Homework #6: ECE 461/661

Error Constants, Routh Criteria, Skething a Root Locus. Due Monday, October 12th

Error Constants

1) Determine the error constants and steady-state error for the following systems

G(s)	System Type	Kp	Kv	Error for a unit step input
$\left(\frac{20}{(s+3)(s+10)}\right)$	**0**	**0.67**	**0**	**0.6**
$\left(\frac{20}{s(s+3)(s+10)}\right)$	**1**	**inf**	**0.67**	**0**
$\left(\frac{20(s+1)}{s^2(s+3)(s+10)}\right)$	**2**	**inf**	**inf**	**0**
$\left(\frac{20}{(s-3)(s+10)}\right)$	**0**	**-0.67**	**0**	**3**

Routh Criteria

Determine the range of k that results in a negative definite polynomial (i.e. a stable system)

2) $(s-1)(s+4)(s+5)+5k=0$

```
>> poly([1,-4,-5])

ans =     1     8    11   -20
```

$$s^3+8s^2+11s+5k-20=0$$

Setting up a Routh table

1	**11**	**0**	
8	**5k-20**	**0**	
$\frac{-\begin{vmatrix} 1 & 11 \\ 8 & 5k-20 \end{vmatrix}}{8} = 13.5-0.625k$	$\frac{-\begin{vmatrix} 1 & 0 \\ 8 & 0 \end{vmatrix}}{8} = 0$		**k < 21.6**
5k-20	**0**		**k > 4**
0	**0**		

result

4 < k < 21.6

3) $(s+1)(s+3)(s+7)(s+8)+5k=0$

multiply out

```
>> poly([-1,-3,-7,-8])

ans =     1    19   119   269   168
```

$$s^4+19s^3+119s^2+269s+168+5k=0$$

1	**119**	**168+5k**	
19	**269**	**0**	
104.842	**168 + 5k**	**0**	
238 - 0.906k	**0**	**0**	**k < 263.27**
168 + 5k	**0**	**0**	**k > -33.6**
0	**0**	**0**	

(3,1) $$\frac{-\begin{vmatrix} 1 & 119 \\ 19 & 269 \end{vmatrix}}{19}=104.842$$

(3,2) $$\frac{-\begin{vmatrix} 1 & 168+5k \\ 19 & 0 \end{vmatrix}}{19}=168+5k$$

(4,1) $$\frac{-\begin{vmatrix} 19 & 269 \\ 104.842 & 168+5k \end{vmatrix}}{104.842}=238.554-0.906k$$

Sketching a Root Locus

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), and asymptotes

4) $(s-1)(s+4)(s+5)+5k=0$

Real Axis Loci:

- (+1, -4), (-5, -infinity)

Breakaway Point:

