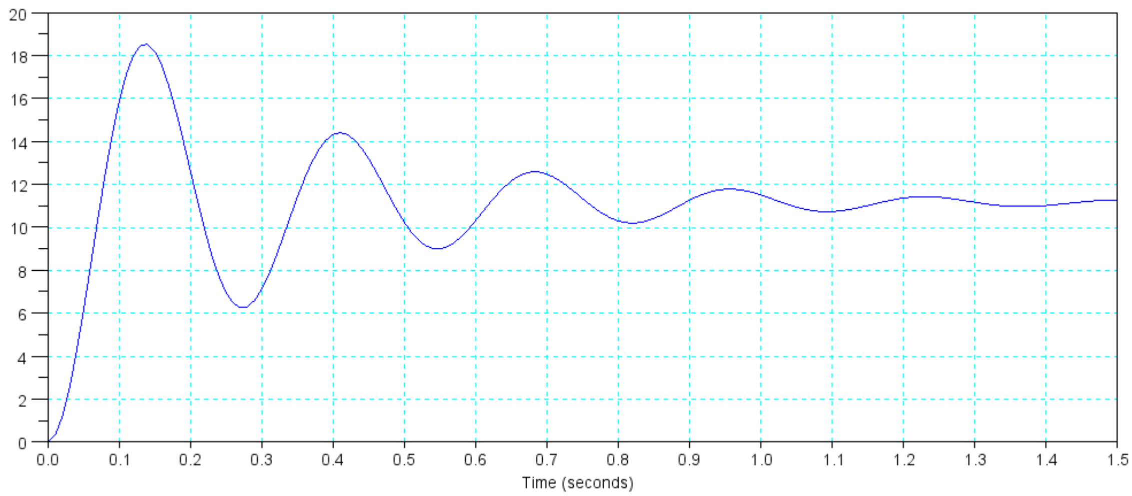
ECE 461 - Final: Name ____________________

Fall - 2020

Problem #1: 2nd Order Approximations

1a) Give the transfer function for a system with the following step response:



DC gain = 11.2

Frequency of oscillation = $\left(\frac{3 \text{ cycles}}{0.82s}\right)2\pi = 22.98\frac{rad}{\text{sec}}$

2% settling time = 1.3 seconds

$$\sigma = \frac{4}{1.3} = 3.07$$

$$G(s) \approx \left(\frac{6,020}{(s+3.07+j22.98)(s+3.7-j22.98)}\right)$$

1b) What is the step response for the following system:

$$Y = \left(\frac{2000}{(s+7+j12)(s+7-j12)(s+40)}\right)X$$

DC Gain	2% Settling Time	% Overshoot
0.2591	**4/7 sec**	**16.0%**

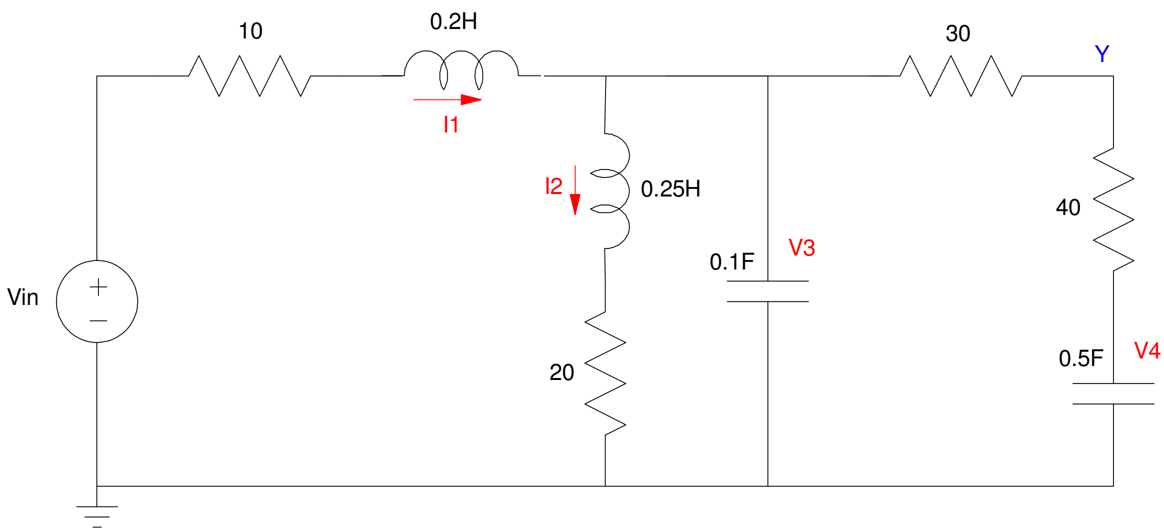
$$\theta = \arctan\left(\frac{12}{7}\right) = 59.74^0$$

$$\zeta = \cos\theta = 0.5039$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 16.0\%$$

