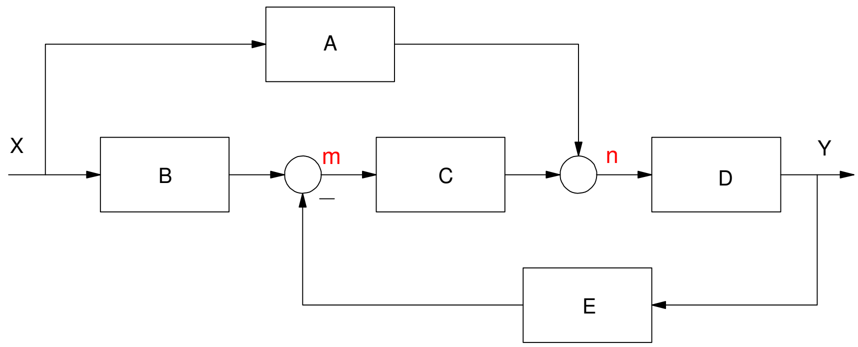
Homework #4: ECE 461/661

Block Diagrams, Canonical Forms, Electrical Circuits.   Due Monday, September 21st

Block Diagrams

1) Determine the transfer function from X to Y



Shortcut

$$Y = \left(\frac{BCD+AD}{1+CDE}\right)X$$

Long Way

$$m = BX - EY$$

$$n = AX + CM$$

$$Y = Dn$$

Solving

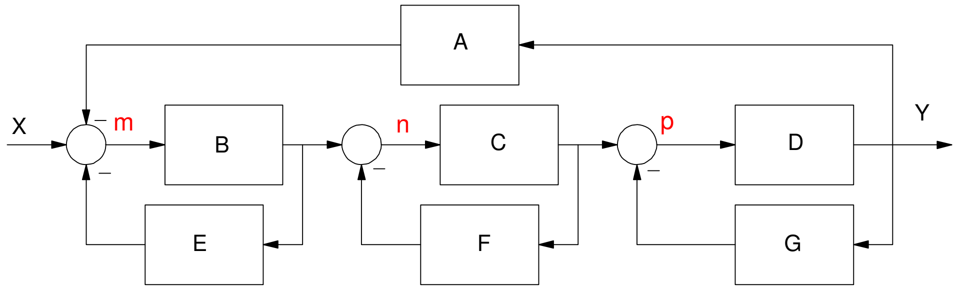
$$Y = DAX + DCm$$

$$Y = DAX + DC(BX - EY)$$

$$(1 + DCE)Y = DAX + DCBX$$

$$Y = \left(\frac{DA+DCB}{1+DCE}\right)X$$

2) Determine the transfer funciton from X to Y



Shortcut doesn't exactly work here due to three feedback loops in series

$$Y = \left(\frac{BCD}{1+BE+CF+DG+ABCD}\right)X$$

What does work is to simplify each feedback loop

$$Y = \left(\frac{\left(\frac{B}{1+BE}\right)\left(\frac{C}{1+CF}\right)\left(\frac{D}{1+DG}\right)}{1+A\left(\frac{B}{1+BE}\right)\left(\frac{C}{1+CF}\right)\left(\frac{D}{1+DG}\right)}\right)X$$

Simlifying

$$Y = \left(\frac{DCB}{(1+GD)(1+FC)(1+EB)+DCBA}\right)X$$

Long Way

$$m = X - EBm - AY$$

$$n = Bm - FCn$$

$$p = Cn - GDp$$

$$Y = Dp$$

Solving

$$m = \left(\frac{1}{1+EB}\right)(X - AY)$$

$$n = \left(\frac{B}{1+FC}\right)m$$

$$p = \left(\frac{C}{1+GD}\right)n$$

$$p = \left(\frac{C}{1+GD}\right)\left(\frac{B}{1+FC}\right)\left(\frac{1}{1+EB}\right)(X - AY)$$

$$Y = Dp = D\left(\frac{C}{1+GD}\right)\left(\frac{B}{1+FC}\right)\left(\frac{1}{1+EB}\right)(X - AY)$$

$$((1+GD)(1+FC)(1+EB)+DCBA)Y = DCBX$$

$$Y = \left(\frac{DCB}{(1+GD)(1+FC)(1+EB)+DCBA}\right)X$$

Canonical Forms

3) Give two different state-space models that produce the following transfer function

$$Y = \left(\frac{200(s+5)}{(s+2)(s+3+j5)(s+3-j5)} \right) U$$

Multiply out

```
>> G = zpk(-5,[-2,-3+j*5,-3-j*5],200)

       200 (s+5)
---------------------
(s+2) (s^2 + 6s + 34)

>> tf(G)

     200 s + 1000
-----------------------
s^3 + 8 s^2 + 46 s + 68
```

