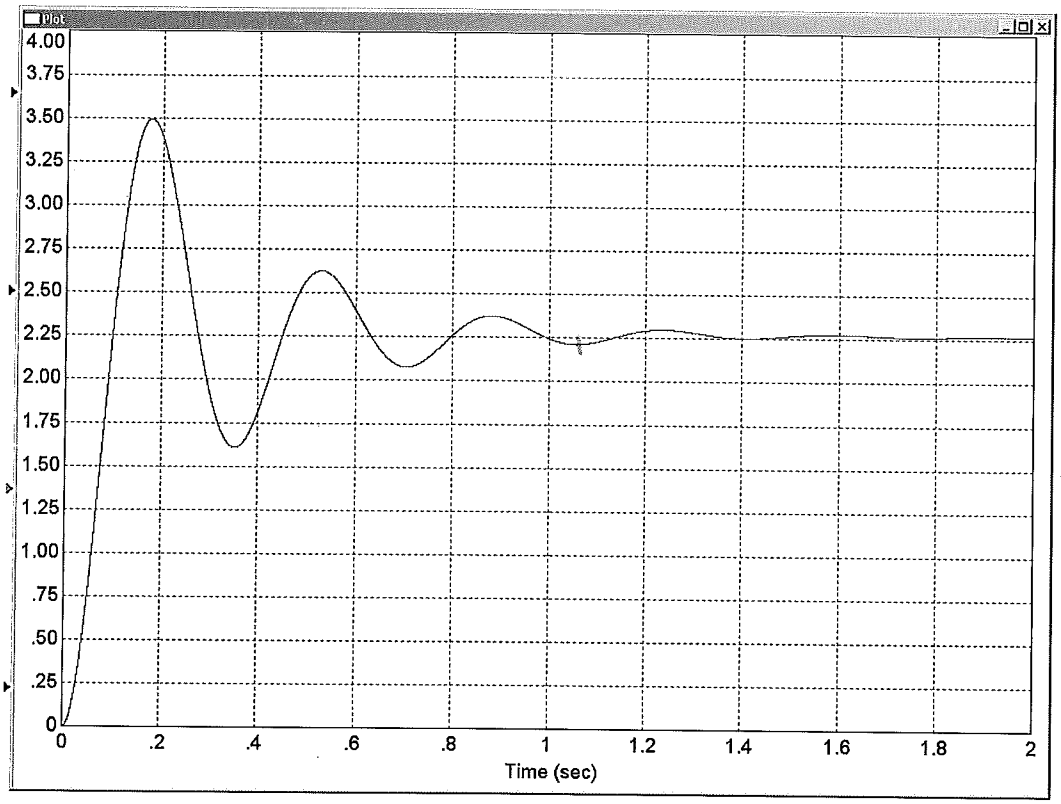
ECE 461/661 - Test #2: Name ____________________

October 19, 2018

1) Give the transfer function for a system with the following response to a unit step input:



$T_s \approx 1.2$ sec $\quad \sigma = \frac{4}{T_s} = 3.33$

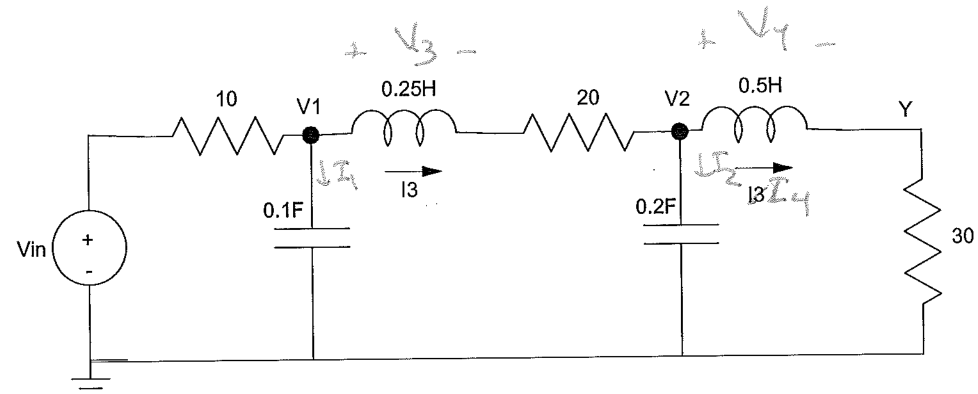
$\omega_d \approx \frac{3 \text{ cycles}}{1.05 \text{ sec}} \cdot 2\pi = 17.95$

