Homework #10: ECE 461/661

Unstable Systems, z-Transform, Converting G(s) to G(z)   Due Wednesday, November 14, 2018

Unstable Systems

Problem 1: Assume

$$G(s) = \left(\frac{200}{(s-2)(s+5)(s+10)(s+15)}\right)$$

Design a which results in the closed-loop system having

- 20% overshoot for a step input.
- No error for a step input, and
- A 2% settling time of 2 seconds

- Design an op-amp circuit to implement K(s)
- Determine the dominant poles of the closed-loop system
- Plot the step response of the closed-loop system using VisSim (or similar program)

Step 1: Stabilize the system

Let

$$K_1(s) = k\left(\frac{s+5}{s+20}\right)$$

$$GK_1 = \left(\frac{200}{(s-2)(s+20)(s+10)(s+15)}\right)$$

Place the closed-loop pole at s = -1

$$\left(\frac{200}{(s-2)(s+20)(s+10)(s+15)}\right)_{s=-1} = -0.0278$$

$$k = \frac{1}{0..0278} = 35.91$$

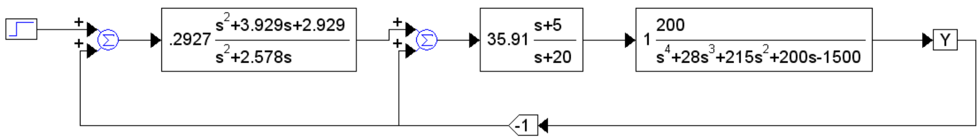
$$\boxed{K_1(s) = 35.91\left(\frac{s+5}{s+20}\right)}$$

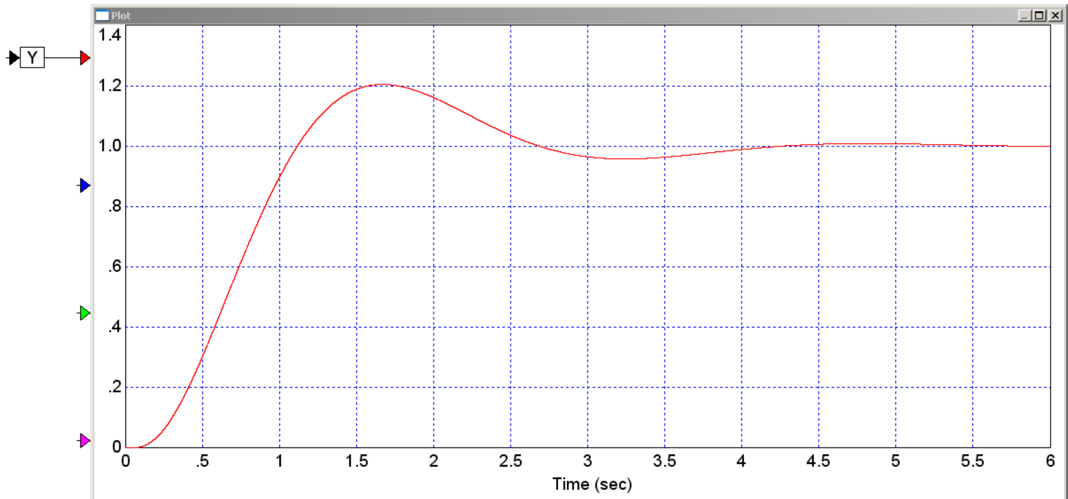
In Matlab:

```
G = zpk([],[2,-5,-10,-15],200);
K1 = zpk(-5,-20,35.91);
GK1 = minreal(G*K1)

            7182
--------------------------
(s-2) (s+10) (s+15) (s+20)

G2 = minreal(G*K1 / (1+G*K1))

                      7182
G2 = --------------------------------------
     (s+2.929) (s+1) (s^2 + 39.07s + 403.6)
```