By inspection, in controller canonical form

$$s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -68 & -46 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1000 & 200 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

Observer form is the transpose

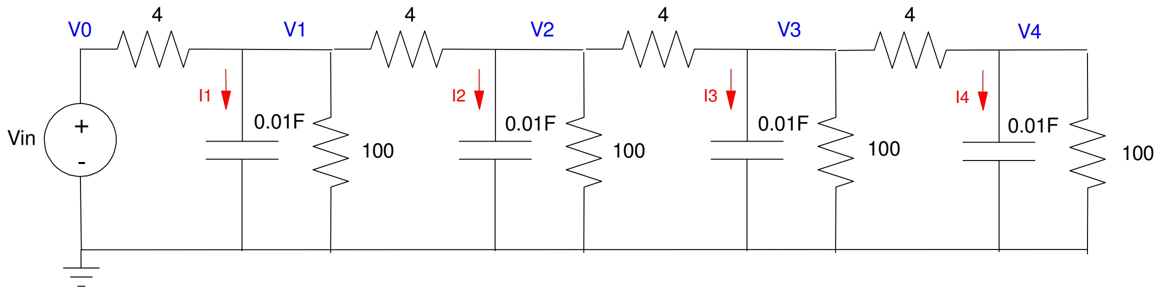
$$s\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -68 \\ 1 & 0 & -46 \\ 0 & 1 & -8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1000 \\ 200 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

There are an infinite number of other ways to express the same transfer function.

Electrical Ciruits

4) Using state-space methods, find the transfer function from Vin to V4



$$I_1 = CsV_1 = \left(\frac{V_0 - V_1}{4}\right) + \left(\frac{V_2 - V_1}{4}\right) - \left(\frac{V_1}{100}\right)$$

the same pattern repeats for V2 and V3. V4 is the oddball

$$I_4 = CsV_4 = \left(\frac{V_3 - V_4}{4}\right) - \left(\frac{V_4}{100}\right)$$

plugging in numbers

$$sV_1 = 25V_0 - 51V_1 + 25V_2$$

$$sV_2 = 25V_1 - 51V_2 + 25V_3$$

$$sV_3 = 25V_2 - 51V_3 + 25V_4$$

$$sV_4 = 25V_3 - 26V_4$$

Put in matrix (state-space) form

$$s\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -51 & 25 & 0 & 0 \\ 25 & -51 & 25 & 0 \\ 0 & 25 & -51 & 25 \\ 0 & 0 & 25 & -26 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} 25 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

$$Y = V_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Input this into matlab and find the transfer function

```
>> A = [-51,25,0,0 ;
25,-51,25,0 ;
0,25,-51,25 ;
0,0,25,-26]

A =

   -51    25     0     0
    25   -51    25     0
     0    25   -51    25
     0     0    25   -26

>> B = [25 ; 0 ; 0 ; 0]

B =

    25
     0
     0
     0

>> C = [0,0,0,1];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

Zero/pole/gain:
              390625
-----------------------------------
(s+89.3) (s+59.68) (s+26) (s+4.015)
```

>>

5) For the previous RC filter, find the transfer function from Vin to V3

```
>> C = [0,0,1,0];
>> G = ss(A,B,C,D);
>> zpk(G)

Zero/pole/gain:
           15625 (s+26)
-----------------------------------
(s+89.3) (s+59.68) (s+26) (s+4.015)
```

