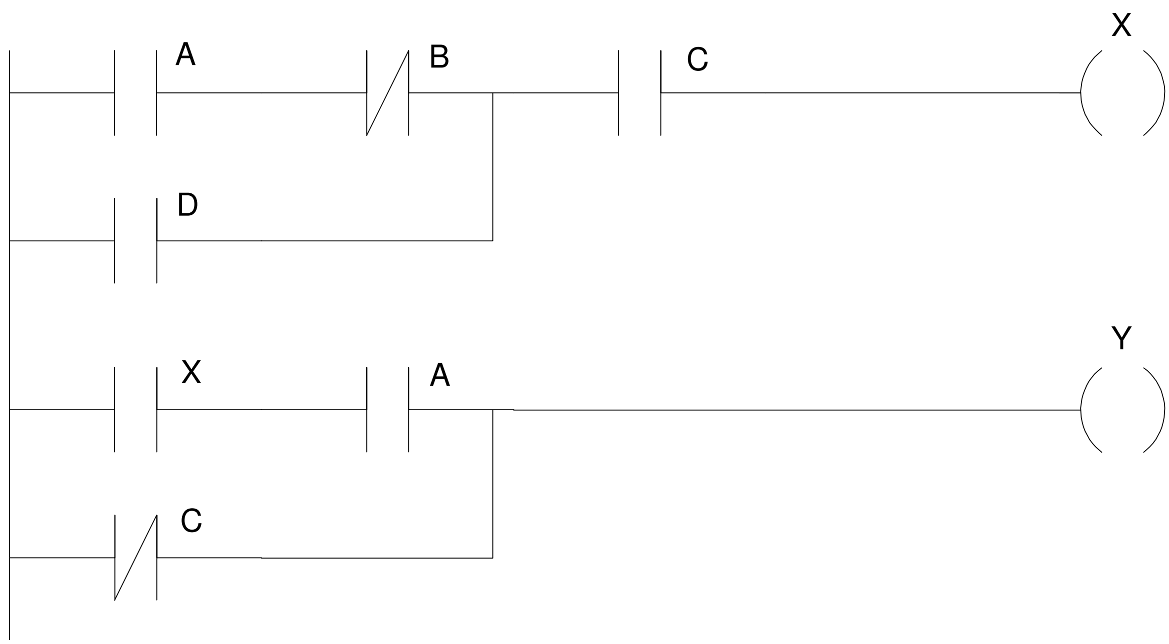
ECE 461/661 - Test #1: Name ___________________

Fall 2020

1) Determine the function Y = f(A, B, C, D) for the following ladder logic circuit. (you don't need to simplify)



