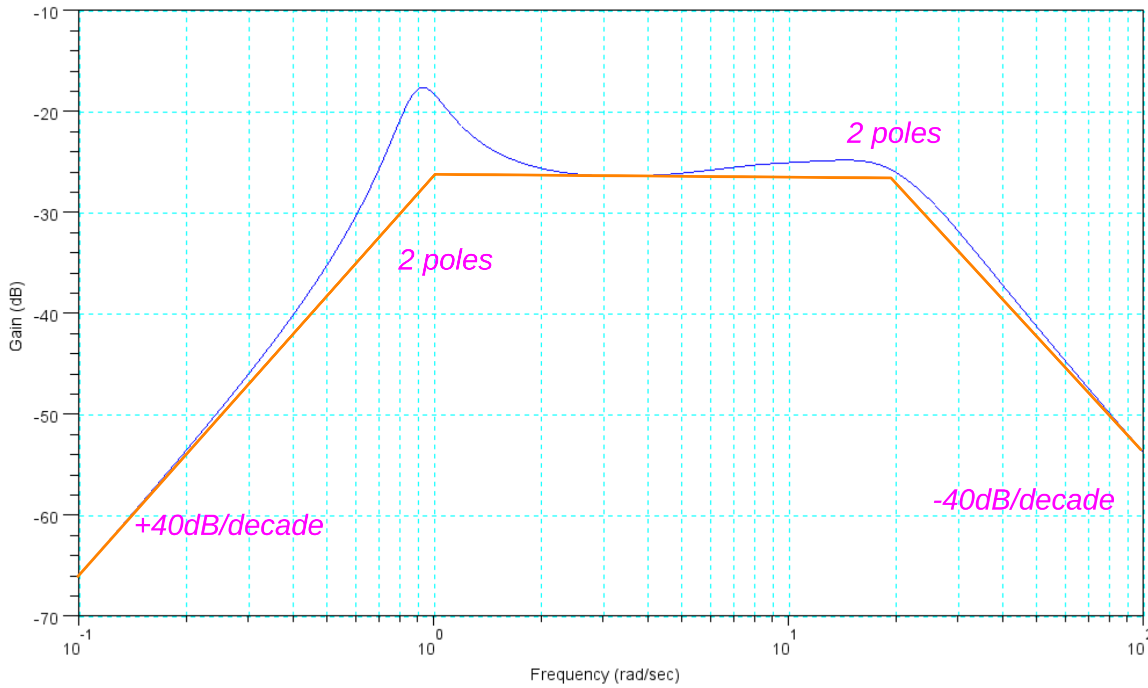
Homework #12: ECE 461/661

Bode Plots, Nichols Charts, Nyquist and Inverse Nyquist Diagrams. Due Monday, December 3rd, 2018

Bode Plots

1) Determine the system which has the following gain vs. frequency (Bode plot)



Start by drawing the straight-line approximation with all slopes multiples of 20dB/decade

There are two corners: 2 poles at s = 1 and 2 poles at s = 20

The damping ratio comes from the gain at the corner (relative to the corner)

s = 1:

gain @ corner = +8dB (above the corner) = 2.5119

$$\frac{1}{2\zeta} = 2.5119 \qquad \zeta = 0.1991 \qquad \theta = 75.8^0$$

s= 20:

gain @ corner = +1dB above the corner = 1.122

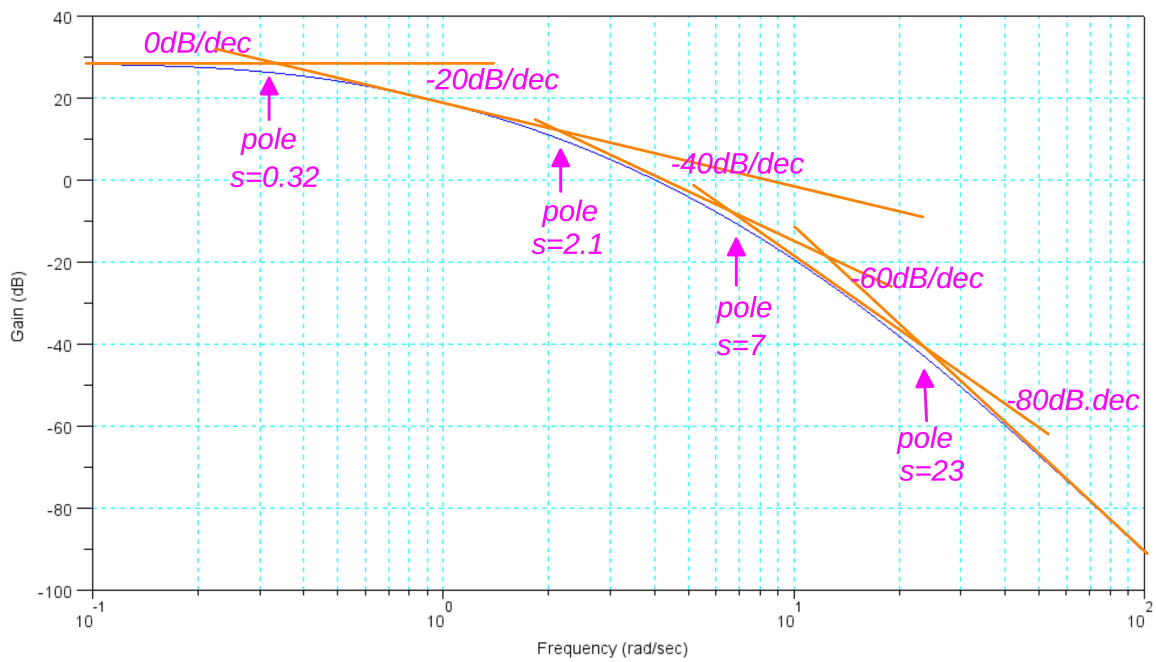
$$\frac{1}{2\zeta} = 1.122 \qquad \zeta = 0.4456 \qquad \theta = 63.5^0$$

so G(s) is as follows. Plug in a point on the curve to get the numerator gain:

$$G(s) \approx \left(\frac{ks^2}{(s+1\angle\pm 75.8^0)(s+20\angle\pm 63.5^0)} \right)_{s=j3} = -26dB = 0.0501$$

$$\boxed{G(s) \approx \left(\frac{20s^2}{(s+1\angle\pm 75.8^0)(s+20\angle\pm 63.5^0)} \right)}$$

2) Determine the system which has the following gain vs. frequency



Draw in the straight-line approximations with lines at multiples of 20dB/decade (shown in orange)

The corners are poles. From this

$$G(s) \approx \left(\frac{k}{(s+0.32)(s+2.1)(s+7)(s+23)}\right)$$

Plug in a point to find 'k'.

$$\left(\frac{k}{(s+0.32)(s+2.1)(s+7)(s+23)}\right)_{s=j0.1} = +28dB = 25.1$$

$$k = 2715$$

resulting in

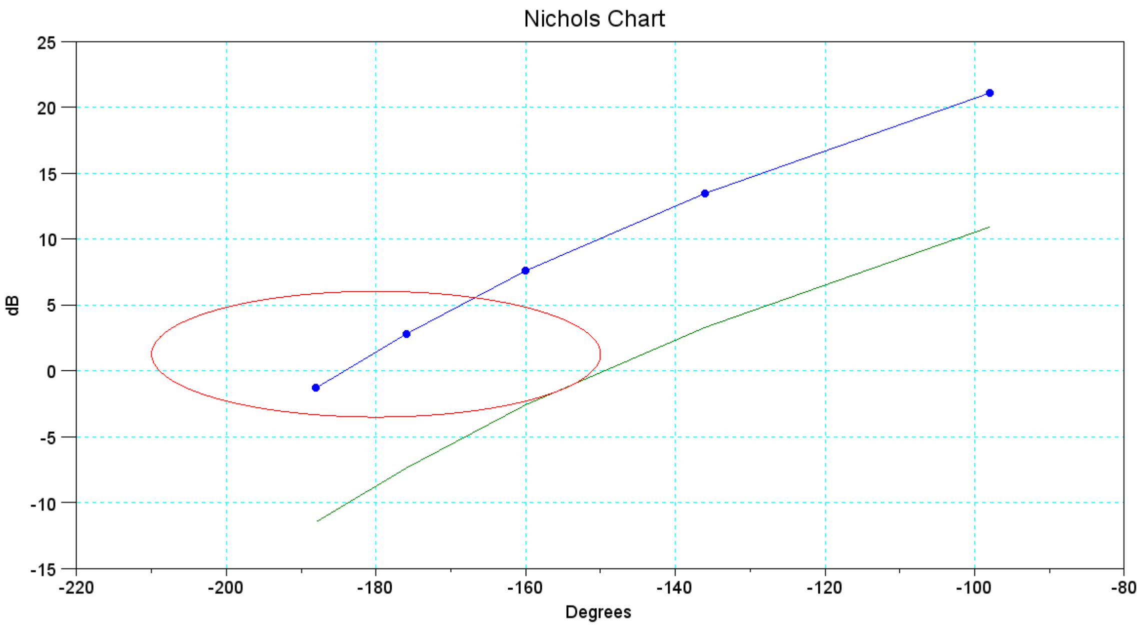
$$G(s) \approx \left(\frac{2715}{(s+0.32)(s+2.1)(s+7)(s+23)}\right)$$