Note that

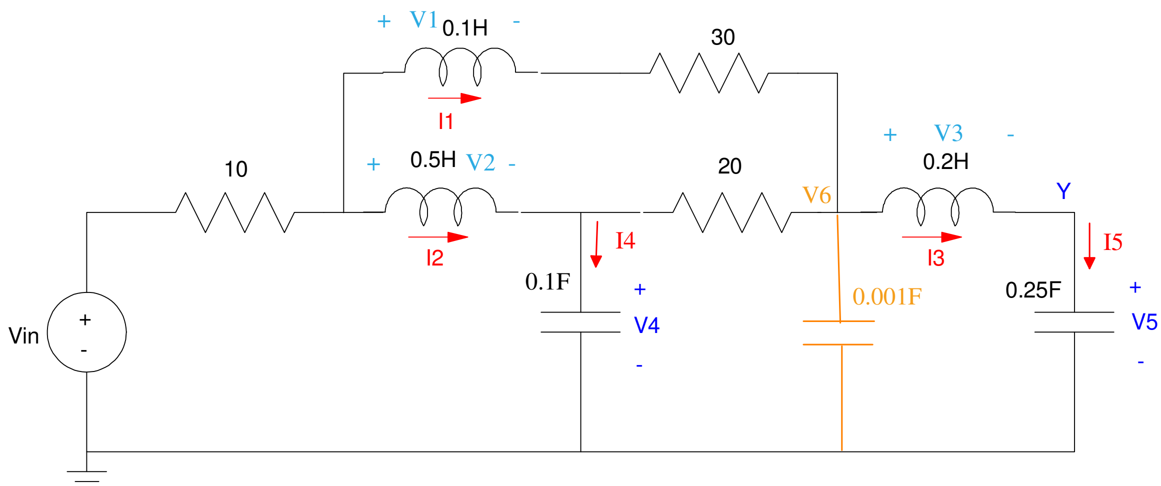
- the poles don't change when you change the output
- the zeros do change, however

Poles tell you how the energy decays. Changing what you measure doesn't change how the energy decays.

Zeroes tell you which energy states you're watching. They do change as you change what you're measuring.

6) Express the dynamics for the following RLC circuit in state-space form.

- Find the transfr function from Vin to V5



Problem 6 & 7

Without the 0.001F capacitor, it gets a little tricky writing the equation for L1 (0.1H). Adding a 0.001F capacitor makes the equations a lot easier to write, and changes the circuit only slightly...

$$V_1 = 0.1sI_1 = V_{in} - 10(I_1 + I_2) - 30I_1 - V_6$$

$$V_2 = 0.5sI_2 = V_{in} - 10(I_1 + I_2) - V_4$$

$$V_3 = 0.2sI_3 = V_6 - V_5$$

$$I_4 = 0.1sV_4 = I_2 - \left(\frac{V_4 - V_6}{20}\right)$$

$$I_5 = 0.2sV_5 = I_3$$

$$I_6 = 0.001sV_6 = \left(\frac{V_4 - V_6}{20}\right) + I_1 - I_3$$

Solving and simplifying

$$sI_1 = 10V_{in} - 400I_1 - 100I_2 - 10V_6$$

$$sI_2 = 2V_{in} - 20I_1 - 20I_2 - 2V_4$$

$$sI_3 = 5V_6 - 5V_5$$

$$sV_4 = 10I_2 - 0.5V_4 + 0.5V_6$$

$$sV_5 = 5I_3$$

$$sV_6 = 50V_4 - 50V_6 + 1000I_1 - 1000I_3$$

Place in matrix form (state-space form)

$$s\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} -400 & -100 & 0 & 0 & 0 & -10 \\ -20 & -20 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 5 \\ 0 & 10 & 0 & -0.5 & 0 & 0.5 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 1000 & 0 & -1000 & 50 & 0 & -50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} + \begin{bmatrix} 10 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

$$Y = V_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

In Matlab

```
>> A = [-400,-100,0,0,0,-10 ;
-20,-20,0,-2,0,0 ;
0,0,0,0,-5,5 ;
0,10,0,-0.5,0,0.5 ;
0,0,5,0,0,0 ;
1000,0,-1000,50,0,-50]

    1.0e+003 *

    -0.4000   -0.1000         0         0         0   -0.0100
    -0.0200   -0.0200         0   -0.0020         0         0
          0         0         0         0   -0.0050    0.0050
          0    0.0100         0   -0.0005         0    0.0005
          0         0    0.0050         0         0         0
     1.0000         0   -1.0000    0.0500         0   -0.0500

>> B = [10;0;0;0;0;0]

    10
     0
     0
     0
     0
     0

C = [0,0,0,0,1,0];
D = 0;
G = ss(A,B,C,D);
zpk(G)

                    250000 (s+19.47) (s+1.027)
--------------------------------------------------------------
(s+376.2) (s+12.53) (s+2.352) (s+0.2036) (s^2 + 79.26s + 5538)
```