Problem 2: Modeling and State Space

2a) Write the differential equations which describe the following circuit (i.e. write the N differential equations which correspond to the voltage node equations)



$$V_1 = 0.2\dot{I}_1 = V_{in} - 10I_1 - V_3$$

$$V_2 = 0.25\dot{I}_2 = V_3 - 20I_2$$

$$I_3 = 0.1\dot{V}_3 = I_1 - I_2 - \left(\frac{V_3 - V_4}{70}\right)$$

$$I_4 = 0.5\dot{V}_4 = \left(\frac{V_3 - V_4}{70}\right)$$

Output

$$\left(\frac{Y - V_3}{30}\right) + \left(\frac{Y - V_4}{40}\right) = 0$$

$$Y = \left(\frac{4}{7}\right)V_3 + \left(\frac{3}{7}\right)V_4$$

2b) Express these dynamics in state-space form

Group terms

$$\dot{I}_1 = 5V_{in} - 50I_1 - 5V_3$$

$$\dot{I}_2 = 4V_3 - 80I_2$$

$$\dot{V}_3 = 10I_1 - 10I_2 - \left(\frac{1}{7}\right)V_3 + \left(\frac{1}{7}\right)V_4$$

$$\dot{V}_4 = \left(\frac{1}{35}\right)V_3 - \left(\frac{1}{35}\right)V_4$$

Place in matrix form

$$s\begin{bmatrix} I_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -50 & 0 & -5 & 0 \\ 0 & -80 & 4 & 0 \\ 10 & -10 & \left(\frac{-1}{7}\right) & \left(\frac{1}{7}\right) \\ 0 & 0 & \left(\frac{1}{35}\right) & \left(\frac{-1}{35}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

$$Y = \begin{bmatrix} 0 & 0 & \left(\frac{4}{7}\right) & \left(\frac{3}{7}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_3 \\ V_4 \end{bmatrix}$$