Nichols Charts

3) The gain vs. frequency for a system is as follows:

Freq (rad/sec)	0	2	1	3	4	5
Gain (dB)	29.11	21.07	13.46	7.58	2.77	-1.31
Phase (deg)	0	-98	-136	-160	-176	-188

a) Plot this data on a Nichols Chart



b) Determine the range of gain, k, which results in a stable closed-loop system

G(jw) = +2dB when the phase is 180 degrees.

k < -2dB for stability

c) From this data, determine the gain, k, which results in a resonance of Mm = +6dB

Shift the curve down until it it stable and tangent to the 6dB m-circle

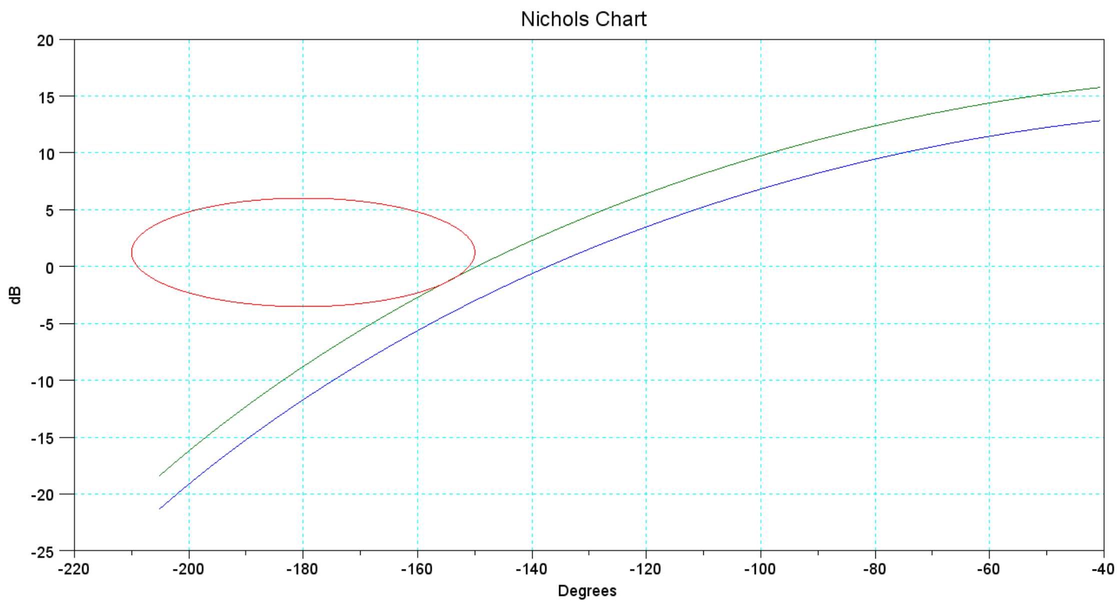
k = 0.31

4) For the system

$$G(s) = \left(\frac{1000}{(s+2)(s+5)(s+20)} \right)$$

a) Plot the gain & phase of G(s) on a Nichols chart

```
w = [1:0.01:10]';
s = j*w;
Gw = 1000 ./ ( (s+2).*(s+5).*(s+20) ) .* exp(-0.2*s);
nichols(Gw*[1,1.4],2);
```



