Homework #8: ECE 461/661

Meeting Specs, Delays, Unstable Systems. Due Monday, October 26th
20 points per problem

Meeting Design Specs

1) Assume

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 6 seconds, and
- 20% overshoot for the step response

Translation:

- Make it a type-1 system
- Place the closed-loop dominant pole at s = -0.667 + j1.333

Solution: Let

$$K(s) = k\left(\frac{(s+0.1617)((s+1.04)}{s(s+a)} \right)$$

resulting in

$$GK = \left(\frac{1.4427k}{s(s+a)(s+2.719)(s+5.05)} \right)$$

Evaluate what you know at s = -1 + j2

$$\left(\frac{1.4427}{s(s+2.719)(s+5.05)} \right)_{s=-0.667+j1.333} = 0.086\angle -166.506^0$$

To bring the phase to 180 degrees

$$\angle(s+a) = 13.494^0$$

'a' is then

$$a = \frac{1.333}{\tan\left(13.494^0\right)} + 0.667$$

$$a = 6.222$$

giving

$$K(s) = k\left(\frac{(s+0.1617)(s+1.04)}{s(s+6.222)} \right)$$

$$GK = \left(\frac{1.4427k}{s(s+2.719)(s+5.05)(6.222)} \right)$$

Evaluate at s = -0.667 + j1.333

$$\left(\frac{1.4427k}{s(s+2.719)(s+5.05)(6.222)}\right)_{s=-0.667+j1.333} = 0.015\angle 180^0$$

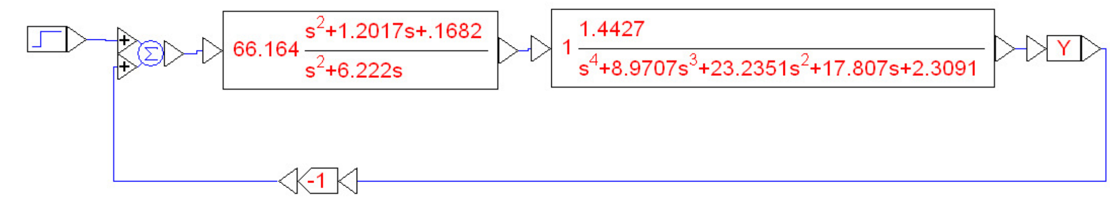
resulting in

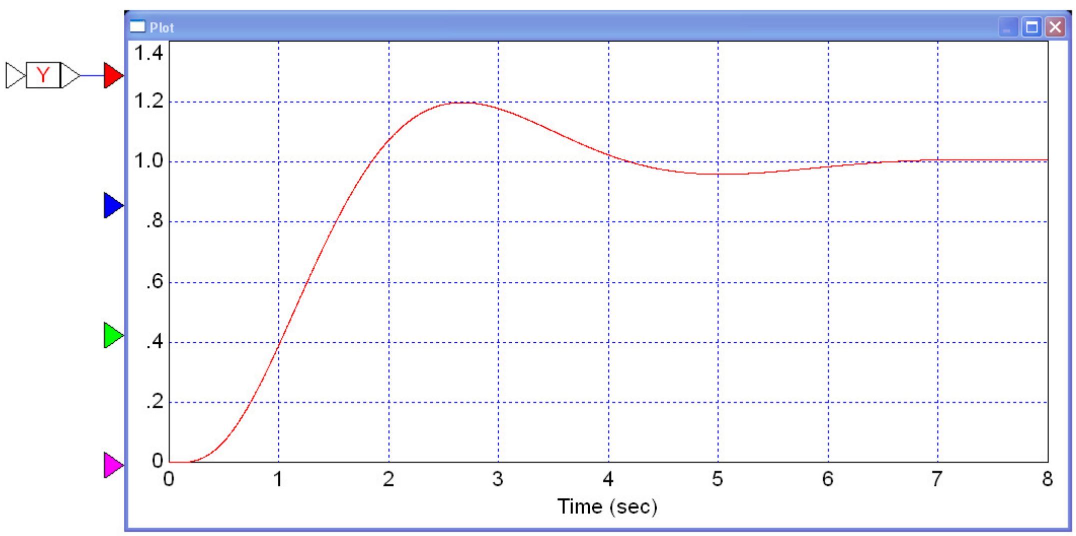
$$k = \frac{1}{0.015} = 66.164$$

and

$$K(s) = 66.164\left(\frac{(s+0.1617)(s+1.04)}{s(s+6.222)}\right)$$

Check your design in Matlab or Simulink or VisSim





Give an op-amp circuit to implement K(s)

$$K(s) = 66.164\left(\frac{(s+0.1617)(s+1.04)}{s(s+6.222)}\right)$$

Rewrite this as

$$K(s) = \left(10\left(\frac{s+1.04}{s+6.222}\right)\right)\left(\frac{6.6164(s+0.1617)}{s}\right)$$