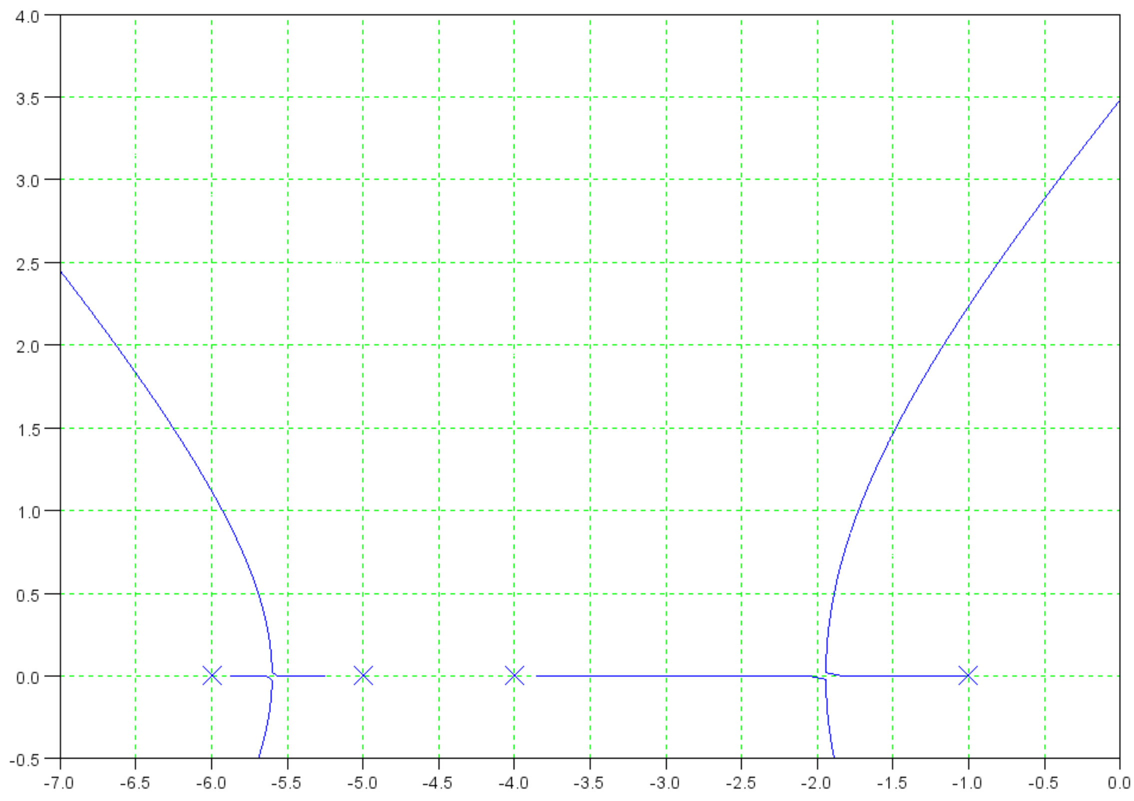
Problem 3: Root Locus

3) Gain Compensation: The root locus for

$$G(s) = \left(\frac{40}{(s+1)(s+4)(s+5)(s+6)} \right)$$

is shown below. Determine the following:

Maximum gain, k, for a stable closed-loop system	**k = 20.3027** s = j3.4821
k for a damping ratio of 0.6	**k = 2.9290**
Closed-loop dominant pole(s) for a damping ratio of 0.6	**s = -1.3224 + j1.7633**
Closed-Loop DC gain for a damping ratio of 0.6	**DC Gain = 0.4940** Kp = (2.9290)(0.3333) = 0.9763 Kp / (1 + Kp) = 0.4940



Problem #4: Compensator Design using Root Locus

4) Given the following stable system

$$G(s) = \left(\frac{100}{(s+0.5)(s+2)(s+6)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- No error for a step input, and
- A closed-loop dominant pole at s = -2 + j5

Let

$$K(s) = k\left(\frac{(s+0.5)(s+2)}{s(s+a)}\right)$$

$$GK = \left(\frac{100k}{s(s+6)(s+a)}\right)$$

Evaluate what we know at s = -2 + j5

$$\left(\frac{100}{s(s+6)}\right)_{s=-2+j5} = 2.9001\angle -163.1416^0$$

$$\angle(s+a) = 16.8584^0$$

$$a = 2 + \frac{5}{\tan\left(16.8584^0\right)} = 18.5$$

$$GK = \left(\frac{100k}{s(s+6)(s+18.5)}\right)$$

Evaluate what we know

$$GK = \left(\frac{100}{s(s+6)(s+18.5)}\right)_{s=-2+j5} = 0.1682\angle 180^0$$

$$k = \frac{1}{0.1682} = 5.945$$

so

$$K(s) = \left(\frac{5.945(s+0.5)(s+2)}{s(s+18.5)}\right)$$

Problem #5: Discrete-Time Compensator Design

5) Given the following stable system

$$G(s) = \left(\frac{100}{(s+0.5)(s+2)(s+6)}\right)$$

Determine a digital compensator, K(z), which results in the closed-loop system having

- No error for a step input,
- A closed-loop dominant pole at s = -2 + j5, and
- A sampling rate of T = 0.2

Let

$$K(s) = k\left(\frac{(s+0.5)(s+2)}{s(s+a)}\right)$$

or in the z-plane

$$K(z) = k\left(\frac{(z-0.9048)(z-0.3679)}{(z-1)(z-a)}\right)$$

G(s) * K(z) * zero order hold is (modeled as a 1/2 sample delay)

$$GK\Delta = \left(\frac{100}{(s+0.5)(s+2)(s+6)}\right) \cdot k\left(\frac{(z-0.9048)(z-0.3679)}{(z-1)(z-a)}\right) \cdot e^{-0.1s}$$

Evaluate what we know at

- s = -2 + j5
- $z = e^{sT} = 0.3622 + j0.5641$

$$\left(\frac{100}{(s+0.5)(s+2)(s+6)}\right)_{s=-2+j5} \cdot \left(\frac{(z-0.9048)(z-0.3679)}{(z-1)}\right)_{z=0.3622+j0.5641} \cdot (e^{-0.1s})_{s=-2+j5} = 0.8433\angle 54^0$$