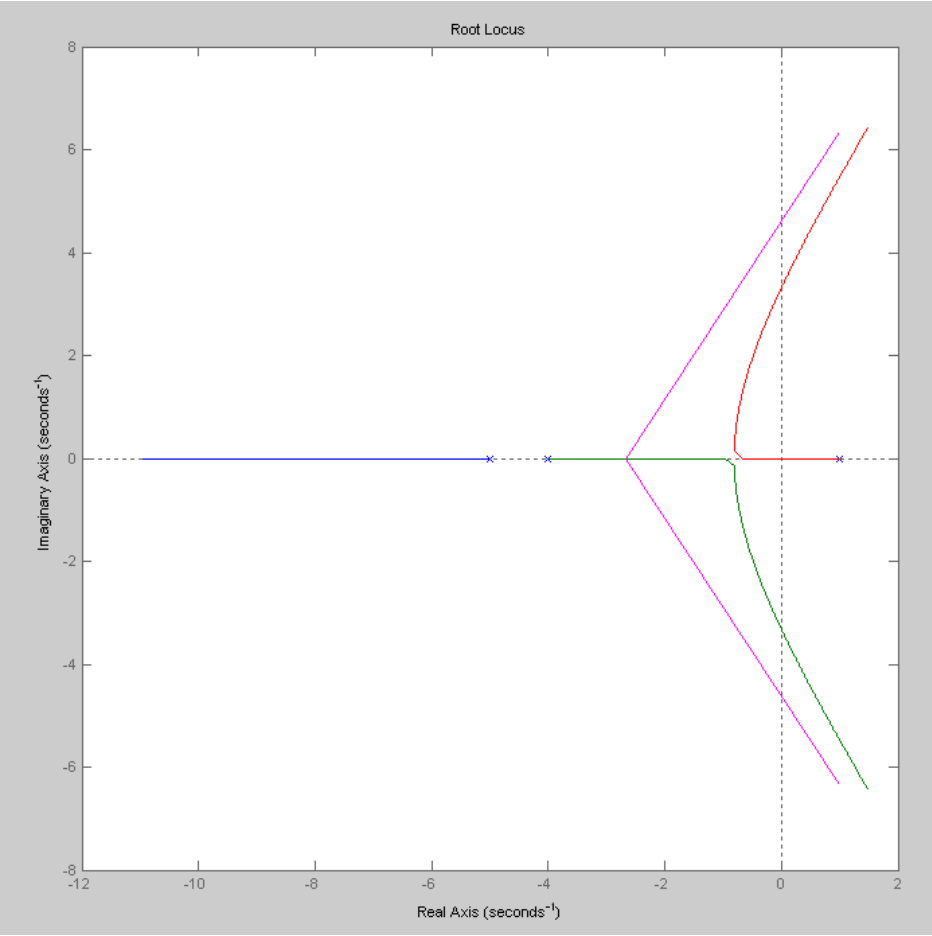
- s = -0.810

jw crossing:

- j3.317

Asymptotes

- 3 asymptotes
- intercept = -8/3
- angles {+60 degrees, -60 degrees, 180 degrees}



5) $(s+1)(s+3)(s+7)(s+8)+5k=0$

Real Axis Loci

(-1, -3), (-7, -8)

