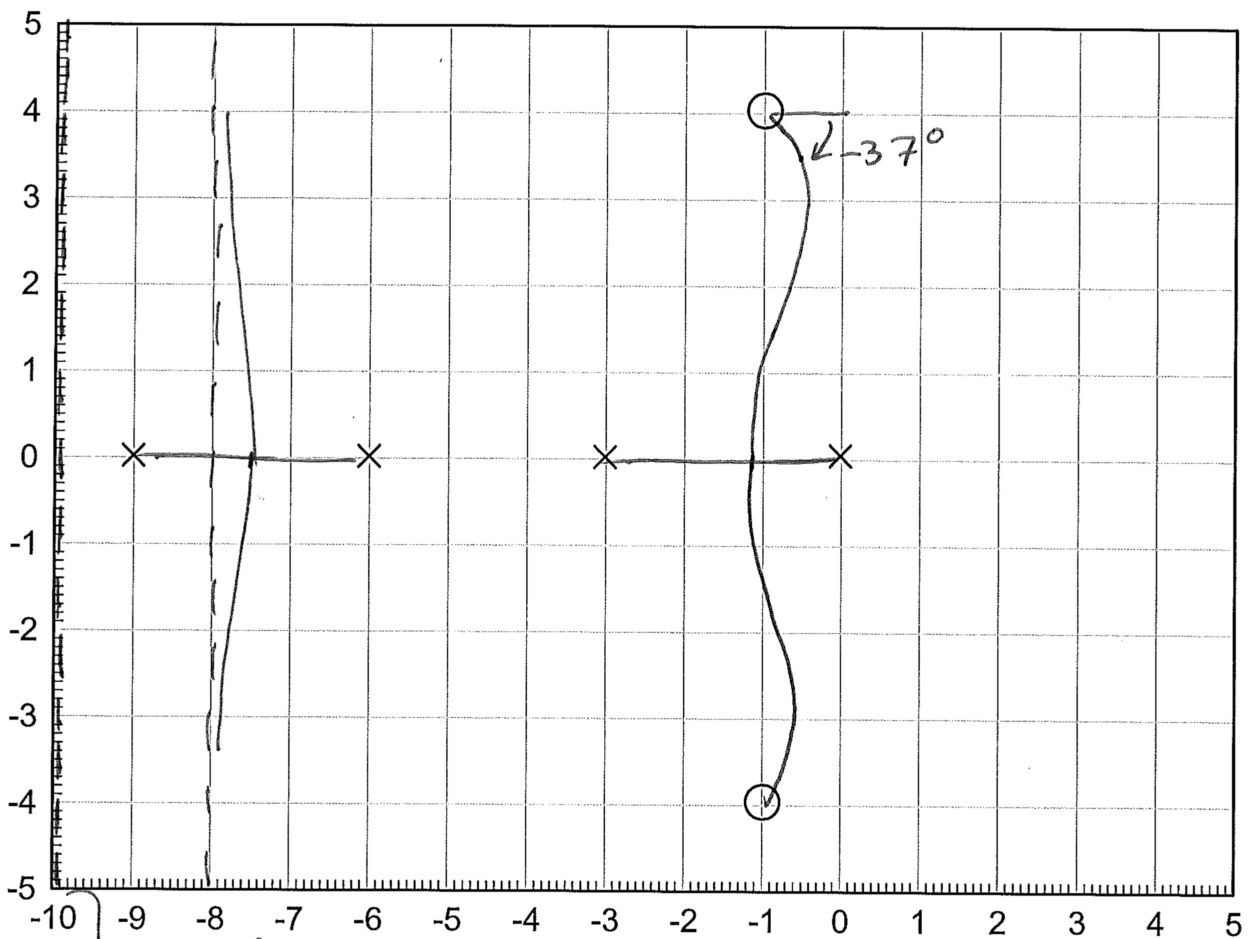
ECE 461/661 - Test #2: Name ____________________

Fall 2019

1) Sketch the root locus of $G(s) = \left(\frac{(s+1+j4)(s+1-j4)}{s(s+3)(s+6)(s+9)}\right)$ on the graph below. Also determine