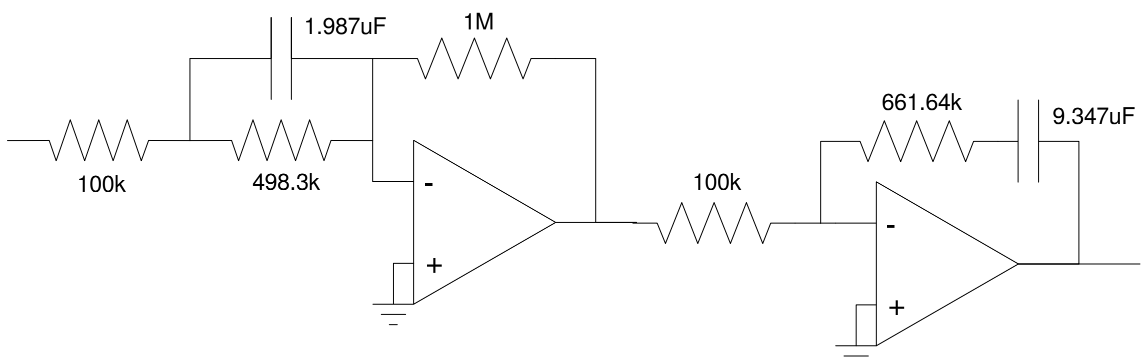
Implement this as a PI * Lead

Lead: Let R1 = 100k

- As s goes to infinity, K(s) = 10. R3 = 1M
- As s goes to zero, the gain is 16.71 = R3 / (R1 + R2).
- R2 = 498.3k
- 1/(R2 C) = 1.04
- C = 1.987uF

PI: Let R1 = 100k

- As s goes to infinity, the gain is 6.6164.
- R2 = 661.64k
- 1/(R2 C) = 0.1617
- C = 9.9347uF



Systems with Delays

2) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) e^{-0.1s}$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 6 seconds, and
- 20% overshoot for the step response

Let K(s) be

$$K(s) = k\left(\frac{(s+0.1617)((s+1.04)}{s(s+a)}\right)$$

$$GK = \left(\frac{1.4427k}{s(s+a)(s+2.719)(s+5.05)}\right) e^{-0.1s}$$

Evaluate what you know at s = -0.667 + j 1.333

$$\left(\left(\frac{1.4427}{s(s+2.719)(s+5.05)}\right) e^{-0.1s}\right)_{s=-0.667+j1.333} = 0.092\angle -174.144^0$$

meaning

$$\angle(s+a) = 5.856^0$$

$$a = \frac{1.333}{\tan\left(5.856^0\right)} + 0.667$$

$$a = 13.664$$

telling you

$$K(s) = k\left(\frac{(s+0.1617)((s+1.04)}{s(s+13.664)}\right)$$

$$GK = \left(\frac{1.4427k}{s(s+2.719)(s+5.05)(s+13.664)}\right) e^{-0.1s}$$

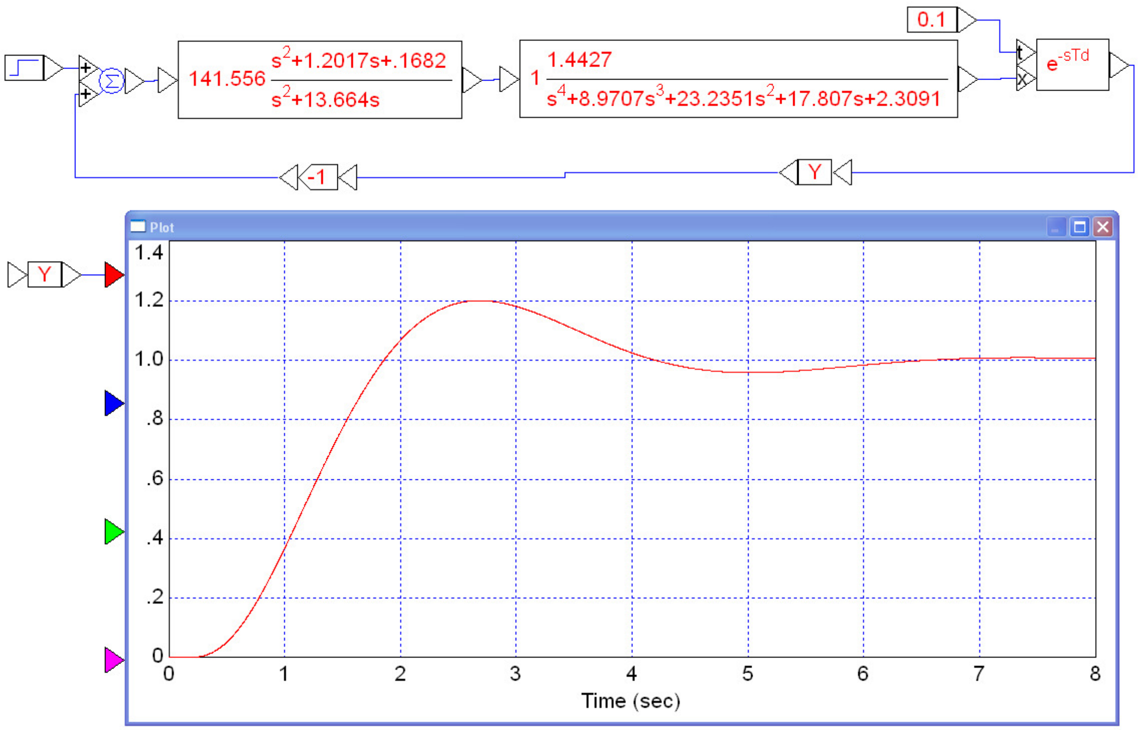
Evaluate at s = -0.667 + j 1.333

$$\left(\left(\frac{1.4427k}{s(s+2.719)(s+5.05)(s+13.664)}\right) e^{-0.1s}\right)_{s=-0.667+j1.333} = 0.007\angle 180^0$$

