ECE 461 / 661 Homework #3

Structured Text,LaPlace Transforms, 1st and 2nd Order Approximations

Due Monday, September 14th

PLC Structured Text

- Will accept problems 1-3 any time before December 1st (so you can use the Micro810 PLC's)

Option #1: If using Allen Bradley PLC's write a structured text program (i.e. a Pascal program) to implement the automated watering system of homework #2.

Option #2: Write a C program (for an Arduino, PIC processor, Rasberry PI) to implement the automated watering system of homework #2

A soil moisture sensor measures the ground moisture

- 0V = dry
- 10V = wet

Start the watering process if

- You press button #0, or
- The moisture sensor reads less than 4.00V for more than 10 seconds,

When watering starts

- Relay #0 turns on for 5 seconds
- One second later, Relay #1 turns on for 5 seconds,
- One second later, Relay #2 turns on for 5 seconds.

1) Write a strtuctured test program to implement the same program

2) Test your program (collect data on its timing)

3) Demo your program (in person or with a video)

LaPlace Transforms

4) Assume Y and X are related by

$$Y = \left(\frac{10(s+2)}{(s+1)(s+7)}\right) X$$

a) What is the differential equation relating X and Y?

Cross multiply

$$(s+1)(s+7)Y = 10(s+2)X$$

$$(s^2+8s+7)Y = (10s+20)X$$

sY means dy/dt

$$y'' + 8y' + 7y = 10x' + 20x$$

$$\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 7y = 10\frac{dx}{dt} + 20x$$

b) Find y(t) assuming

$$x(t) = 2 + 3\sin(4t)$$

Use phasors and superposition

DC: x(t) = 2

$$X = 2$$

$$s = 0$$

$$Y = \left(\frac{10(s+2)}{(s+1)(s+7)}\right)_{s=0} (2 + j0)$$

$$Y = 5.714$$

meaning y(t) = 5.714

AC: x(t) = 3 sin(4t)

$$X = 0 - j3$$

$$s = j4$$

$$Y = \left(\frac{10(s+2)}{(s+1)(s+7)}\right)_{s=j4} (0 - j3)$$

$$Y = -2.715 - j2.986$$

meaning y(t) = -2.715 cos(4t) + 2.986 sin(4t). The total Answer: DC + AC

$$y(t) = 5.714 - 2.715\cos(4t) + 2.986\sin(4t)$$

c) Find y(t) assuming

$$x(t) = 2u(t)$$

Use LaPlace transforms

$$X(s) = \left(\frac{2}{s}\right)$$

$$Y = \left(\frac{10(s+2)}{(s+1)(s+7)}\right)\left(\frac{2}{s}\right)$$

using partial fractions

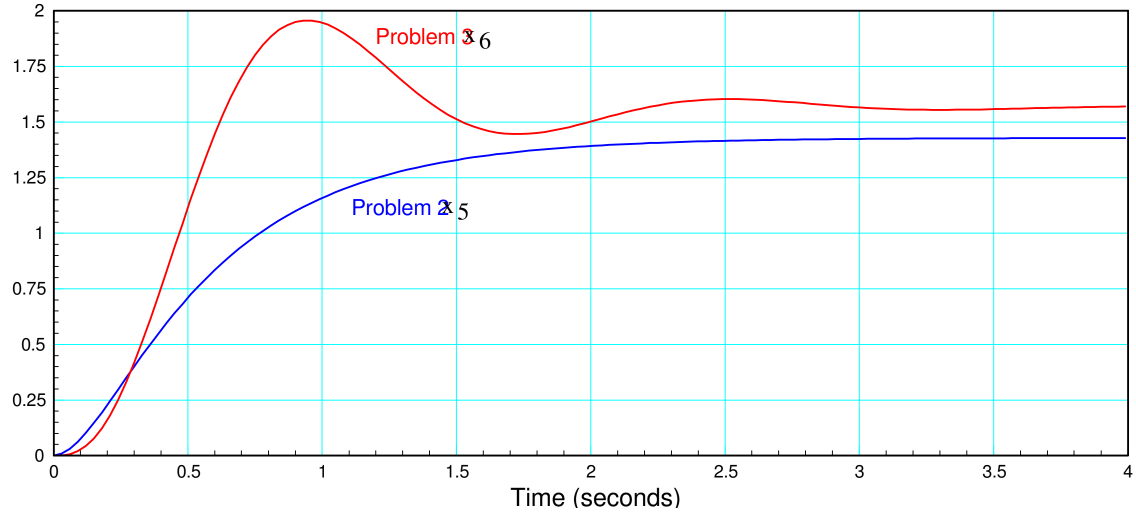
$$Y = \left(\frac{5.714}{s}\right) + \left(\frac{-3.333}{s+1}\right) + \left(\frac{-2.831}{s+7}\right)$$

taking the inverse LaPlace transform

$$y(t) = (5.714 - 3.333e^{-t} - 2.831e^{-7t})u(t)$$

1st and 2nd Order Approximations:

5) Determine the transfer function for a system with the following step response (blue)



Problem 2 (blue - no oscillations) & Problem 3 (red - oscillations)

This is a 1st-order system (no oscillations).

$$Y = \left(\frac{a}{s+b}\right)X$$

To find the transfr function, we need two measurements.