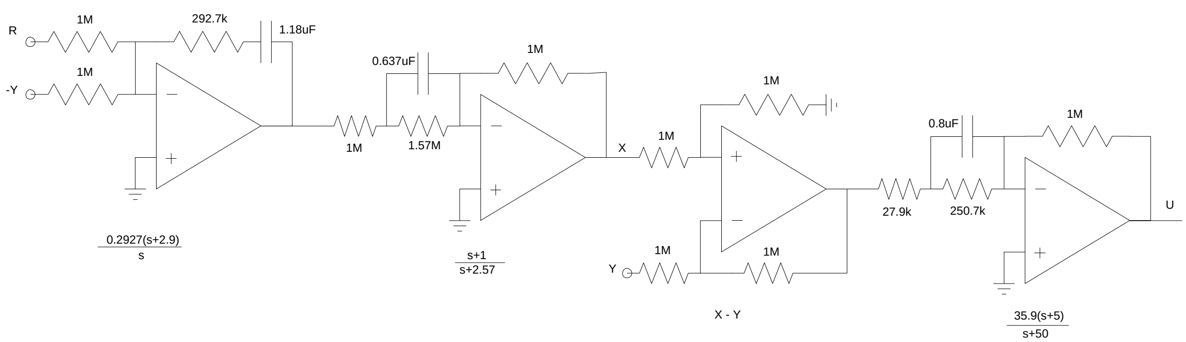
Now add a second compensator to meet the design specs

```
>> K2 = zpk([-1,-2.929],[0,-1000],1);
>> evalfr(G2*K2, -1+j*2)

  -0.0054 - 0.0068i

>> angle(ans)*180/pi

 -128.2719

>> 180 + ans

   51.7281

>> K2 = zpk([-1,-2.929],[0,-2.5779],1);

>> evalfr(G2*K2, -1+j*2)

  -3.4167 - 0.0068i

>> 1/abs(ans)

    0.2927

>> K2 = zpk([-1,-2.929],[0,-2.5779],0.2927)

     0.2927 (s+1) (s+2.929)
K2 = ----------------------
          s (s+2.578)


G3 = minreal(G2*K2 / (1+G2*K2))


                  2102.1714
-------------------------------------------------------
 (s^2 + 2.005s + 5.008) (s^2 + 39.64s + 419.8)
```


.2927
s^2+3.929s+2.929
s^2+2.578s
35.91
s+5
s+20
1
200
s^4+28s^3+215s^2+200s-1500
Y
-1


Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
Time (sec)

Circuit:


R
-Y
1M
1M
292.7k
1.18uF
0.2927(s+2.9)
s
0.637uF
1M
1M
1.57M
s+1
s+2.57
X
1M
1M
Y
1M
1M
X - Y
0.8uF
1M
27.9k
250.7k
U
35.9(s+5)
s+50

z-Transform and Converting G(s) to G(z)

Problem 2: Given the following system

$$G(s) = \left(\frac{200}{(s+2)(s+5)(s+10)(s+15)} \right)$$

a) Determine a discrete-time system, G(z), which has approximately the same step response as G(s). Assume a sampling rate of 100ms

First, convert the poles in the s-plane to the z-plane:

```
s = [-2,-5,-10,-15]'

    -2
    -5
   -10
   -15

T = 0.1;
z = exp(s*T)

    0.8187
    0.6065
    0.3679
    0.2231
```

G(z) is then

```
Gz = zpk([],z,1)

                     1
-------------------------------------------
(z-0.8187) (z-0.6065) (z-0.3679) (z-0.2231)
```

Add a gain to match the DC gain (s=0, z=1)