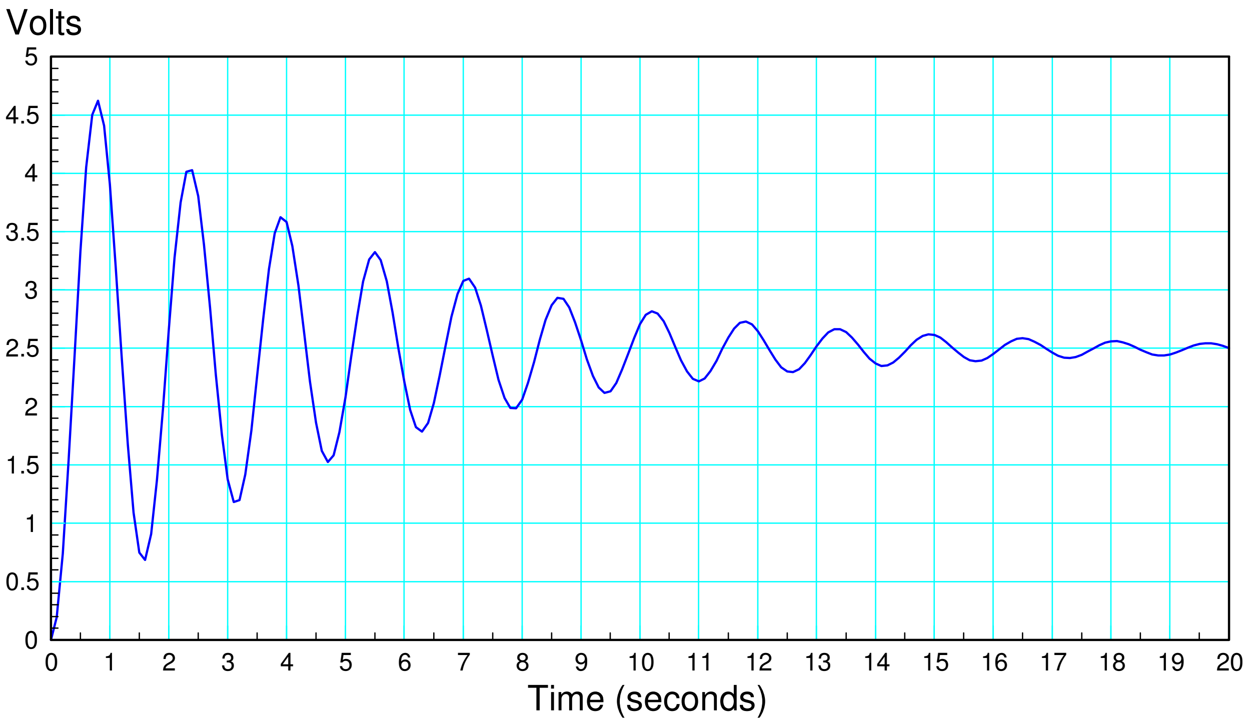
$$X = C\left(A\overline{B} + D\right)$$

$$Y = AX + \overline{C}$$

substituting

$$Y = A\left(C\left(A\overline{B} + D\right)\right) + \overline{C}$$

2) Give the transfer function for a system with the following response to a unit step input:



DC gain = 2.5

$$f = \left(\frac{7 \text{ cycles}}{11 \text{ seconds}}\right) = 0.636Hz$$

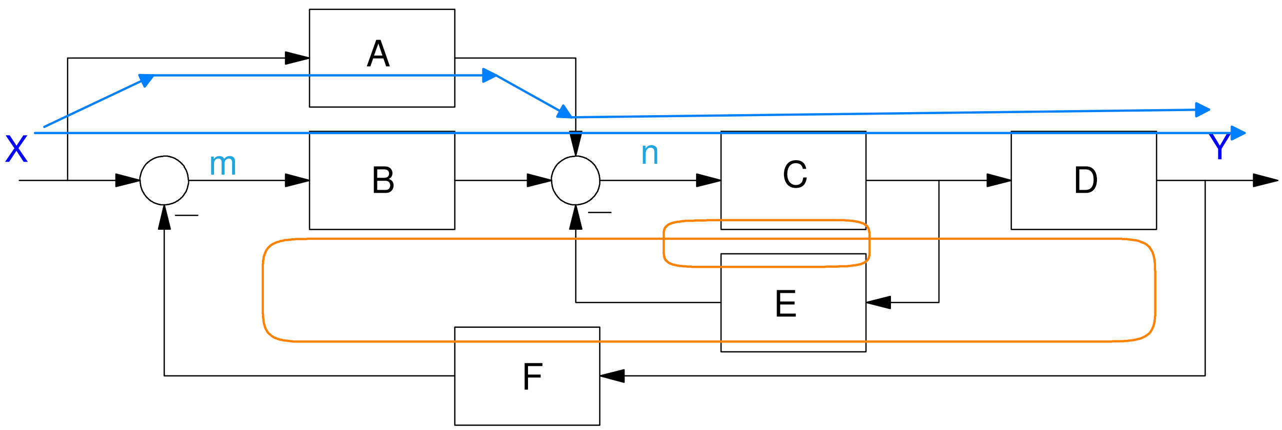
$$\omega = 2\pi f = 3.998$$

$$t_s \approx 20$$

$$\sigma = \frac{4}{t_s} = 0.2$$

$$G(s) \approx \left(\frac{40.060}{(s+0.2+j3.998)(s+0.2-j3.998)}\right)$$

3) Find the transfer funciton from X to Y



Shortcut (usually works)

$$Y = \left(\frac{ACD+BCD}{1+CE+BCDF}\right)X$$

Long Way (always works)