b) Determine the gain, k, which results in a resonance of Mm = +6dB

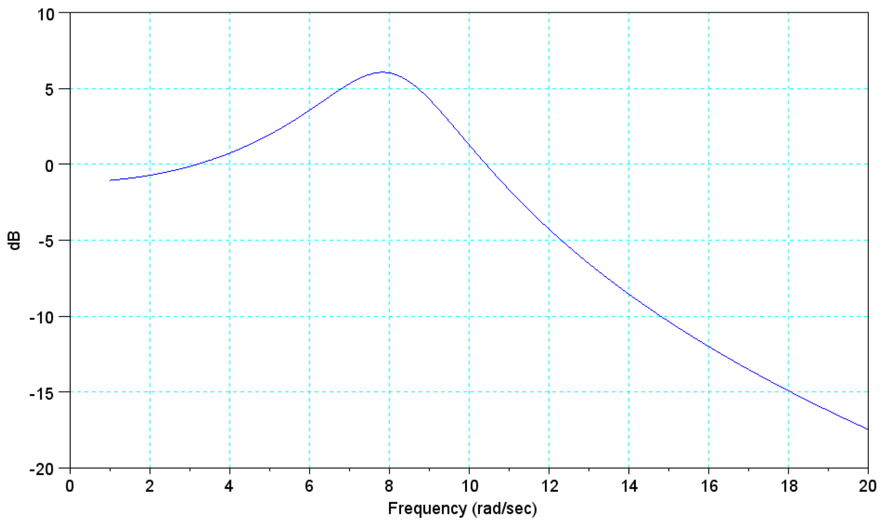
Trial and error: keep guessing gains until you're tangent to the 6dB M-circle (shown in green)

k = 1.4

c) Check your answer by plotting the closed-loop gain vs. frequency

$$G_{cl} = \left(\frac{Gk}{1+Gk} \right)$$

```
k = 1.4;
Gcl = Gw*k ./ (1 + Gw*k);
plot(w,20*log10(abs(Gcl)));
```



10
5
0
-5
-10
-15
-20
dB
0
2
4
6
8
10
12
14
16
18
20
Frequency (rad/sec)