DC gain ≈ 2.25

$$G(s) \approx \frac{2.25 \cdot (3.33^2 + 17.85^2)}{(s+3.33+j17.85)(s+3.33-j17.85)}$$

$$G(s) \approx \frac{741}{s^2 + 6.66s + 329}$$

2) For the following mass-spring system:



a) Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, V2, I3, I4}

$$I_1 = 0.1\dot{V}_1 = \frac{V_{in} - V_1}{10} - I_3 \qquad \dot{V}_1 = -V_1 - 10I_3 + V_{in}$$

$$I_2 = 0.2\dot{V}_2 = I_3 - I_4 \qquad \dot{V}_2 = 5I_3 - 5I_4$$

$$V_3 = 0.25\dot{I}_3 = V_1 - 20I_3 - V_2 \qquad \dot{I}_3 = 4V_1 - 80I_3 - 4V_2$$

$$V_4 = 0.5\dot{I}_4 = V_2 - 30I_4 \qquad \dot{I}_4 = 2V_2 - 60I_4$$

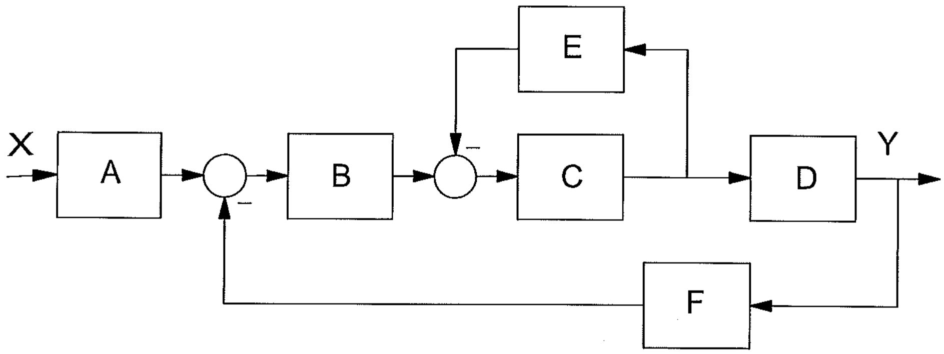
$$y = 30I_4$$

b) Express these dynamics in state-space form

$$\begin{bmatrix} sV1 \\ sV2 \\ sI3 \\ sI4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -10 & 0 \\ 0 & 0 & 5 & -5 \\ 4 & -4 & -80 & 0 \\ 0 & 2 & 0 & -60 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} Vin$$

$$Y = \begin{bmatrix} 0 & 0 & 0 & 30 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ I3 \\ I4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} Vin$$