DC gain = 1.40

$$\left(\frac{a}{b}\right) = 1.4$$

2% settling time = 2.5 seconds (approx)

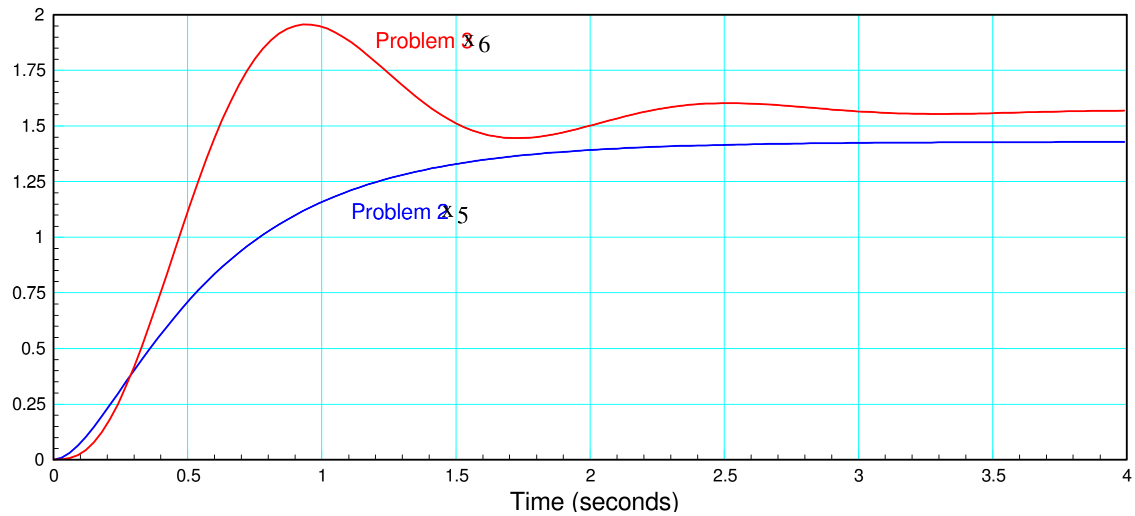
$$b = \frac{4}{2.5} = 1.60$$

meaning

$$G(s) = \left(\frac{a}{s+1.6}\right) = \left(\frac{2.24}{s+1.6}\right)$$

Pick 'a' to make the DC gain equal to 1.40

6) Determine the transfer function for a system with the following step response (red)



This is a 2nd-order system (it has oscillations)

$$G(s) = \left(\frac{k}{(s+\sigma+j\omega)(s+\sigma-j\omega)}\right)$$

The 2% settlimg time is 3 seconds (approx)

$$\sigma = \frac{4}{3} = 1.333$$

The period is 1.7 seconds

$$f = \frac{1}{T} = \frac{1}{1.7} = 0.75Hz$$

$$\omega = 2\pi f = 4.71$$

meaning

$$G(s) = \left(\frac{k}{(s+1.333+j4.71)(s+1.333-j4.71)}\right)$$

The DC gain is 1.6. Pick 'k' to make the DC gain 1.60

$$G(s) = \left(\frac{38.34}{(s+1.333+j4.71)(s+1.333-j4.71)}\right)$$

7) Find a 2nd-order approximation for G(s). Plot the step response of both G(s) and its second order approximation