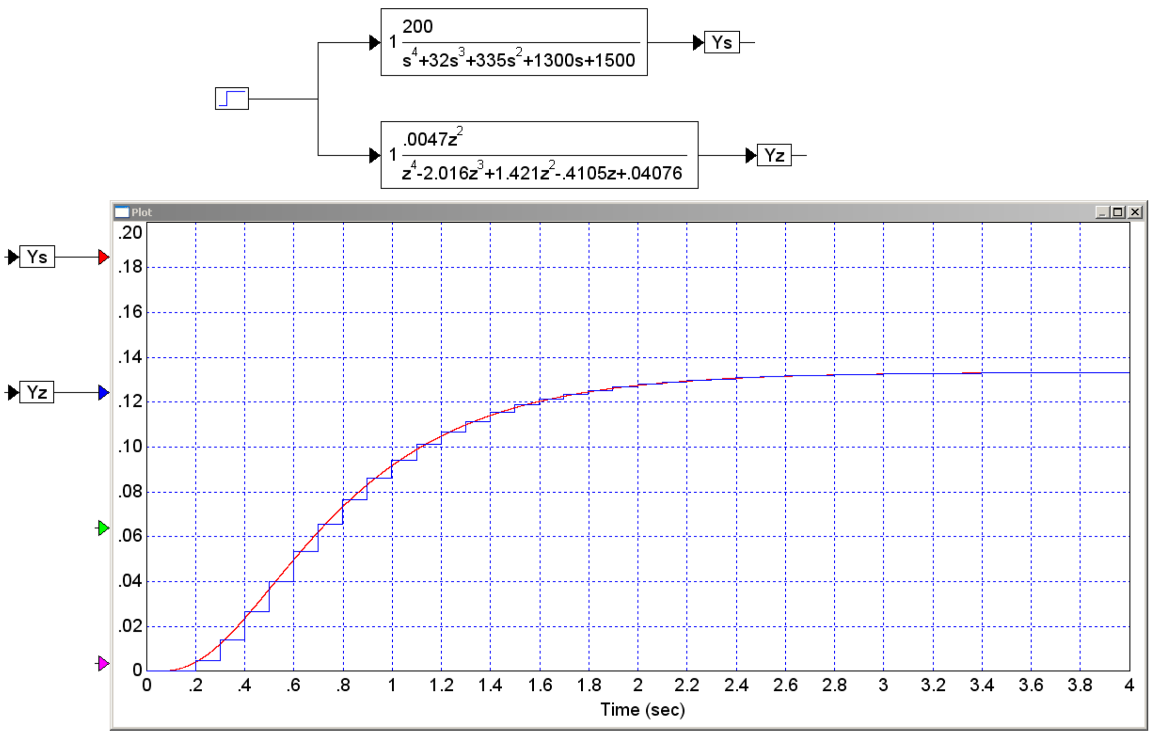
```
evalfr(Gz,1)

   28.5507

Gs = zpk([],[-2,-5,-10,-15],200);
evalfr(Gs,0)

    0.1333

k = 0.1333 / 28.5507

k =    0.0047
```

This gives G(z):

```
Gz = zpk([],z,0.0047)

                         0.0047
G(z) = -------------------------------------------
       (z-0.8187) (z-0.6065) (z-0.3679) (z-0.2231)
```

b) Plot the step response of G(s) and G(z) using VisSim (or similar program)

```
>> tf(Gs)

                200
--------------------------------------
s^4 + 32 s^3 + 335 s^2 + 1300 s + 1500

>> tf(Gz)

                     0.0047 z^2
------------------------------------------------
z^4 - 2.016 z^3 + 1.421 z^2 - 0.4105 z + 0.04076
```



c) Write a program (c-like or matlab-like) to implement G(z)

$$Y = \left(\frac{0.0047z^2}{z^4 - 2.016z^3 + 1.421z^2 - 0.4105z + 0.04076} \right) X$$

```
while(1) {
   x2 = x1;
   x1 = x0;
   x0 = A2D_Read(0);

   y4 = y3;
   y3 = y2;
   y2 = y1;
   y1 = y0;
   y0 = 2.016*y1 - 1.421*y2 + 0.4105*y3 - 0.04076*y4 + 0.0047*x2;

   D2A(y0);
   Wait100ms();
   }
```

Problem 3: Given the following system

$$G(s) = \left(\frac{200}{(s^2+2s+10)(s+10)}\right)$$

a) Determine a discrete-time system, G(z), which has approximately the same step response as G(s). Assume a sampling rate of 100ms

Convert the poles to the z-plane

```
s = [-1+j*3, -1 - j*3, -10]'

  -1.0000 - 3.0000i
  -1.0000 + 3.0000i
 -10.0000

>> T = 0.1;
>> z = exp(s*T)

   0.8644 - 0.2674i
   0.8644 + 0.2674i
   0.3679
```

The denominator polynomial is then:

```
poly(z)

    1.0000   -2.0967    1.4547   -0.3012

Gz = zpk([],z,1)

                1
----------------------------------
(z-0.3679) (z^2 - 1.729z + 0.8187)
```

Add a gain to match the DC gain. G(s=0) = 2.

```
evalfr(Gz,1)
   17.6005

k = 2/17.6005

    k = 0.1136
```