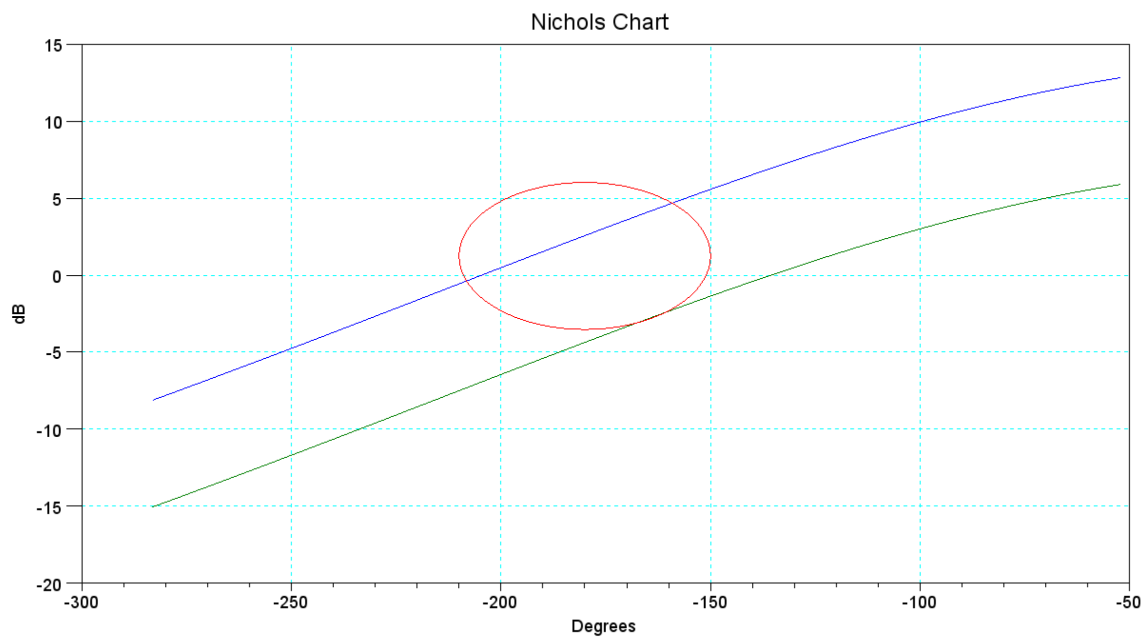
5) For the system with a 200ms delay

$$G(s) = \left(\frac{1000}{(s+2)(s+5)(s+20)}\right) \cdot e^{-0.2s}$$

a) Plot the gain & phase of G(s) on a Nichols chart

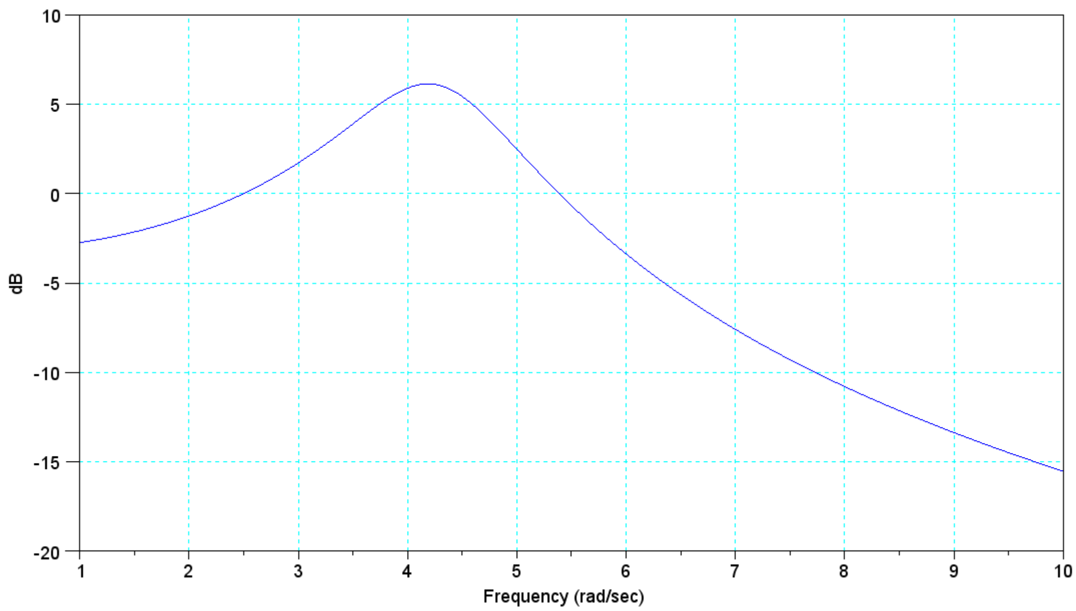
```
w = [1:0.01:10]';
s = j*w;
Gw = 1000 ./ ( (s+2).*(s+5).*(s+20) ) .* exp(-0.2*s);
nichols(Gw*[1,0.45],2);
```

b) Determine the gain, k, which results in a resonance of Mm = +6dB



c) Check your answer by plotting the closed-loop gain vs. frequency

```
k = 0.45;
Gcl = Gw*k ./ (1 + Gw*k);
plot(w,20*log10(abs(Gcl)));
```


10
5
0
-5
-10
-15
-20
dB
1
2
3
4
5
6
7
8
9
10
Frequency (rad/sec)

Nyquist Plots

6) Using a Nyquist plot, determine the gain, k, which results in a resonance of Mm = +6dB

$$G(s) = \left(\frac{1000}{(s+2)(s+5)(s+20)}\right)$$

Plot the open-loop gain of G(jw) as real vs. complex.

```
Gw = 1000 ./ ( (s+2).*(s+5).*(s+20) );
plot(real(Gw),imag(Gw))
```

Add in the M-circle for a closed-loop gain of 2.00 (6dB)