Real Axis Loci	(0, -3) (-6, -9)
Asymptotes	show on graph
Breakaway Point (approx)	-1.1298 -7.6404
Approach Angle to the zero at s + -1 + j4	-37.3039°

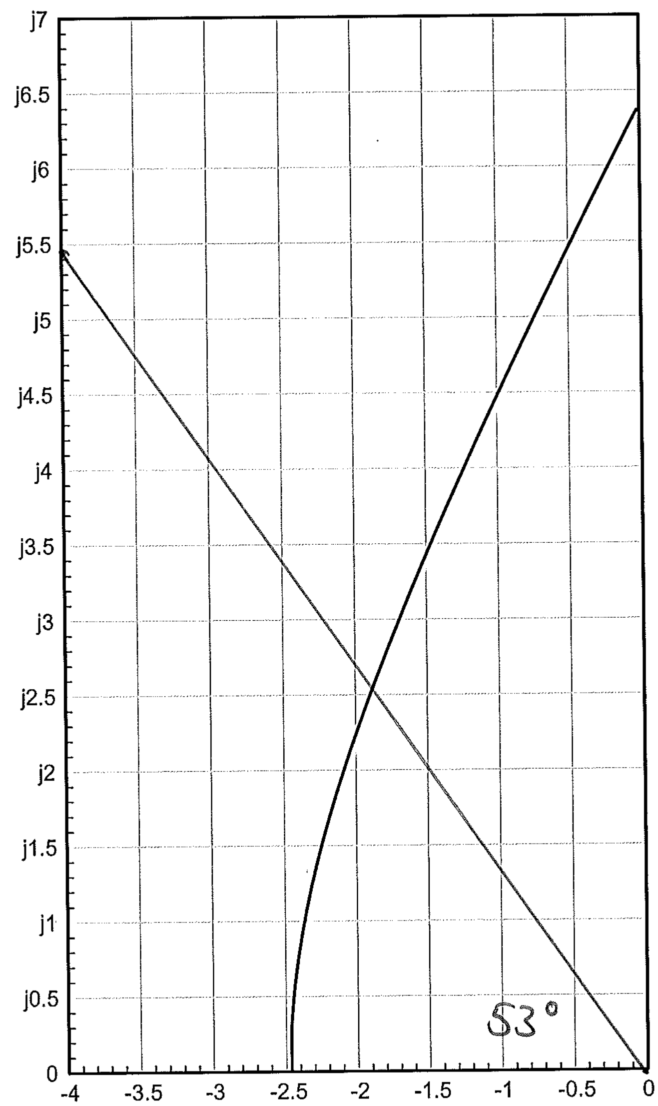


asymptote intersect $= \dfrac{\Sigma\text{poles} - \Sigma\text{zeros}}{\#\text{poles} - \#\text{zeros}} = \dfrac{(-18)-(-2)}{2} = -8$

2) The root locus of G(s) is shown below. Determine k for 10% overshoot. With that value of k, determine the following

$$G(s) = \left(\frac{10}{(s+1)(s+5)(s+6)}\right)$$

Closed-Loop domionant pole(s)	$-1.87 \pm j2.55$
k	5.30
Error Constant: Kp	1.77
2% Settling Time	2.13s



