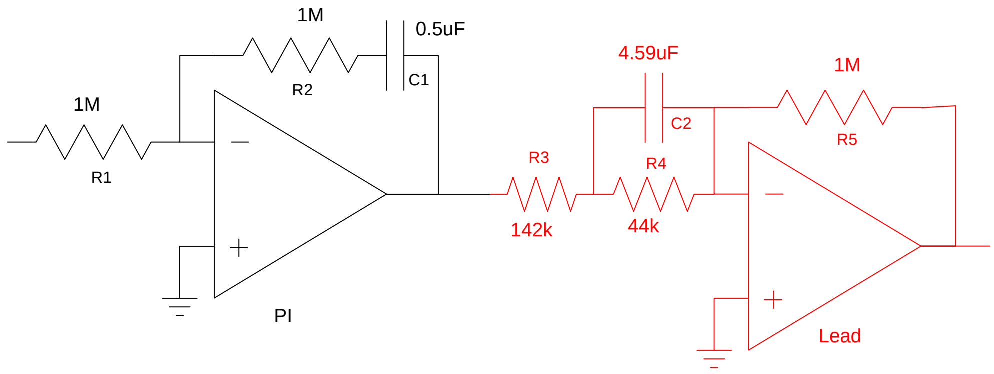
$$GK = \left(\left(\frac{200}{s(s+6.5377)(s+10)(s+15)}\right) \cdot e^{-0.2s}\right)_{s=-1+j2} = 0.1423\angle 180^0$$

$$k = \frac{1}{0.1423} = 7.0271$$

$$K(s) = 7.0271\left(\frac{(s+2)(s+5)}{s(s+6.5377)}\right)$$

Design an op-amp circuit to implement K(s)

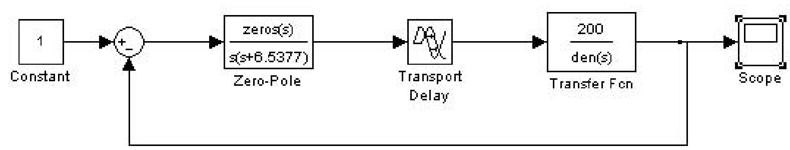
$$K(s) = \left(\frac{s+2}{s}\right)\left(\frac{7.02(s+5)}{(s+6.5377)}\right)$$

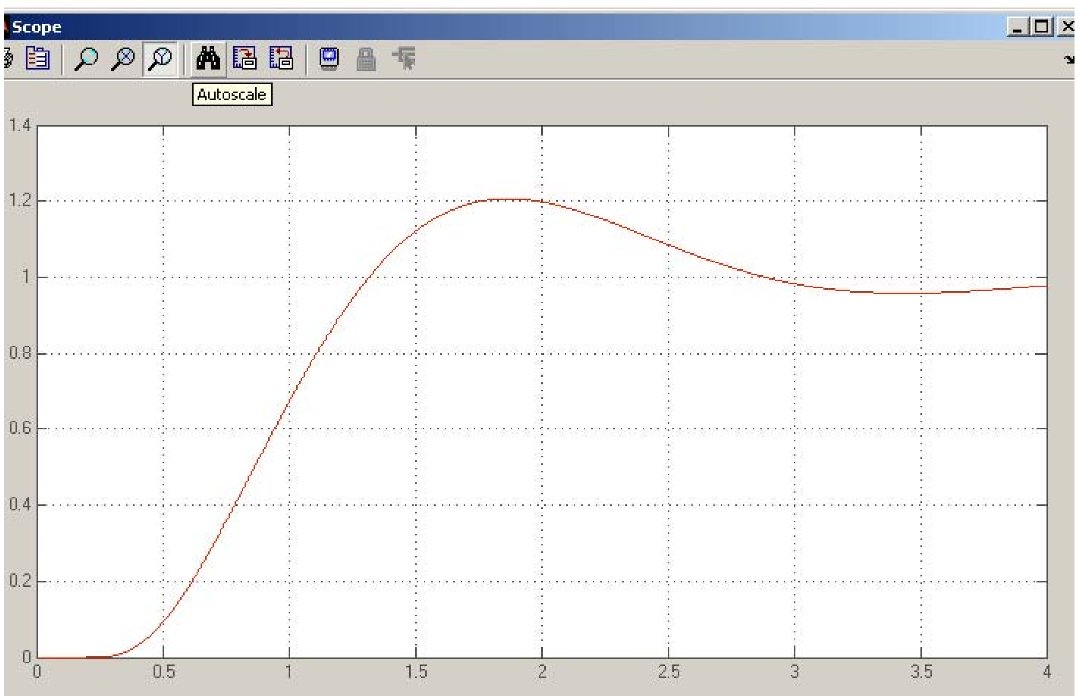


Determine the dominant poles of the closed-loop system

$s = -1 + j2$

Plot the step response of the closed-loop system using VisSim (or similar program)





Problem 2: Lightly Damped Systems: Assume you have a system which is lightly damped:

$$G(s) = \left(\frac{200}{(s+2)(s+j4)(s-j4)}\right)$$

Design a compensator which results in the closed-loop system having

- 20% overshoot for a step input.
- No error for a step input, and
- A 2% settling time of 4 seconds

