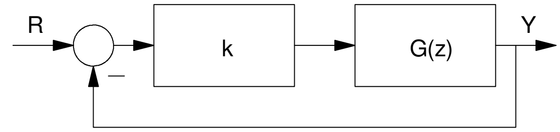
ECE 461/661 - Test #3: Name ____________________

Digital Control & Frequemncy Domain techniques - Fall 2020

Root Locus in the z-Plane

1) Assume a unity feedback system

$$G(z) = \left(\frac{0.04z}{(z-0.9)(z-0.8)} \right)$$

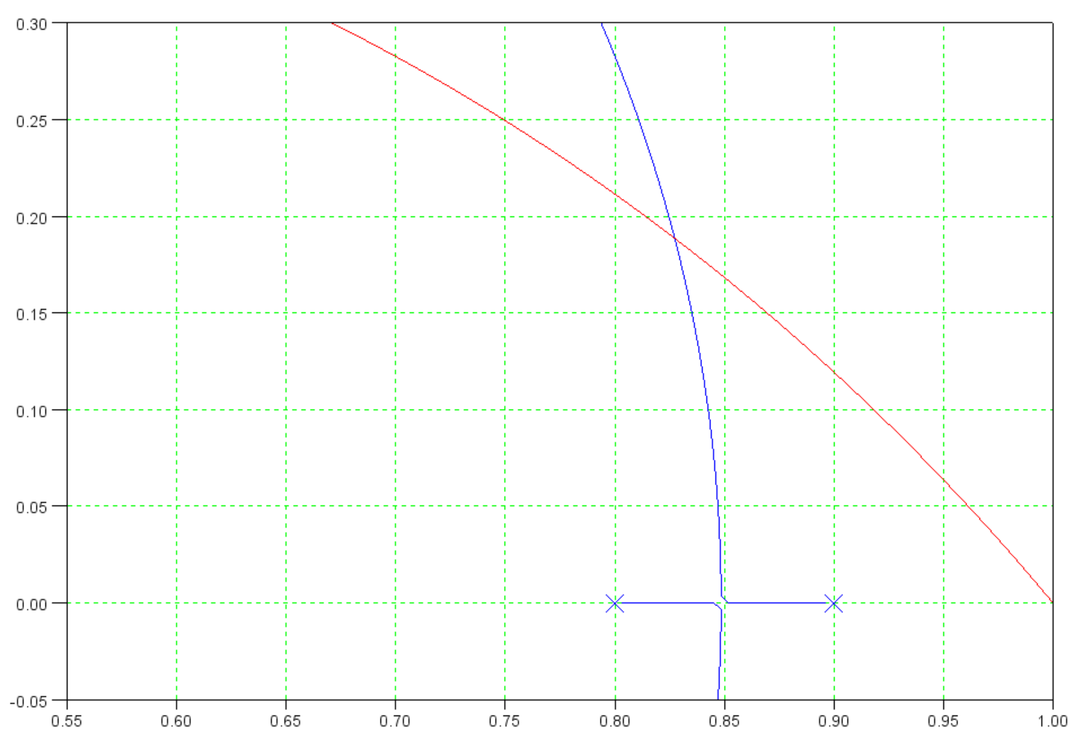


Determine a gain compesator, K(z) = k, which results in 10% overshoot for a step input ($\zeta = 0.5910$). Specify

- The resulting gain, k
- The closed-loop dominant pole(s)
- The resulting 2% settling time (in terms of samples), and
- The error constant, Kp

Step 1: Draw the root locus plot along with the 0.5910 damping line in the z-plane:

```
G = zpk(0,[0.9,0.8],0.04);
k = logspace(-2,2,1000)';
rlocus(G,k);
hold on
s = (-1 + j*1.36492) * [0:0.01:10]';
T = 1;
z = exp(s*T);
plot(real(z),imag(z),'r');
```



Find the point which intersects the damping line:

$$z = 0.8273 + j0.1886$$

At any point on the root locus, GK = -1

$$\left(\frac{0.04z}{(z-0.9)(z-0.8)}\right)_{z=0.8273+j0.1886} = 0.8812\angle 180^0$$

meaning

$$k = \frac{1}{0.8812} = 1.1349$$

Answers:

Resulting Gain, k:

k = 1.1349

Closed-Loop Dominant Poles:

z = 0.8273 + j0.1886 (and its complex conjugate)

2% Settling Time:

24 samples

$$z = 0.8485\angle 12.845^0$$

$$(0.8485)^k = 0.02$$

$$k = 23.81$$

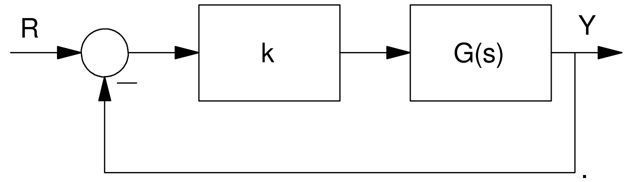
Kp = 0.2270

$$\left(\frac{0.04z}{(z-0.9)(z-0.8)}\right)_{z=1} = 2.00$$

$$K_p = (G \cdot k)_{s=0} = 2k = 2.2697$$

Compensator Design in the z-Plane

2) Assume a unity feedback system with a sampling rate of T = 0.1 second



$$G(s) = \left(\frac{10}{(s+2)(s+10)}\right)$$

Design a digital compensator, K(z), which results in

- No error for a step input
- 10% overshoot ($\zeta = 0.5910$), and
- A 2% settling time of 2 seconds

The closed-loop dominant pole is

- $s = -2 + j2.7299$
- $z = 0.7884 + j0.2207$

Pick K(z) to

- Cancel the poles at s = -2 and s = -10
- Add a pole at s = 0, and
- Add a pole to place these points on the root locus (angle adds up to 180 degrees)

$$K(z) = k\left(\frac{(z-0.8187)(z-0.3679)}{(z-1)(z-a)}\right)$$

To find 'a', evaluate what we know

$$G(s) \cdot K(z) \cdot ZOH = -1$$

$$\left(\left(\frac{10}{(s+2)(s+10)}\right)\left(\frac{(z-0.8187)(z-0.3679)}{(z-1)(z-a)}\right)(e^{-sT/2})\right)_{s=-2+j2.7299} = 0.1657\angle -124.94^0$$

$$\angle(z-a) = 55.0596^0$$

$$a = 0.7884 - \frac{0.2207}{\tan\left(55.0596^0\right)} = 0.6342$$

meaning

$$K(z) = k\left(\frac{(z-0.8187)(z-0.3679)}{(z-1)(z-0.6342)}\right)$$

To find k

$$\left(\left(\frac{10}{(s+2)(s+10)}\right)\left(\frac{(z-0.8187)(z-0.3679)}{(z-1)(z-a)}\right)(e^{-sT/2})\right)_{s=-2+j2.7299} = 0.6155\angle 180^0$$