10% overshoot means

$\xi = 0.5912$

$\theta = \text{acos}(\xi) = 53.76°$

$\tan\theta = 1.3644$

$s = -1.8751 + j\,2.5583$

$G(s) = .1887\angle 180°$

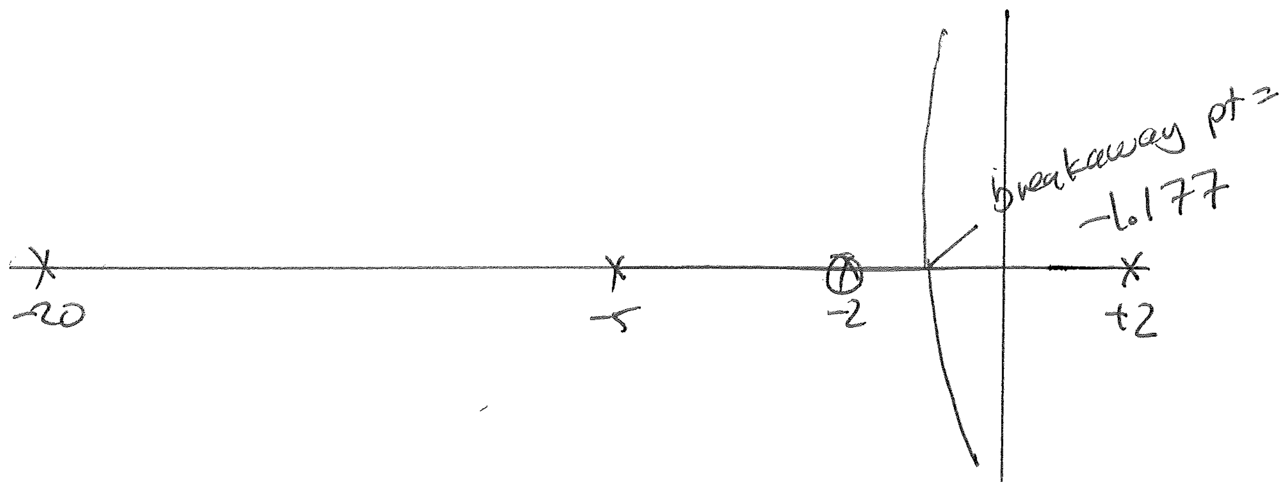
$k = \frac{1}{.1887} = 5.3002$

$K_p = (GK)\big|_{s=0} = \left(\frac{1}{3}\right)(5.3) = 1.77$

$t_s = \frac{4}{1.87} = 2.13s$

3) Design a compensator, K(s), to place the closed-loop dominant pole at s = -1

$$G(s) = \left(\frac{10}{(s-2)(s+2)(s+5)}\right)$$



$$k(s) = \frac{k\,(s+2)}{(s+20)}$$

$$Gk = \left.\frac{10}{(s-2)(s+5)(s+20)}\right|_{s=-1} = -.0439$$

$k = 22.8$

$$\boxed{k(s) = 22.8\left(\frac{s+2}{s+20}\right)}$$

4) Assume

$$G(s) = \left(\frac{100}{(s+2)(s+5)(s+10)}\right)$$

Design a compensator, K(s), so that the closed-loop system

- Has no error for a step input, and
- Has closed-loop dominant poles at s = -2 + j3