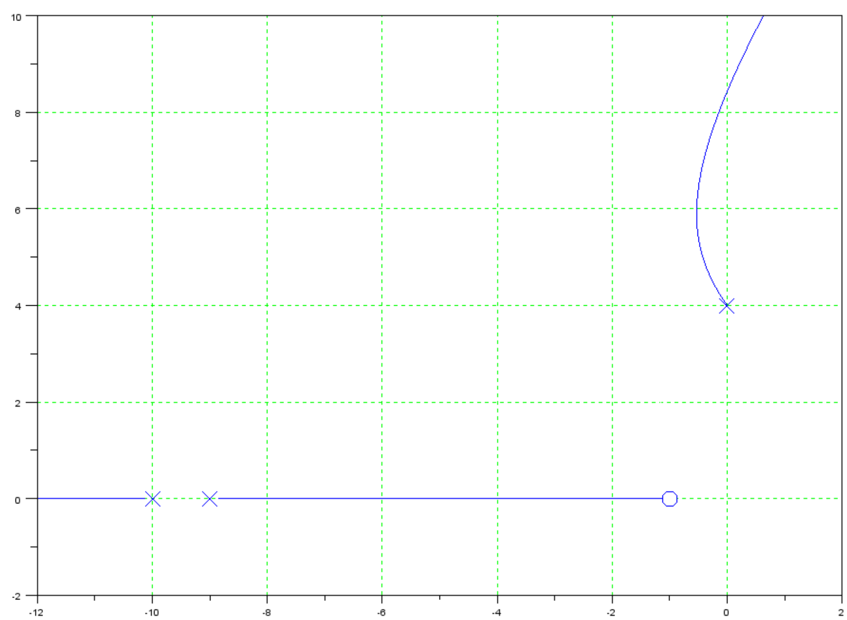
- Design an op-amp circuit to implement K(s)
- Determine the dominant poles of the closed-loop system
- Plot the step response of the closed-loop system using VisSim (or similar program)

Step 1: Stabilize the System. Ignore the requirements - just come up with a compensator to stabilize the closed-loop system.

$$K_1(s) = \left(\frac{(s+1)(s+2)}{(s+10)(s+11)}\right)$$

$$GK_1 = \left(\frac{200(s+1)}{(s+j4)(s-j4)(s+10)(s+11)}\right)$$

Sketch the root locus



It's not great, but at least it's stable. Pick a spot on the root locus and find k:

```
>> G = zpk([],[-2,j*4,-j*4],200);
>> tf(G)

             200
-----------------------
s^3 + 2 s^2 + 16 s + 32

>> K1 = zpk([-1,-2],[-10,-11],1);
>> GK1 = G*K1;
```

```
>> s = -0.52 + j*6;
>> evalfr(GK1,s)

  -0.4272 - 0.0430i

>> 1/abs(ans)

    2.3288

>> K1 = zpk([-1,-2],[-10,-11],2.3288)

      2.3288 (s+1) (s+2)
K1 = ------------------
        (s+10) (s+11)
```

$$K_1 = 2.3288\left(\frac{(s+1)(s+2)}{(s+10)(s+11)}\right)$$

```
>> G2 = minreal( G*K1 / (1 + G*K1) )

                      465.76 (s+1)
G2 = ------------------------------------------
     (s+3.902) (s+15.64) (s^2 + 1.459s + 36.47)

>> roots([1,1.459,36.47])


  -0.7295 + 5.9948i
  -0.7295 - 5.9948i
```

Add a second compensator to meet the requirements

$$K_2 = \left(\frac{(s+3.902)(s+0.7295+j6)(s+0.7295-j6)}{s(s+1)(s+a)}\right)$$

$$G_2K_2 = \left(\frac{400}{s(s+a)(s+15.64)}\right)$$

Pick 'a' so that s = -1 + j2 is on the root locus

$$\left(\frac{400}{s(s+15.64)}\right)_{s=-1+j2} = 12.106\angle -124.34^0$$

For the angles to add to 180 degrees

$$\angle(s+a) = 55.66^0$$

$$a = 1 + \frac{2}{\tan(55.66^0)} = 2.367$$

meaning

$$G_2K_2 = \left(\frac{400}{s(s+2.367)(s+15.64)}\right)$$

```
>> K2 = zpk([-3.902,-0.7295+j*6,-0.7295-j*6],[0,-1,-2.367],1);
>> evalfr(G2*K2,-1+j*2)

  -5.8299 - 0.0012i

>> 1/abs(ans)
```

```
    0.1715

>> K2 = zpk([-3.902,-0.7295+j*6,-0.7295-j*6],[0,-1,-2.367],0.1715)

     0.1715 (s+3.902) (s^2 + 1.459s + 36.53)
K2 = --------------------------------------
                s (s+1) (s+2.367)

>> tf(K2)

0.1715 s^3 + 0.9194 s^2 + 7.242 s + 24.45
-----------------------------------------
        s^3 + 3.367 s^2 + 2.367 s
```

Checking in VisSim (or Simulink)