The fast pole (-376.2) is due to the 0.001F capacitor. Remove it and you have the original circuit.

7) Assime Vin = 0. Specify the initial conditions so that the total energy at t = 0 is 1.0 Joules and

- The transients decay as slow as possible
- The transients decay as fast as possible

This is an eigenvector problem

```
>> [M,V] = eig(A)

M =

    0.3213            -0.0254 + 0.0064i  -0.0254 - 0.0064i   0.1607            -0.0461             0.0108
    0.0180            -0.0040 - 0.0069i  -0.0040 + 0.0069i  -0.5624             0.1560             0.0232
    0.0126            -0.0360 - 0.0567i  -0.0360 + 0.0567i   0.2080            -0.1048             0.0270
    0.0008            -0.0041 - 0.0048i  -0.0041 + 0.0048i   0.4925            -0.9159            -0.3384
   -0.0002            -0.0019 + 0.0041i  -0.0019 - 0.0041i  -0.0830             0.2229            -0.6643
   -0.9467             0.9973             0.9973            -0.6043             0.2722            -0.6654

   -376.15            -39.63 - 62.99i     -39.63 + 62.99i    -12.53             -2.35              -0.020
```

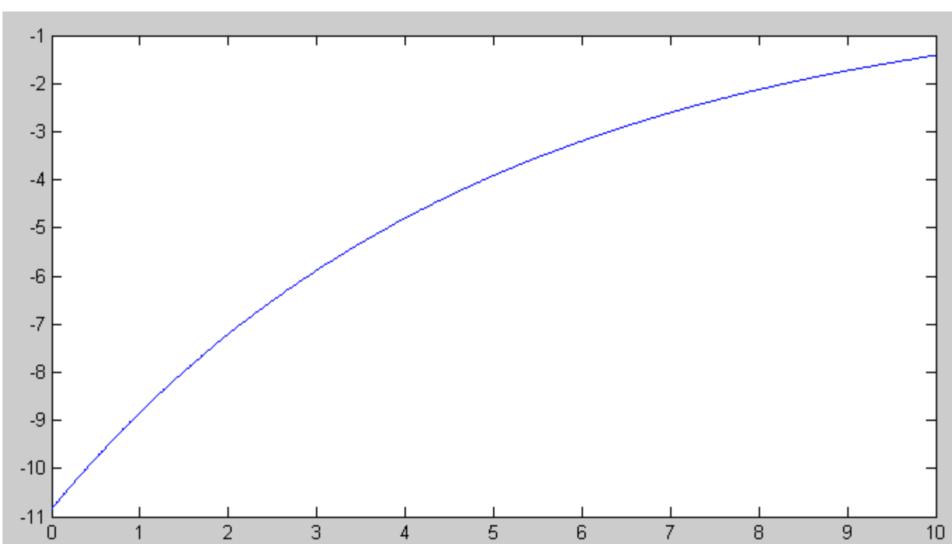
The eigenvector associated with the eigenvalue at -376 is just the voltage on the 0.001F capacitor (ignore)

Pole at s = -0.2036

```
X0 = M(:,6);

Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =    0.0613

X0 = X0 / Joules
```