$$k = \frac{1}{0.007} = 141.556$$

$$K(s) = 141.556\left(\frac{(s+0.1617)((s+1.04)}{s(s+13.664)}\right)$$

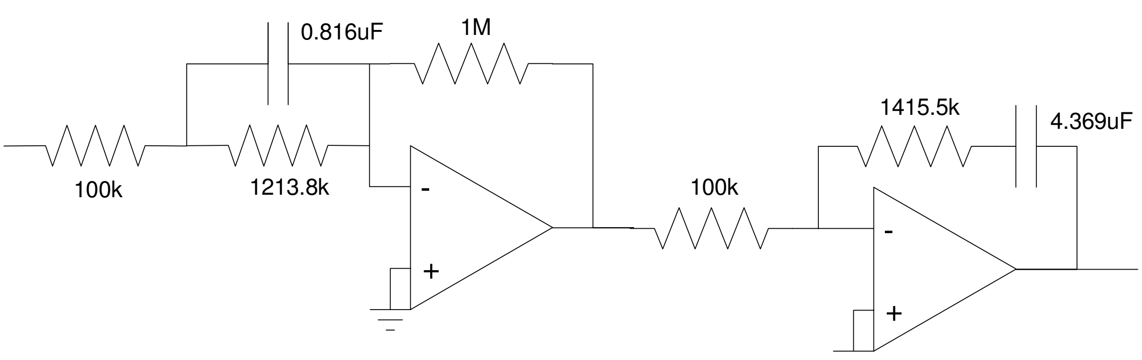
Check your design in Matlab or Simulink or VisSim



Give an op-amp circuit to implement K(s)

$$K(s) = 141.556\left(\frac{(s+0.1617)((s+1.04)}{s(s+13.664)}\right)$$

$$K(s) = \left(10\left(\frac{s+1.04}{s+13.664}\right)\right)\left(\frac{14.1556(s+0.1617)}{s}\right)$$



Unstable Systems

3) Assume the slow pole was unstable

$$G(s) = \left(\frac{1.4427}{(s-0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)$$

Design a compensator, K(s), For the 4th-order model that results in

- No error for a step input
- A 2% settling time of 6 seconds, and
- 20% overshoot for the step response

Check your design in Matlab or Simulink or VisSim

Step 1: Stabilize the system

$$K_1(s) = k\left(\frac{s+1.04}{s+10}\right)$$

Place the closed-loop poles at s = -0.5

$$GK_1 = \left(\frac{1.4427k}{(s-0.1617)(s+2.719)(s+5.05)(s+10)}\right)_{s=-0.5} = -0.0227k$$

$$k = \frac{1}{0.0227} = 43.9924$$

and

$$K_1(s) = 43.9924\left(\frac{s+1.04}{s+10}\right)$$

The closed-loop system is then (from Matlab)

```
>> G = zpk([],[0.1617,-1.04,-2.719,-5.05],1.4427);
>> K1 = zpk(-1.04,-10,43.9927);

>> G2 = minreal(G*K1 / (1+G*K1))

                 63.4683
------------------------------------
(s+0.5) (s+1.435) (s+5.86) (s+9.812)
```

$$G_2 = \left(\frac{GK_1}{1+GK_1}\right) = \left(\frac{63.4683}{(s+0.5)(s+1.435)(s+5.86)(s+9.812)}\right)$$

Now add a second feedback loop to meet the design specs

$$K_2 = k\left(\frac{(s+0.5)(s+1.435)}{s(s+a)}\right)$$

$$G_2K_2 = \left(\frac{63.4683k}{s(s+a)(s+5.86)(s+9.812)}\right)$$

Evaluate what you know at s = -0.667 + j1.333

$$\left(\frac{63.4683}{s(s+5.86)(s+9.812)}\right)_{s=-0.667+j1.333} = 0.8729\angle -139.2719^0$$

meaning

$$\angle(s+a) = 40.7281^0$$

$$a = \frac{1.333}{\tan\left(40.7281^0\right)} + 0.667$$

$$a = 2.2152$$

meaning

$$K_2 = k\left(\frac{(s+0.5)(s+1.435)}{s(s+2.2152)}\right)$$

$$G_2K_2 = \left(\frac{63.4683k}{s(s+2.2152)(s+5.86)(s+9.812)}\right)_{s=-0.667+j1.333} = 0.4206k\angle 180^0$$

$$k = \frac{1}{0.4206} = 2.3773$$

$$K_2 = 2.3773\left(\frac{(s+0.5)(s+1.435)}{s(s+2.2152)}\right)$$