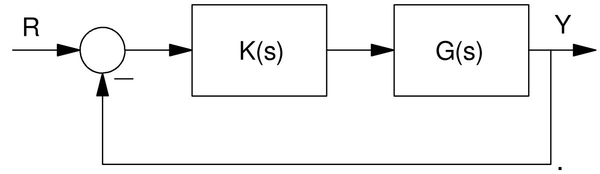
$$k = \frac{1}{0.6155} = 1.6246$$

$$K(z) = 1.6246\left(\frac{(z-0.8187)(z-0.3679)}{(z-1)(z-0.6342)}\right)$$

Nichols Charts

3) Assume a unity feedback system with

$$G(s) = \left(\frac{10}{s(s+2)(s+10)} \right)$$



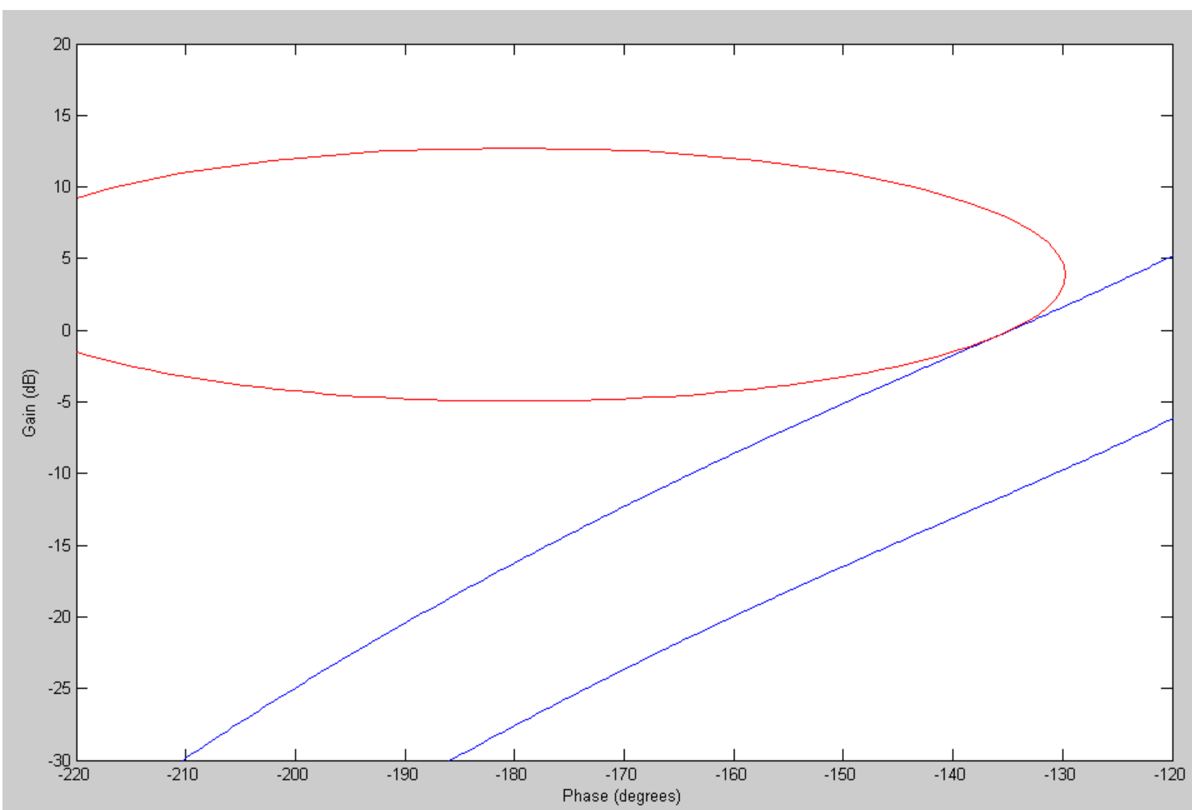
Determine a gain compensator, K(s) = k, which results in a resonance of Mm = 1.3 (2.279dB).

Plot the resulting Nichols chart for the G(s) * k

```
>> G = zpk([],[0,-2,-10],10);
>> w = logspace(-2,2,1000)';
>> s = j*w;
>> Gw = Bode2(G,w);
>> Nichols2(G2,1.3);
??? Undefined function or variable 'G2'.

>> Nichols2(Gw,1.3);
>> Nichols2(Gw*[1,3],1.3);
>> Nichols2(Gw*[1,4],1.3);
>> Nichols2(Gw*[1,3.6],1.3);
>> Nichols2(Gw*[1,3.7],1.3);
>>
```

answer: k = 3.70



Compensator Design in the Frequency Domain

4) Assume a unity feedback system with

$$G(s) = \left(\frac{10}{s(s+2)(s+10)}\right)$$

Determine a compensator, K(s), which results in

- No error for a step input (closed-loop gain at DC = 1.000)
- A 60 degree phase margin, and
- A 0dB gain frequency of 2 rad/sec

Assume K(s) is in the form of

$$K(s) = k\left(\frac{s+2}{s+a}\right)$$

For a 60 degree phase margin at 2 rad/sec

$$GK(j2) = \left(\left(\frac{10}{s(s+2)(s+10)}\right)\left(\frac{k(s+2)}{s+a}\right)\right)_{s=j2} = 1\angle -120^0$$

Evaluating what we know

$$\left(\left(\frac{10}{s(s+2)(s+10)}\right)\left(\frac{s+2}{1}\right)\right)_{s=j2} = 0.4903\angle -101.3099^0$$

For the phase to add up to -120 degrees

$$\angle(s+a) = 18.6901^0$$

$$a = \frac{2}{\tan\left(18.6901^0\right)} = 5.9121$$

Going back to GK

$$\left(\left(\frac{10}{s(s+2)(s+10)}\right)\left(\frac{s+2}{s+5.9121}\right)\right)_{s=j2} = 0.0786\angle -120^0$$

meaning

$$k = \frac{1}{0.0786} = 12.7297$$

and

$$K(s) = 12.7297\left(\frac{s+2}{s+5.9121}\right)$$

Other answers work:

- $G(j0.9214) = 0.4909\angle -120^0$
- The zero can be 1..3 x 0.9214 rad/sec