3) For the following block diagram, find the transfer funciton from X to Y



$$\frac{ABCD}{1+CE+BCDF}$$

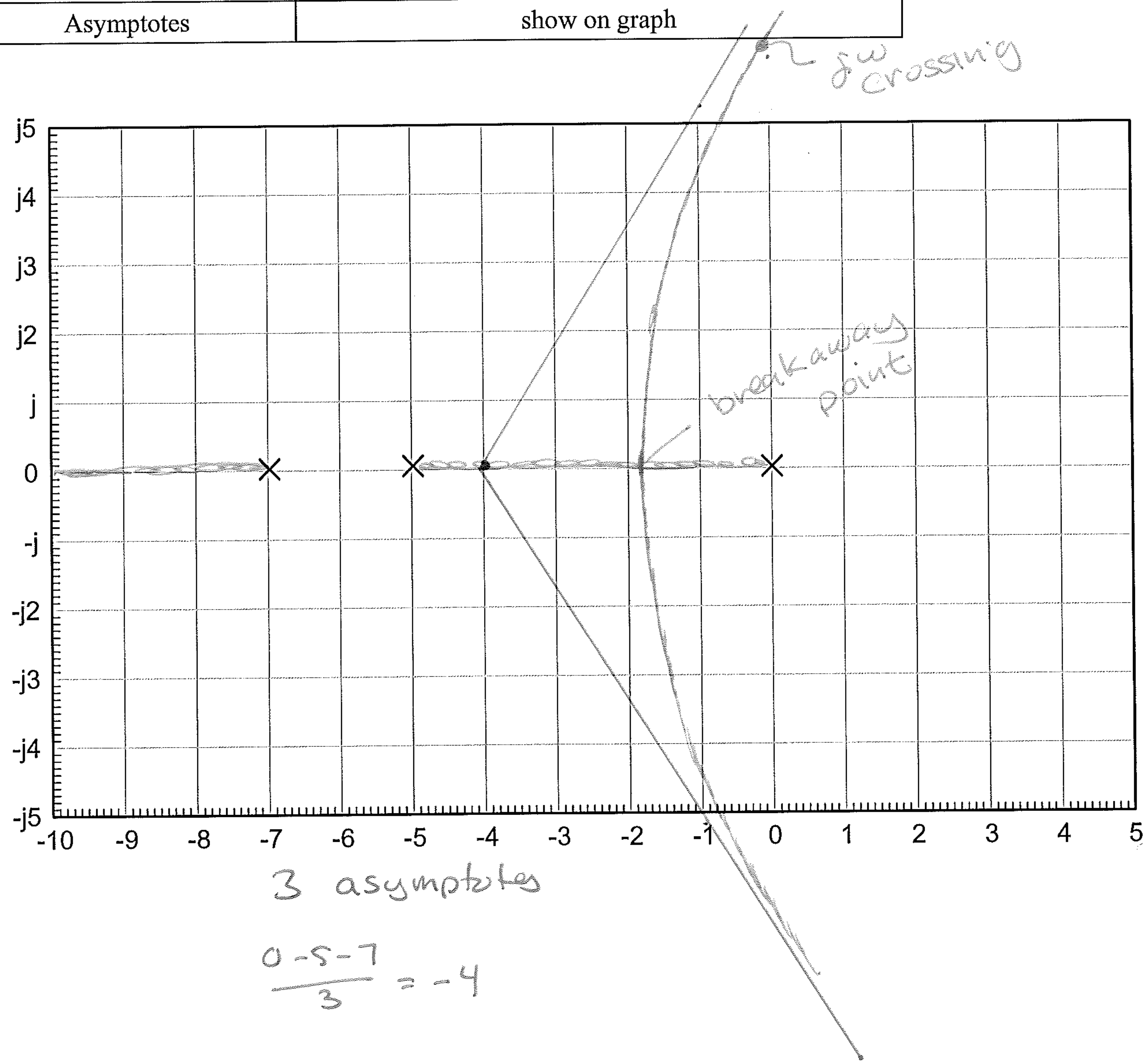
4) Sketch the root locus for

$$G(s) = \left(\frac{10}{s(s+4)(s+7)}\right)$$

5

Determine the following as well:

Real Axis Loci	(0, -5) (-7 -∞)
Breakaway Points (approx)	-1.9175
jw Crossing (approx)	j5.9161
Asymptotes	show on graph