Too much phase. Try

$$K(s) = k\left(\frac{(s+0.5)(s+2)(s+6))}{s(s+a)^2}\right)$$

$$K(z) = k\left(\frac{(z-0.9048)(z-0.3679)(s-0.3012)}{(z-1)(z-a)^2}\right)$$

Now

$$(G(s) \cdot K(z) \cdot e^{-sT/2})_{s=-2+j5} = 0.2150\angle -106.897^0$$

$$\angle(z-a)^2 = 73.1026^0$$

$$\angle(z-a) = 36.5513^0$$

$$a = 0.3622 - \frac{0.5641}{\tan\left(36.5513^0\right)} = -0.3987$$

and

$$K(z) = k\left(\frac{(z-0.9048)(z-0.3679)(s-0.3012)}{(z-1)(z+0.3977)^2}\right)$$

Evaluate what we know

$$(G(s) \cdot K(z) \cdot e^{-sT/2})_{s=-2+j5} = 0.2401\angle 180^0$$

$$k = \frac{1}{0.2410} = 4.1655$$

and

$$K(z) = 4.1655\left(\frac{(z-0.9048)(z-0.3679)(s-0.3012)}{(z-1)(z+0.3977)^2}\right)$$

Problem #6: Compensator Design using Bode Plots

6) Given the following stable system

$$G(s) = \left(\frac{100}{(s+0.5)(s+2)(s+6)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- A closed-loop DC gain of 1.000 (i.e. no error for a step input),
- A 0dB gain frequency of 5 rad/sec, and
- Mm = 1.45

Convert Mm to a phase margin

$$\frac{1}{M_m^2} = 2 + 2\cos\phi$$

$$\phi = -139.6575^0$$

Pick K(s) so that

$$G(j5)K(j5) = 1\angle -139.6575^0$$

Let

$$K(s) = k\left(\frac{(s+0.5)(s+2)}{s(s+a)}\right)$$

$$GK = \left(\frac{100k}{s(s+6)(s+a)}\right)$$

Evaluating what we know at 5 rad/sec

$$\left(\frac{100}{s(s+6)}\right)_{s=j5} = 0.2561\angle -129.8056^0$$

To make the phase add up to -139.6575 degrees

$$\angle(s+a) = 9.8519^0$$

$$a = \frac{5}{\tan\left(9.8619^0\right)} = 28.7913$$

and

$$GK = \left(\frac{100k}{s(s+6)(s+28.7913)}\right)$$

Evaluate what we know

$$\left(\frac{100}{s(s+6)(s+28.7913)}\right)_{s=j5} = 0.0876\angle -139.6575^0$$

meaning

$$k = \frac{1}{0.0876} = 11.4116$$

and

$$K(s) = 11.4116\left(\frac{(s+0.5)(s+2)}{s(s+28.7913}\right)$$

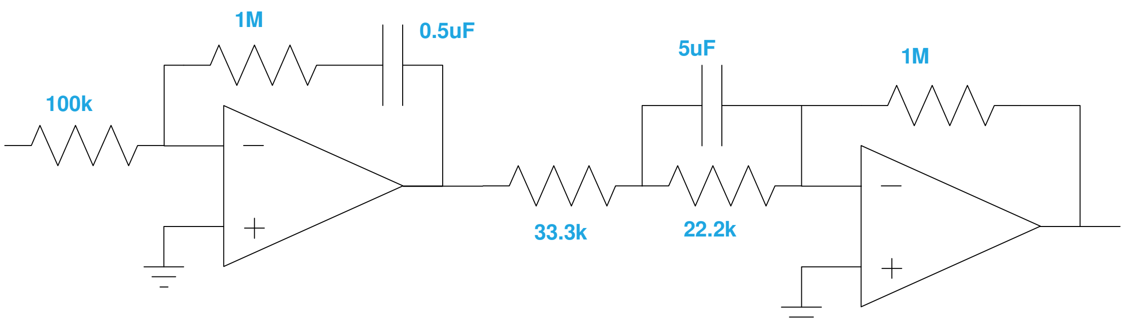
Problem #8: Implementation

7) Determine R and C so that the following compensator has the transfer function of

$$K(s) = 300\left(\frac{(s+2)(s+9)}{s(s+15)}\right)$$

Rewrite as

$$K(s) = \left(\frac{10(s+2)}{s}\right)\left(30\left(\frac{s+9}{s+15}\right)\right)$$



There are other solutions.