$$m = X - FY$$

$$n = AX + Bm - CEn$$

$$Y = DCn$$

substituting

$$n = AX + B(X - FY) - CEn$$

$$(1 + CE)n = AX + B(X - FY)$$

$$n = \frac{AX+B(X-FY)}{(1+CE)}$$

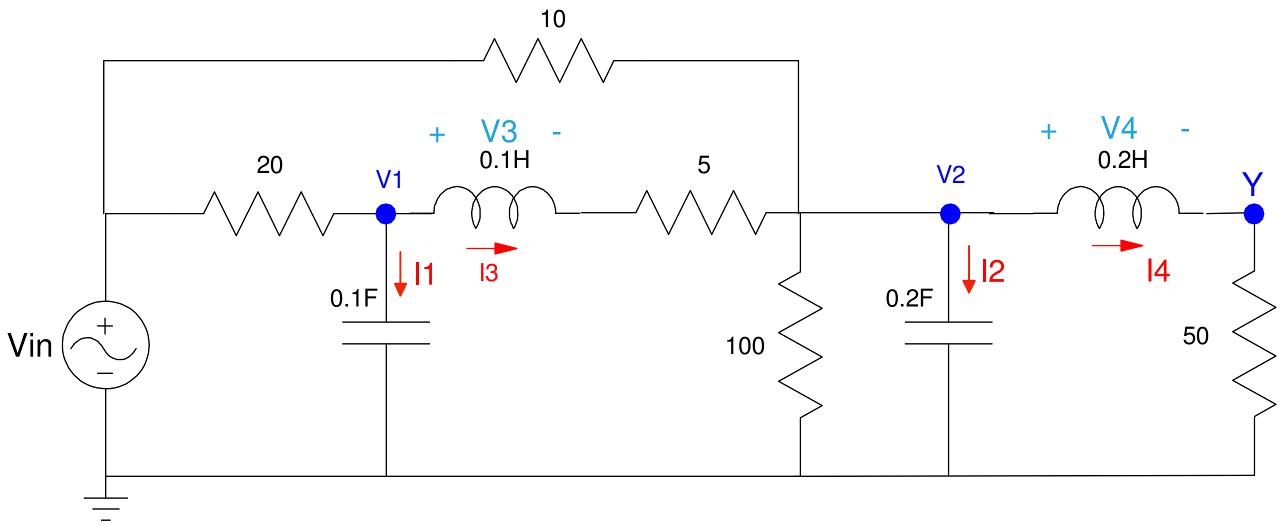
$$Y = DC\left(\frac{AX+B(X-FY)}{(1+CE)}\right)$$

$$(1 + CE)Y = DC(AX + B(X - FY))$$

$$(1 + CE + DCBF)Y = DCAX + DCBX$$

$$Y = \left(\frac{DCA+DCB}{1+CE+DCBF}\right)X$$

4) For the following RLC circuit:



- Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, V2, I3, I4}

$$I_1 = 0.1sV_1 = \left(\frac{V_{in}-V_1}{20}\right) - I_3$$

$$I_2 = 0.2sV_2 = I_3 + \left(\frac{V_{in}-V_2}{10}\right) - \left(\frac{V_2}{100}\right) - I_4$$