$$\boxed{K(s) = 1.71 \frac{(s+2)(s+5)}{s(s+6.67)}}$$

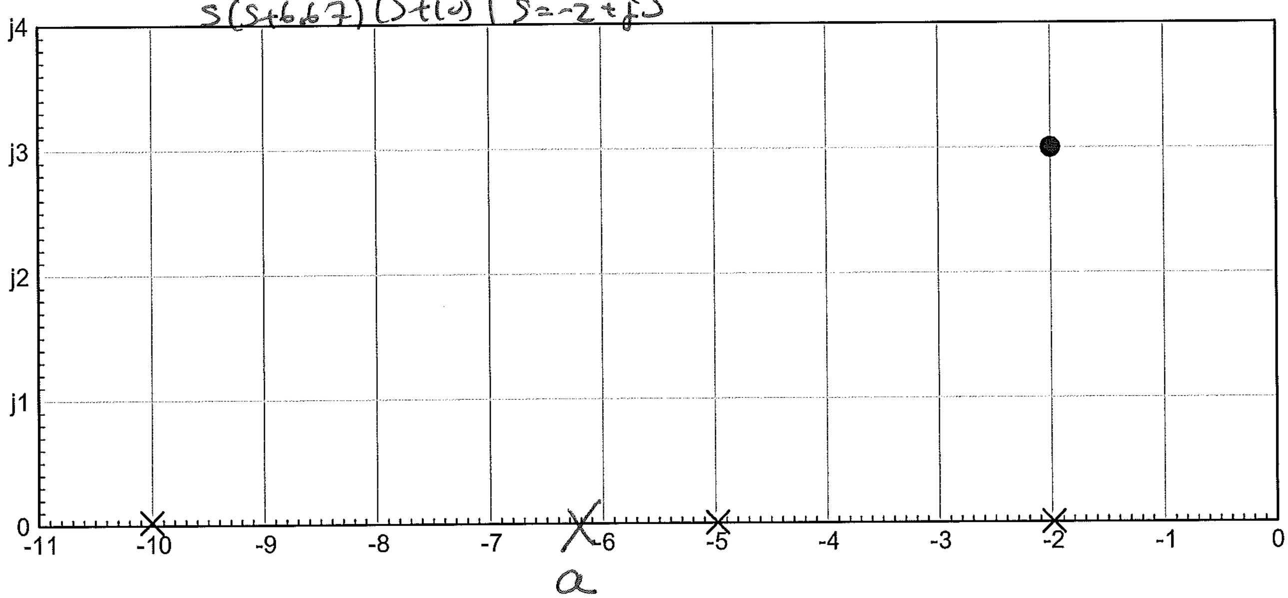
$$K(s) = \frac{k(s+2)(s+5)}{s(s+a)}$$

$$GK = \left.\frac{100k}{s(s+10)(s+a)}\right|_{s=-2+j3} = 1\angle 180^\circ$$

$$\angle s+a = 35.75^\circ$$

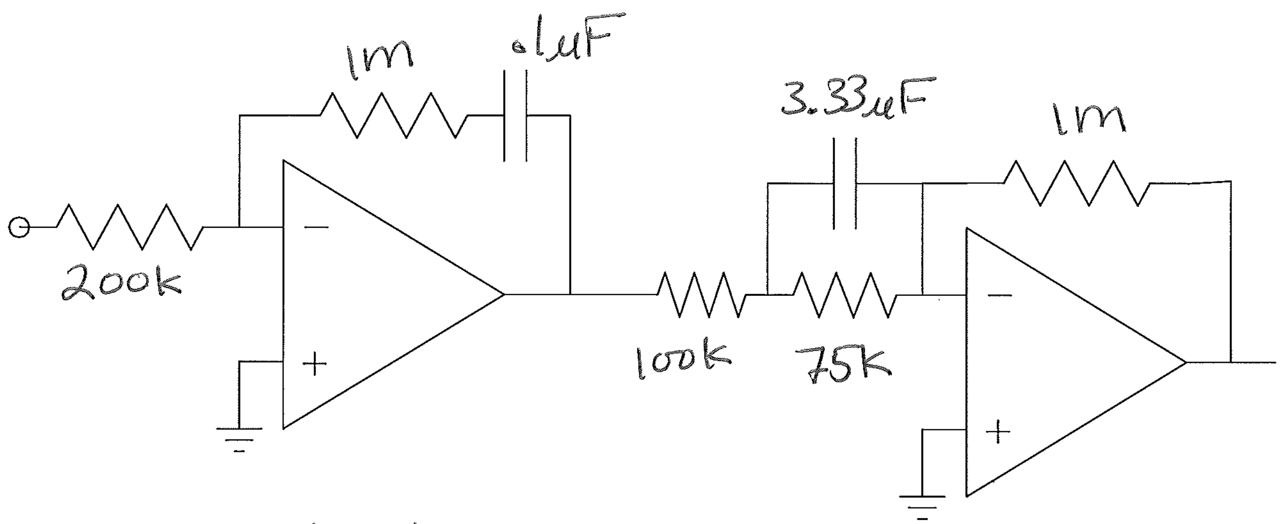
$$a = \frac{3}{\tan(35.75^\circ)} + 2 = 6.167$$

$$\left.\frac{100}{s(s+6.67)(s+10)}\right|_{s=-2+j3} = .5843\angle 180^\circ$$



5) Determine R and C to implement K(s)

$$K(s) = \left(\frac{50(s+4)(s+10)}{s(s+7)} \right)$$



$$\frac{5\,(s+10)}{s} \qquad 10\,\frac{s+4}{s+7}$$

≈ 50 years old

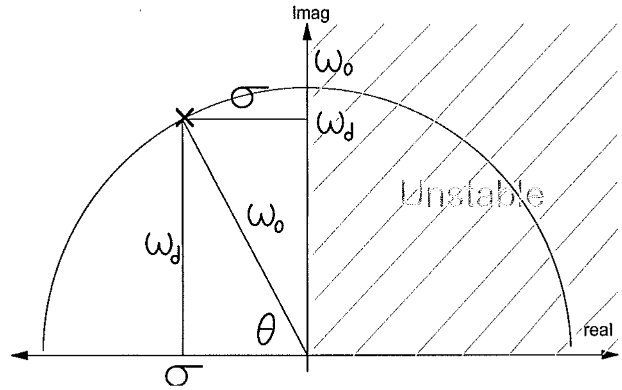
Colbert Bonus! Who is older? The Count (from Sesame Street) or Stephen Colbert?