a) $$G(s) = \left(\frac{2000}{(s+0.5)(s+2)(s+5)(s+10)}\right)$$

$$G(s) \approx \left(\frac{k}{(s+0.5)(s+2)}\right)$$

Pick 'k' so that the DC gain is the same

$$\left(\frac{2000}{(s+0.5)(s+2)(s+5)(s+10)}\right)_{s=0} = 40$$

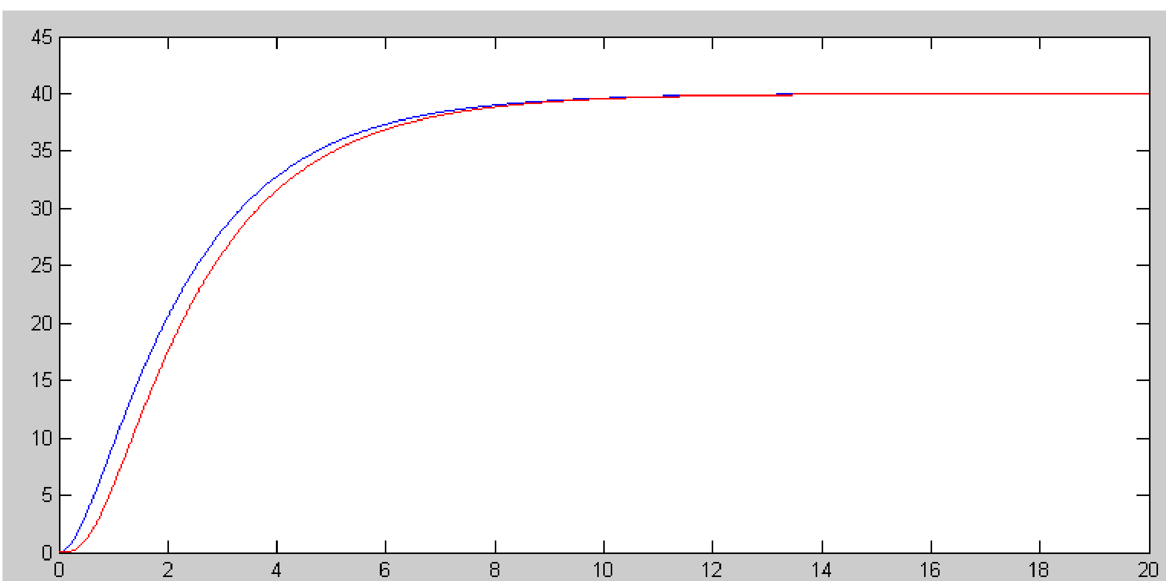
$$\left(\frac{k}{(s+0.5)(s+2)}\right)_{s=0} = 40$$

$$k = 40$$

$$G(s) \approx \left(\frac{40}{(s+0.5)(s+2)}\right)$$

Checking in Matlab

```
>> G4 = zpk([],[-0.5,-2,-5,-10],2000);
>> G2 = zpk([],[-0.5,-2],40);
>> t = [0:0.01:20]';
>> y2 = step(G2,t);
>> y4 = step(G4,t);
>> plot(t,y2,'b',t,y4,'r');
```



2nd-order system (blue) & 4th-order system (red)

7b) $$G(s) = \left(\frac{200{,}000}{(s+2+j10)(s+2-j10)(s+8+j20)(s+8-j20)}\right)$$

$$G(s) \approx \left(\frac{k}{(s+2+j10)(s+2-j10)}\right)$$

Pick 'k' to match the DC gain

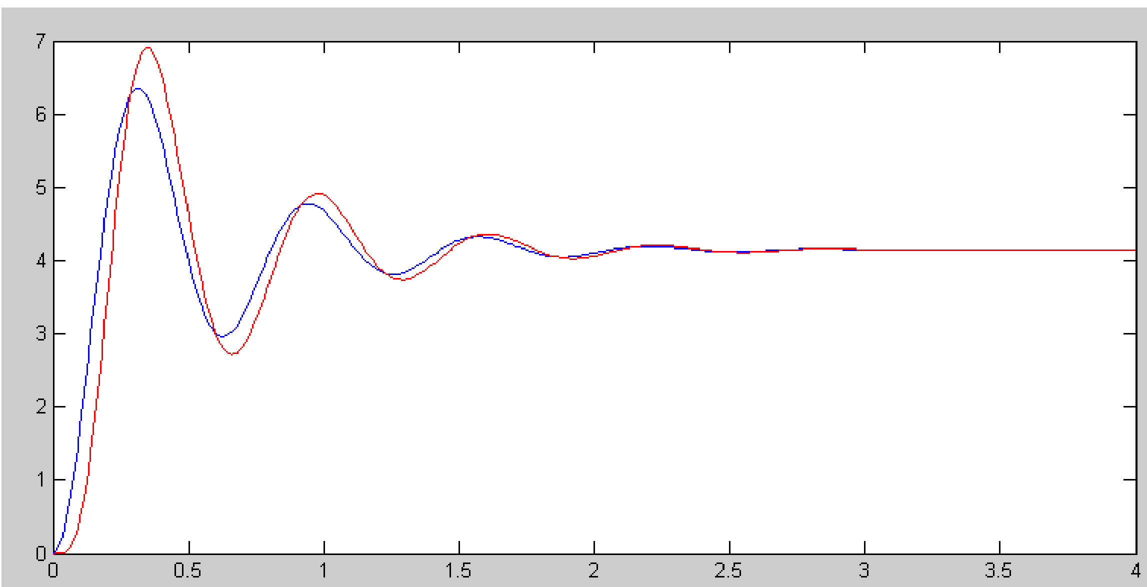
$$\left(\frac{200{,}000}{(s+2+j10)(s+2-j10)(s+8+j20)(s+8-j20)}\right)_{s=0} = 4.145$$

$$\left(\frac{k}{(s+2+j10)(s+2-j10)}\right)_{s=0} = 4.145$$

$$k = 431.03$$

$$G(s) \approx \left(\frac{431.03}{(s+2+j10)(s+2-j10)}\right)$$

```
>> G4 = zpk([],[-2+j*10,-2-j*10,-8+j*20,-8-j*20],200000);
>> G2 = zpk([],[-2+j*10,-2-j*10],431.03);
>> t = [0:0.01:4]';
>> y2 = step(G2,t);
>> y4 = step(G4,t);
>> plot(t,y2,'b',t,y4,'r');
```



2nd-order system (blue) & 4th-order system (red)