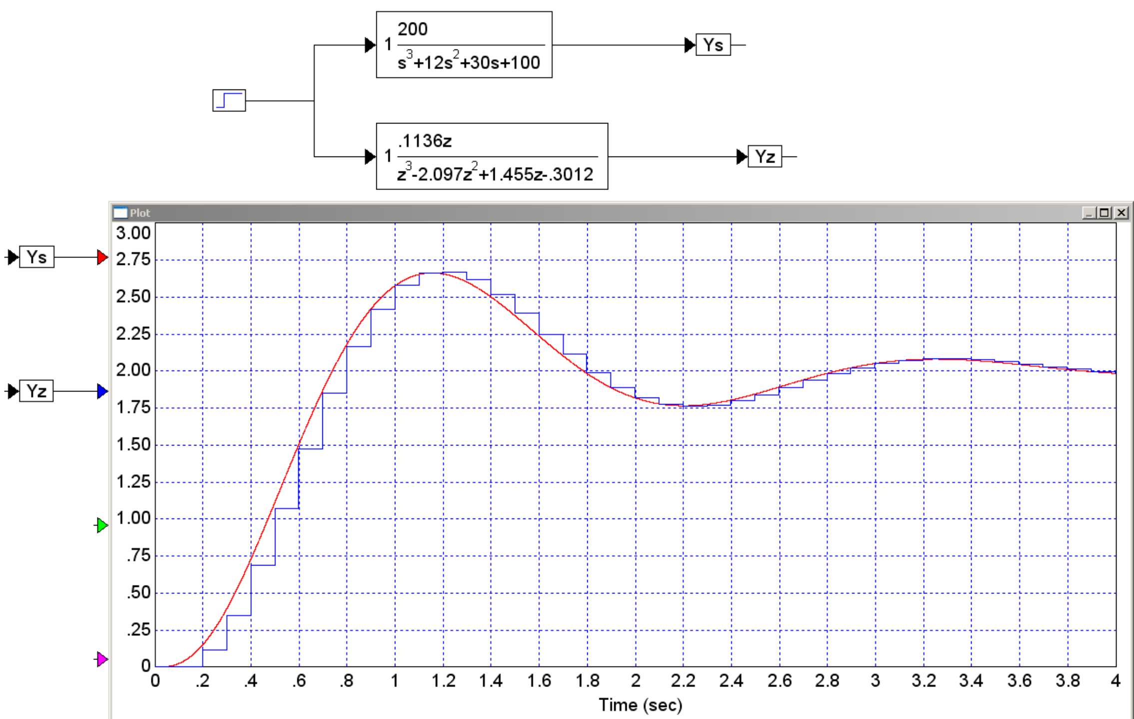
This gives G(z):

```
>> Gz = zpk([],z,0.1136)

                   0.1136
G(z) = ----------------------------------
       (z-0.3679) (z^2 - 1.729z + 0.8187)
```

note: A zero at z=0 is added to adjust the delay to better match up with the step response of G(s)

b) Plot the step response of G(s) and G(z) using VisSim (or similar program)

```
tf(Gs)

                     200
G(s) = -------------------------
        s^3 + 12 s^2 + 30 s + 100

>> tf(Gz)

                     0.1136z
G(z) = ---------------------------------
        z^3 - 2.097 z^2 + 1.455 z - 0.3012
```



c) Write a program (c-like or matlab-like) to implement G(z)

$$Y = \left(\frac{0.1136z}{z^3 - 2.097z^2 + 1.455z - 0.3012}\right) X$$

```
while(1) {
   x2 = x1;
   x1 = x0;
   x0 = A2D_Read(0);

   y3 = y2;
   y2 = y1;
   y1 = y0;
   y0 = 2.097*y1 - 1.455*y2 + 0.3012*y3 + 0.1136*x2;

   D2A(y0);
   Wait100ms();
   }
```

Problem 4: Given the following feedback controller

$$K(s) = 6\left(\frac{(s+2)(s+5)}{s(s+20)}\right)$$

a) Determine a discrete-time system, K(z), which has approximately the same step response as K(s). Assume a sampling rate of 100ms

Convert the zeros to the z-plane

```
s = [-2, -5]';
T = 0.1;
z = exp(s*T)

     0.8187
     0.6053
```