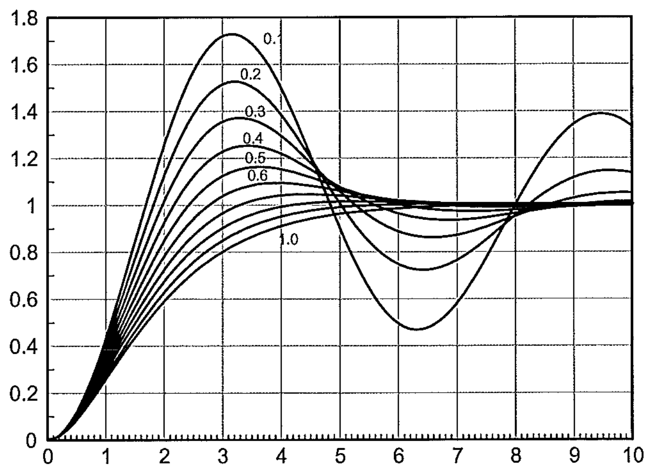
1.6 million years old

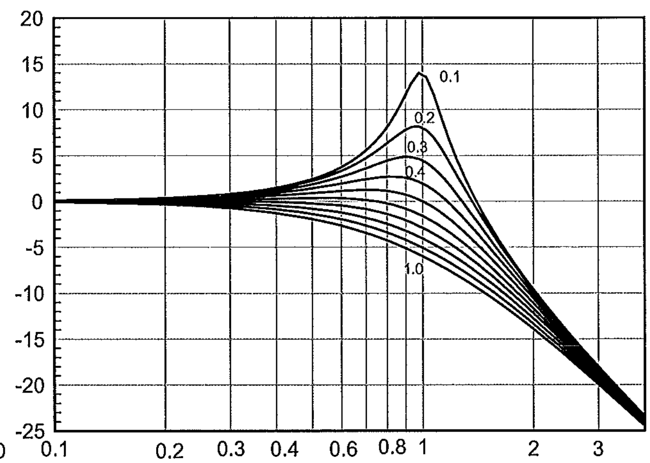
2nd-Order Approximations

$$G(s)=\left(\frac{k\cdot\omega_o^2}{s^2+2\zeta\omega_o s+\omega_o^2}\right)=\left(\frac{k\cdot\left(\sigma^2+\omega_d^2\right)}{(s+\sigma+j\omega_d)(s+\sigma-j\omega_d)}\right)$$

$$s=-\sigma\pm j\omega_d=\omega_o\angle\pm\theta$$







$\zeta=\cos\theta$	damping ratio	$T_p=\frac{\pi}{\omega_0\sqrt{1-\zeta^2}}$	time to peak
$\%OS=\exp\left(-\left(\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)\right)$	% Overshoot	$T_s=T_{2\%}=\frac{4}{\sigma}$	2% Settling Time
$\omega_m=\omega_o\sqrt{1-2\zeta^2}$	Max gain frequency	$M_m=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$	Max gain
		$\frac{1}{2\zeta}$	Gain at corner freq

ζ	Tp	%OS	ω_m	Mm	Mm (dB)
0.1	3.15	72.9%	0.99	5.03	14.02
0.2	3.21	52.7%	0.96	2.55	8.14
0.3	3.29	37.2%	0.91	1.75	4.85
0.4	3.43	25.4%	0.82	1.36	2.7
0.5	3.63	16.3%	0.71	1.15	1.25
0.6	3.93	9.5%	0.53	1.04	0.35
0.7	4.4	4.6%	0.14	1	0
0.8	5.24	1.5%	0	1	0
0.9	7.21	0.2%	0	1	0
1.0	-	0	0	1	0