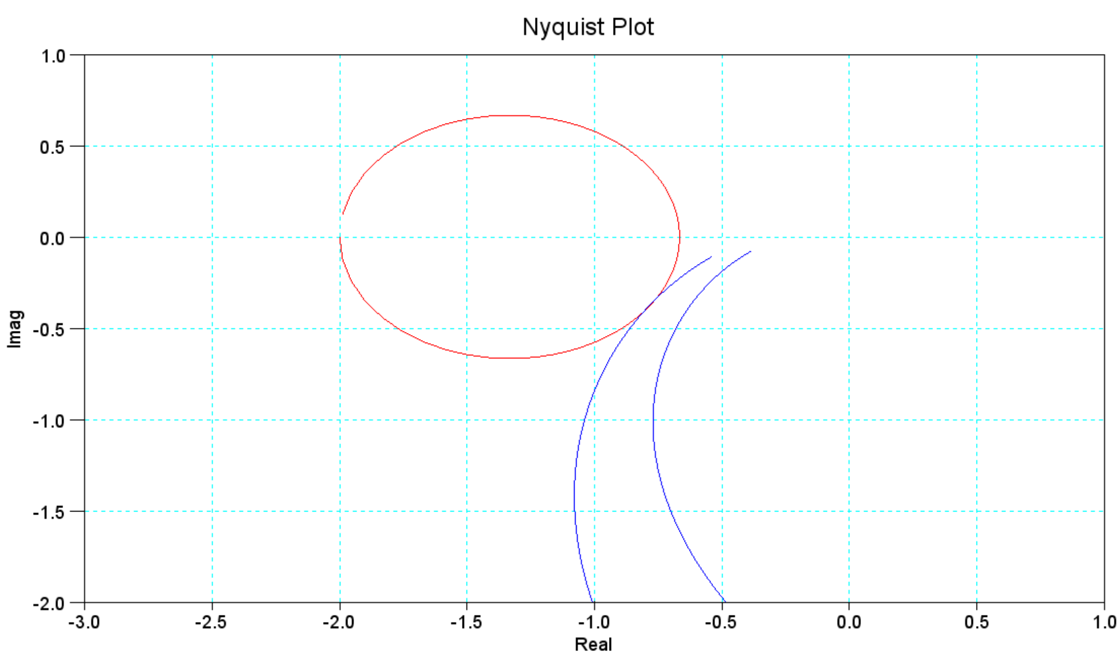
```
P = [1:100]' * 2 * %pi / 100;
Mcl = 2*exp(j*P);
Mol = Mcl ./ (1-Mcl);

plot(real(Mol),imag(Mol),'r')
```

Guess the gain (k) until G*k is tangent to the m-circle

```
k = 1.4;

plot(real(Gw*k),imag(Gw*k))
xlabel('Real');
ylabel('Imag');
```



Nyquist Plot of G(jw) and 1.4*G(jw) along with the 6dB M-circle

Inverse Nyquist Plots

7) Using a Nyquist plot, determine the gain, k, which results in a resonance of Mm = +6dB

$$G(s) = \left(\frac{1000}{(s+2)(s+5)(s+20)} \right)$$

Plot the open-loop gain of 1 / G(jw)

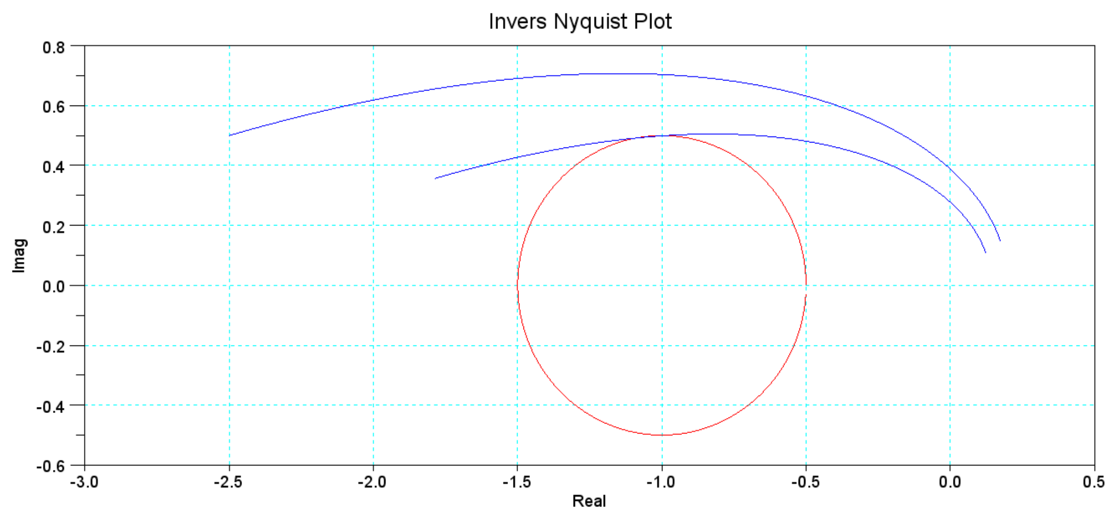
```
Gwi = 1 ./ Gw;
plot(real(Gwi),imag(Gwi))
```

Add the M-circle (noting that we're plotting the inverse gain)

```
Moli = 1 ./ Mol;
plot(real(Moli),imag(Moli),'r')
```

Guess the gain, k, until you're tangent to the M-circle

```
plot(real(Gwi/1.4),imag(Gwi/1.4))
xgrid(4)
xlabel('Real');
ylabel('Imag');
title('Invers Nyquist Plot');
```



Inverse-Nyquist Plot of G(jw) and 1.4*G(jw) along with the 6dB M-circle