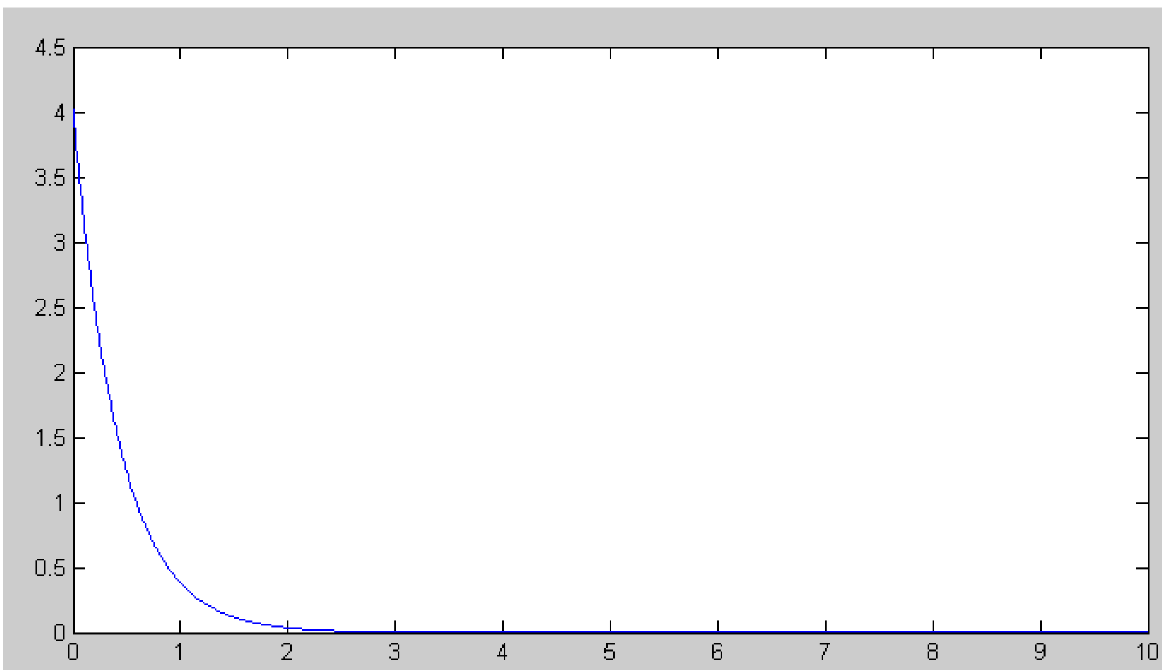
```
I1    0.1766
I2    0.3790
I3    0.4411
V4   -5.5174
V5  -10.8327
V6  -10.8507
```

```
>> G = ss(A, X0, C, D);
>> t = [0:0.01:10]';
>> y = impulse(G,t);
>> plot(t,y)
>>
```



Pole at s = -2.35

```
>> X0 = M(:,5);
>> Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =    0.0555

>> X0 = X0 / Joules

I1   -0.8306
I2    2.8123
I3   -1.8898
V4  -16.5092
V5    4.0175
V6    4.9065

>> G = ss(A, X0, C, D);
>> y = impulse(G,t);
>> plot(t,y)
>>
```