3 asymptotes

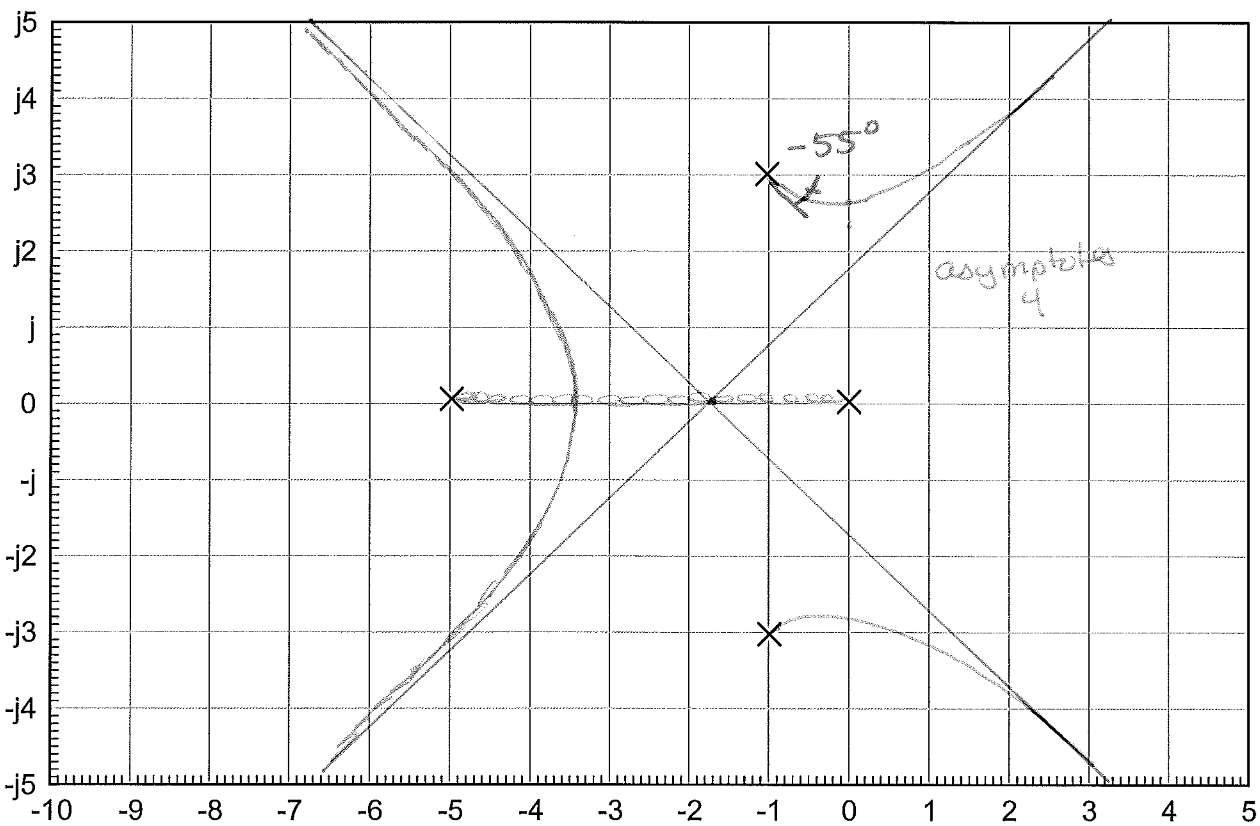
$$\frac{0-5-7}{3} = -4$$

5) Sketch the root locus for

$$s(s+5)(s+1+j3)(s+1-j3)+5k=0$$

Determine the following as well:

Real Axis Loci	(0, -5)	4
Breakaway Points (approx)	-3.3891	4
jw Crossing (approx)	j2.6726	4
Departure Angle from pole at s = -2 + j3	-55°	4
Asymptotes	show on graph	



4 asymptotes

intersect = $\frac{0-1-1-5}{4}$

$= -1.75$

Bonus! Early voting in North Dakota Starts October 29th. What do you need to bring to the polls to vote?