$$V_3 = 0.1sI_3 = V_1 - V_2 - 5I_3$$

$$V_4 = 0.2sI_4 = V_2 - 50I_4$$

Simplify

$$sV_1 = 0.5V_{in} - 0.5V_1 - 10I_3$$

$$sV_2 = 5I_3 + 0.5V_{in} - 0.55V_2 - 5I_4$$

$$sI_3 = 10V_1 - 10V_2 - 50I_3$$

$$sI_4 = 5V_2 - 250I_4$$

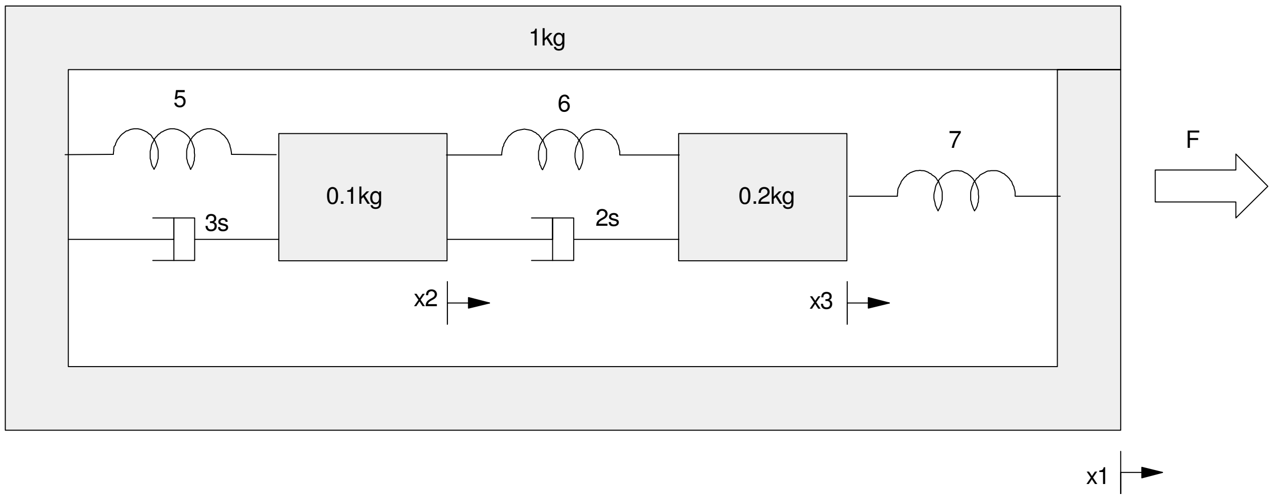
In matrix (state-space) form

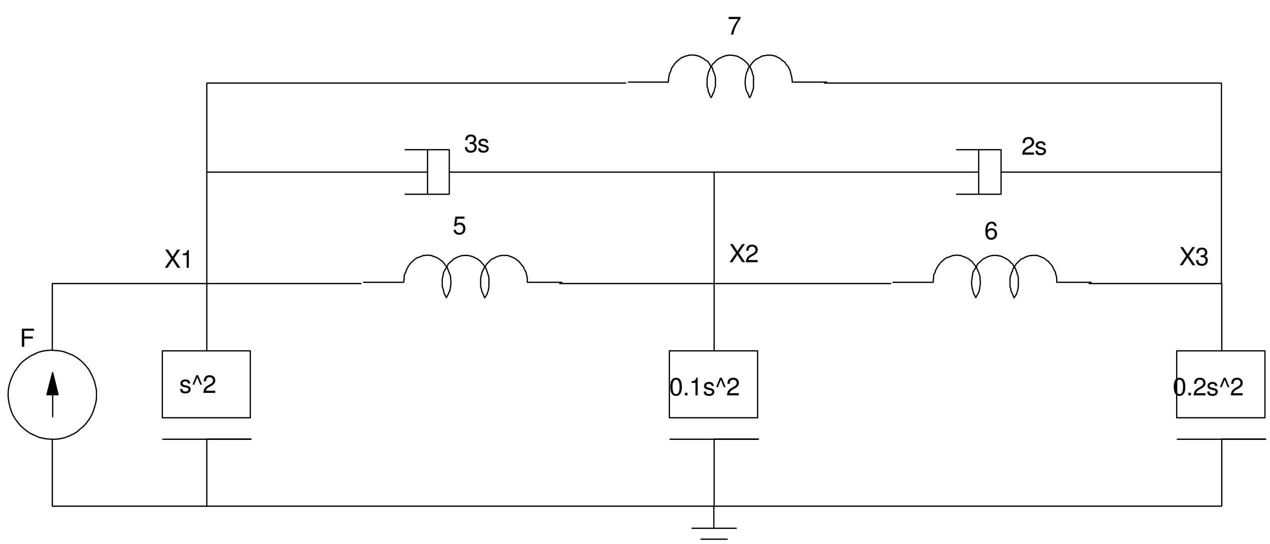
$$s\begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -0.5 & 0 & -10 & 0 \\ 0 & -0.55 & 5 & -5 \\ 10 & -10 & -50 & 0 \\ 0 & 5 & 0 & -250 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

$$Y = 50I_4 = \begin{bmatrix} 0 & 0 & 0 & 50 \end{bmatrix} X + [0]V_{in}$$

5) For the following mass-spring system

- Draw the circuit equivalent for the following mass-spring system
- Write the equations of motion (i.e. write the voltage node equations)





$$(s^2 + 3s + 12)X_1 - (3s + 5)X_2 - (7)X_3 = F$$

$$(0.1s^2 + 5s + 11)X_2 - (3s + 5)X_1 - (2s + 6)X_3 = 0$$

$$(0.2s^2 + 2s + 13)X_3 - (7)X_1 - (2s + 6)X_2 = 0$$