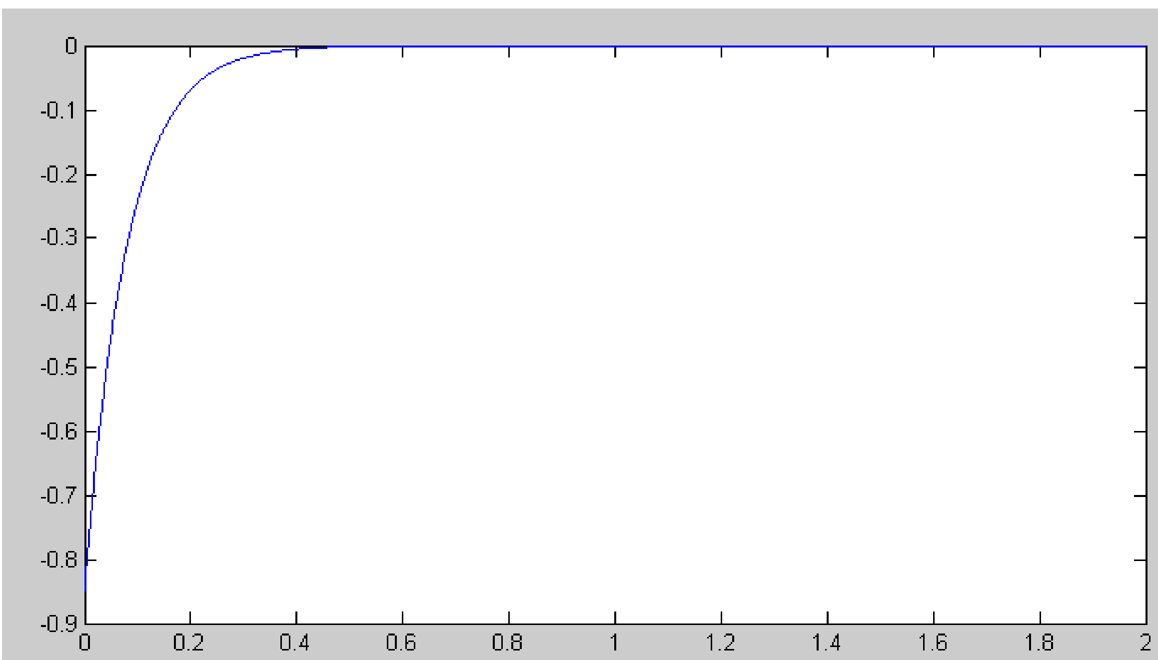


pole at s = -12.53

```
>> V(4,4)

  -12.5321

>> t = [0:0.001:2]';
>> X0 = M(:,4);
>> Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =    0.0979

>> X0 = X0 / Joules
```

```
I1    1.6426
I2   -5.7471
I3    2.1257
V4    5.0331
V5   -0.8481
V6   -6.1759
```

```
>> G = ss(A, X0, C, D);
>> y = impulse(G,t);
>> plot(t,y)
>>
```

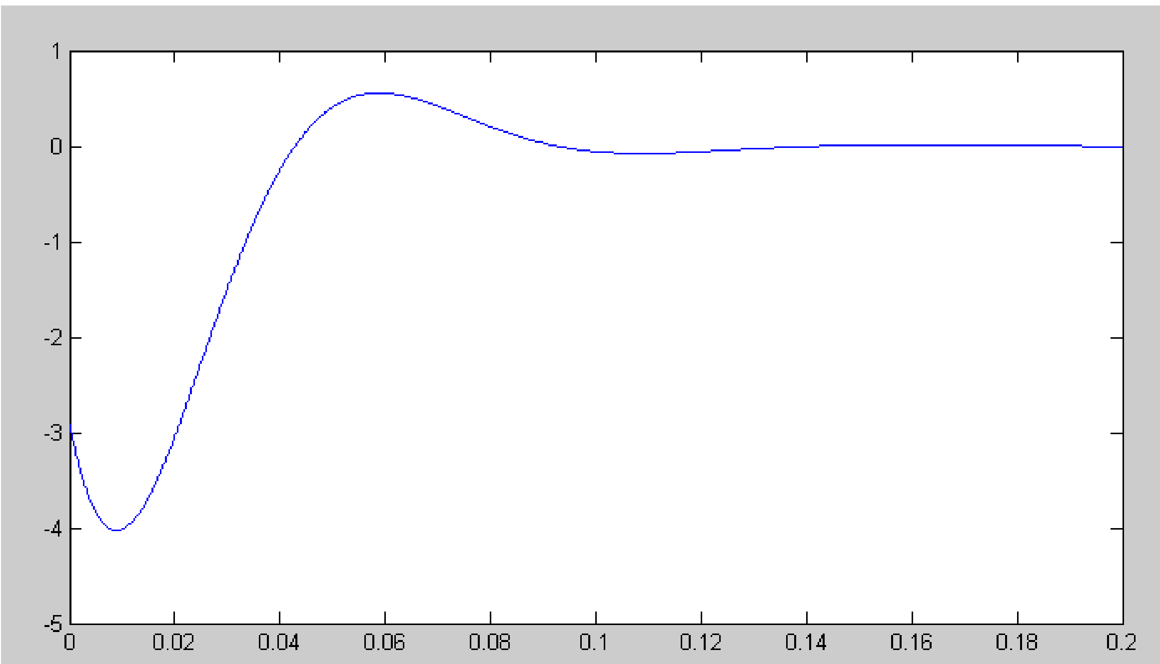


pole at s = -39.62 + j62.98

```
>> V(3,3)

ans =

 -39.6295 -62.9870i

>> X0 = real( M(:,3) );
>> t = [0:0.001:1]' * 0.2;
>> Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =

  6.6457e-004

>> X0 = X0 / Joules
```