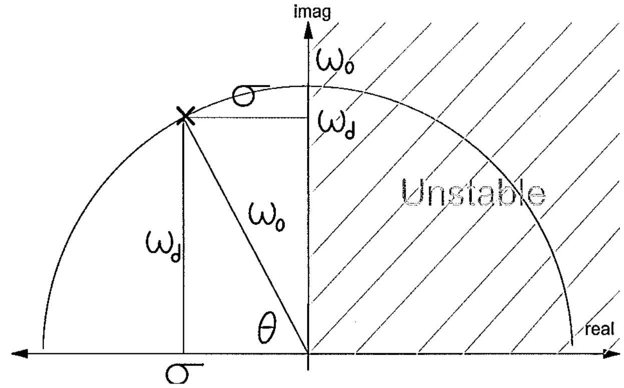
State issued photo ID

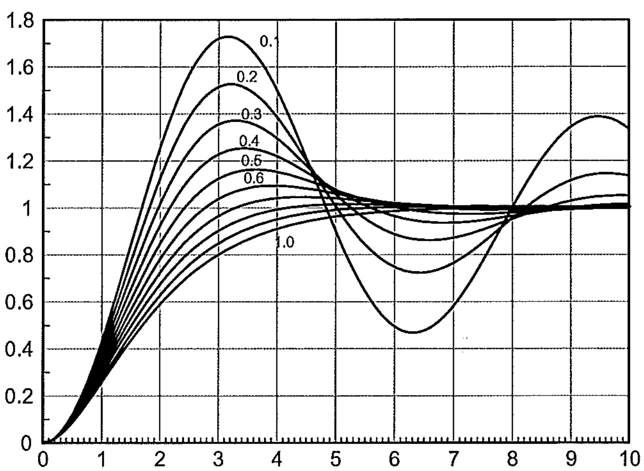
proof of residence (utility bill w/ address, bank statement)

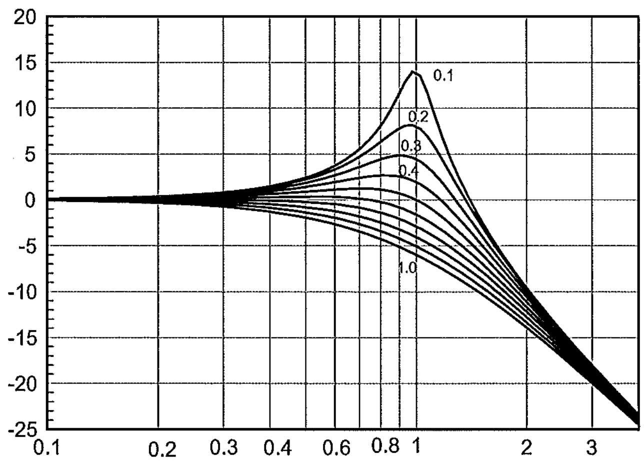
2nd-Order Approximations

$$G(s) = \left(\frac{k\cdot\omega_o^2}{s^2+2\zeta\omega_o s+\omega_o^2}\right) = \left(\frac{k\cdot\left(\sigma^2+\omega_d^2\right)}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

$$s = -\sigma \pm j\omega_d = \omega_o \angle \pm\theta$$







$\zeta = \cos\theta$	damping ratio	$T_p = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$	time to peak
$\%OS = \exp\left(-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)\right)$	% Overshoot	$T_s = T_{2\%} = \frac{4}{\sigma}$	2% Settling Time
$\omega_m = \omega_o\sqrt{1-2\zeta^2}$	Max gain frequency	$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$	Max gain
		$\frac{1}{2\zeta}$	Gain at corner freq

ζ	Tp	%OS	ω_m	Mm	Mm (dB)
0.1	3.15	72.81	0.99	5.03	14.02
0.2	3.21	51.97	0.96	2.55	8.14
0.3	3.29	35.5	0.91	1.75	4.85
0.4	3.43	22.4	0.82	1.36	2.7
0.5	3.63	12.31	0.71	1.15	1.25
0.6	3.93	5.26	0.53	1.04	0.35
0.7	4.4	1.34	0.14	1	0
0.8	5.24	0.09	0	1	0
0.9	7.21	0	0	1	0
1.0	-	0	0	1	0