Convert the poles to the z-plane

```
s = [0, 20]';
z = exp(s*T)

     1.0000
     0.1353
```

So....

```
Gz = zpk(exp([-2,-5]*T),exp([0,-20*T]),1)

        (z-0.8187) (z-0.6065)
G(z) = ---------------------
          (z-1) (z-0.1353)
```

Add a gain so that the DC gain matches the DC gain of G(s). Since s = 0 doesn't work, pick a point close to s=0 (s = 0.01, $z = e^{sT}$)

```
evalfr(Gs,0.01)

  301.9520

evalfr(Gz,exp(0.01*T))

   83.0159

k = 301.95/83.01

   k =  3.6375
```

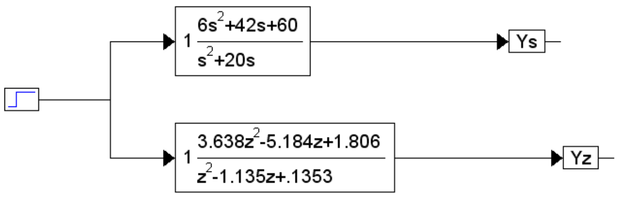
Giving....

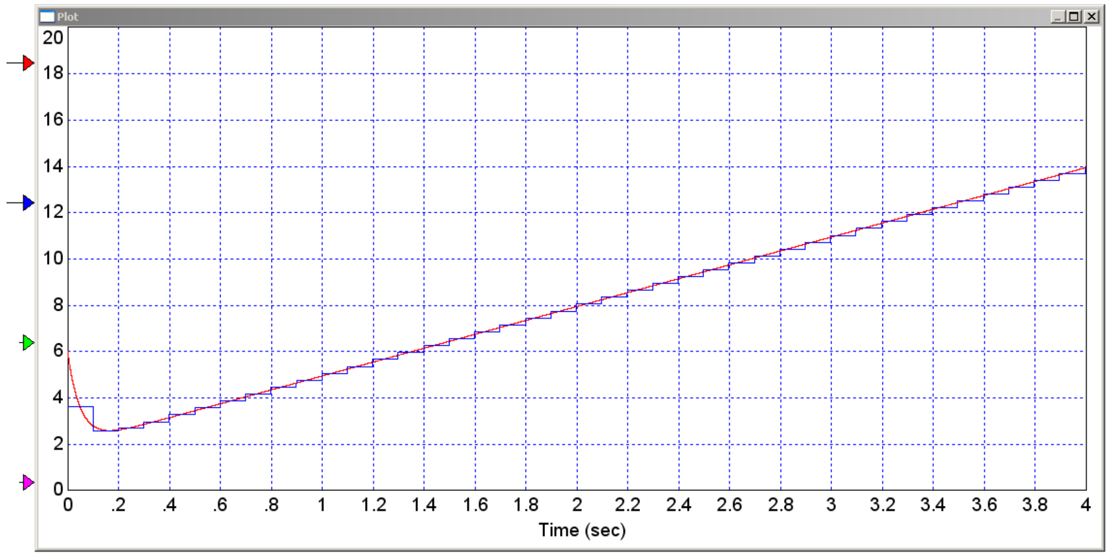
```
Gz = zpk(exp([-2,-5]*T),exp([0,-20*T]),3.6375)
```

```
        3.6375 (z-0.8187) (z-0.6065)
G(z) = ----------------------------
               (z-1) (z-0.1353)
```

b) Plot the step response of K(s) and K(z) using VisSim (or similar program)

```
        6 s^2 + 42 s + 60
G(s) = -----------------
            s^2 + 20 s

        3.638 z^2 - 5.184 z + 1.806
G(z) = --------------------------
          z^2 - 1.135 z + 0.1353
```





c) Write a program (c-like or matlab-like) to implement K(z)

$$Y = \left(\frac{3.638z^2 - 5.184z + 1.806}{z^2 - 1.135z + 0.1353}\right)X$$

```
while(1) {
   x2 = x1;
   x1 = x0;
   x0 = A2D_Read(0);

   y2 = y1;
   y1 = y0;
   y0 = 1.135*y1 - 0.1353*y2 + 3.638*x0 - 5.184*x1 + 1.806*x2;

   D2A(y0);
   Wait100ms();
   }
```