I1	**-38.3**
I2	**-6.1**
I3	**-54.1**
V4	**-6.1**
V5	**-2.9**
V6	**1500.7**

```
>> G = ss(A, X0, C, D);
>> y = impulse(G,t);
>> plot(t,y)
>>
```

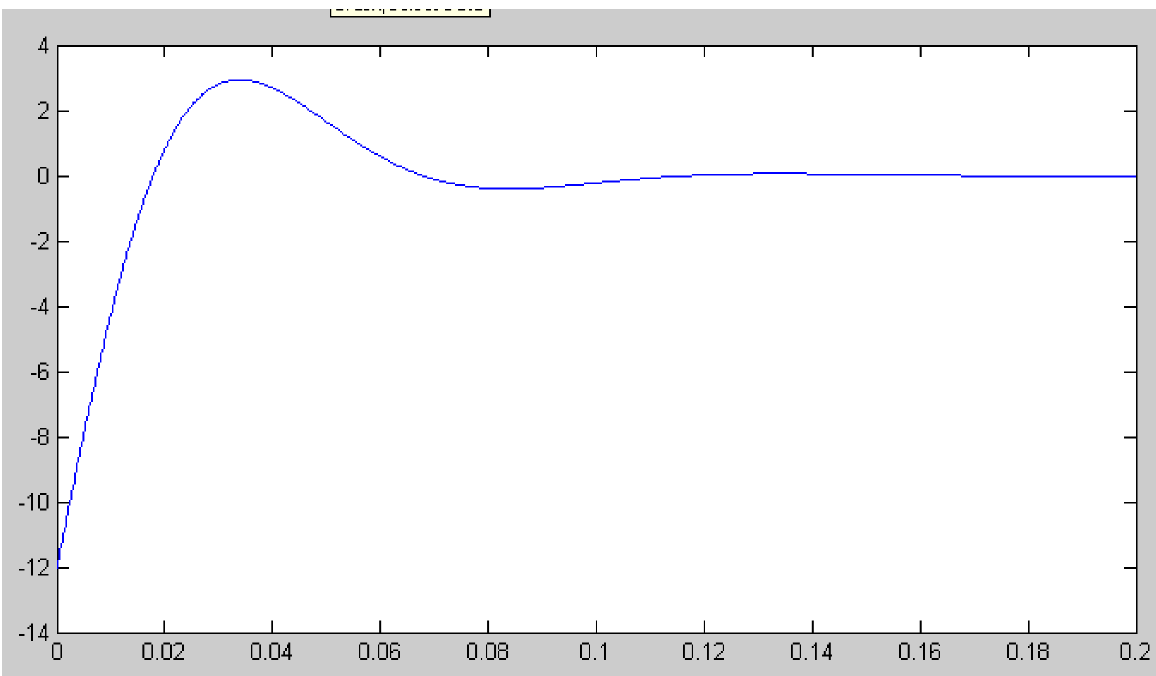


pole at -39.62 + j62.98 (take 2)

```
>> V(3,3)

 -39.6295 -62.9870i

>> X0 = imag( M(:,3) );
>> Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =  3.3860e-004

>> X0 = imag( M(:,3) );
>> X0 = X0 / Joules
```

```
I1  -18.8268
I2   20.5208
I3  167.4041
V4   14.0443
V5  -12.0339
V6         0
```

```
>> G = ss(A, X0, C, D);
>> y = impulse(G,t);
>> plot(t,y)
>>
```



pole at -376

```
>> V(1,1)

 -376.1534

>> X0 = M(:,1);
>> Joules = sum ( 0.5 * [0.1 ; 0.5 ; 0.2 ; 0.1 ; 0.25 ; 0.001] .* X0.^2 )

Joules =    0.0057

>> X0 = X0 / Joules

I1   56.2956
I2    3.1621
I3    2.2044
V4    0.1366
V5   -0.0293
V6 -165.8670

>> G = ss(A, X0, C, D);
>> y = impulse(G,t);
>> plot(t,y)
>>
```