Breakaway Points

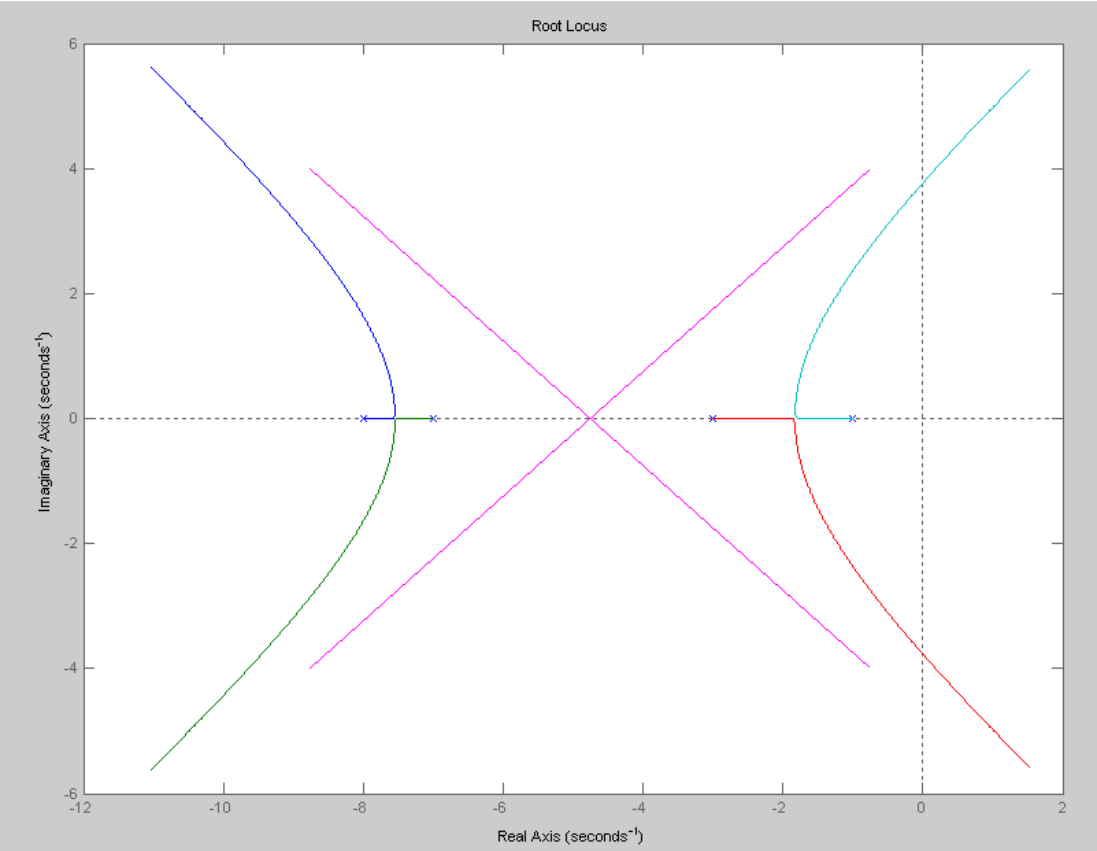
-1.826, -7.548

jw crossing

j3.763, -j3.763

Asymptotes

- 4 asymptotes
- +/- 45 degrees
- +/- 135 degrees
- intercept = -19/4



Root Locus with Complex Poles

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), asymptotes, and departure/approach angle

6) $G(s) = \left(\frac{s}{(s+5)\left(s^2+2s+10\right)} \right)$

Real Axis Loci

- (0, -5)

Asymptotes

- 2 asymptotes
- intercept = -3.5
- +/- 90 degrees

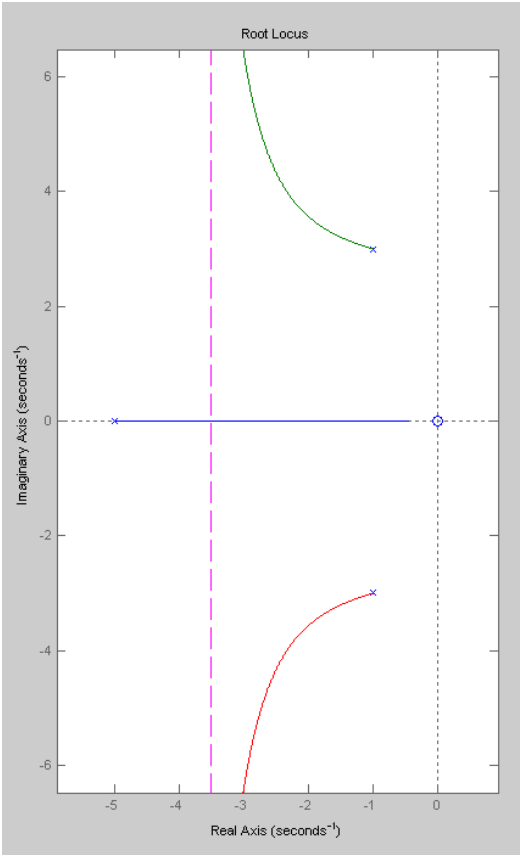
Departure Angle

$$\left(\frac{s}{(s+5)(s+1+j3)(s+1-j3)} \right)_{s=-1+j3} = x\angle 180^0$$

$$\left(\frac{s}{(s+5)(s+1+j3)} \right)_{s=-1+j3} = 0.104\angle -18.435^0$$

$$angle\left(\frac{1}{s+1-j3} \right) = -161.565^0$$

$$angle(s+1-j3) = +161.565^0$$ = departure angle



7) $G(s) = \left(\frac{s^2+4}{s(s+2)(s+5)(s+6)} \right)$

Real Axis Loci

- (0, -2)
- (-5, -6)

Asymptotes

- 2 asymptotes
- +/- 90 degrees
- intercept = -13/2

Approach Angle

$$\left(\frac{(s+j2)(s-j2)}{s(s+2)(s+5)(s+6)} \right)_{s=j2} = x\angle 180^0$$

$$\left(\frac{(s+j2)}{s(s+2)(s+5)(s+6)} \right)_{s=j2} = 0.021\angle -85.236^0$$

$angle(s-j2) = 94.764^0$ approach angle