Homework #10: ECE 461/661

Digital PID Control.   Due Monday, November 16th

I Control

Assume T = 0.5 seconds:

$$G(s) = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right)$$

1) Design a digital I controller

$$K(s) = k\left(\frac{z}{z-1} \right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Option #1: Design in the z-domain. Convert G(s) to G(z)

```
T = 0.5;
s = [-0.1617, -1.04, -2.719, -5.05]
Gz = zpk([0,0],exp(s*T),1, T);
Gs = zpk([],s,1.4427);
k = evalfr(Gs,0) / evalfr(Gz,1)
     k = 0.0135
Gz = zpk([0,0],exp(s*T),k, T);
zpk(Gz)

                   0.0135 z^2
--------------------------------------------
(z-0.9223) (z-0.5945) (z-0.2568) (z-0.08006)

Sampling time (seconds): 0.5
```

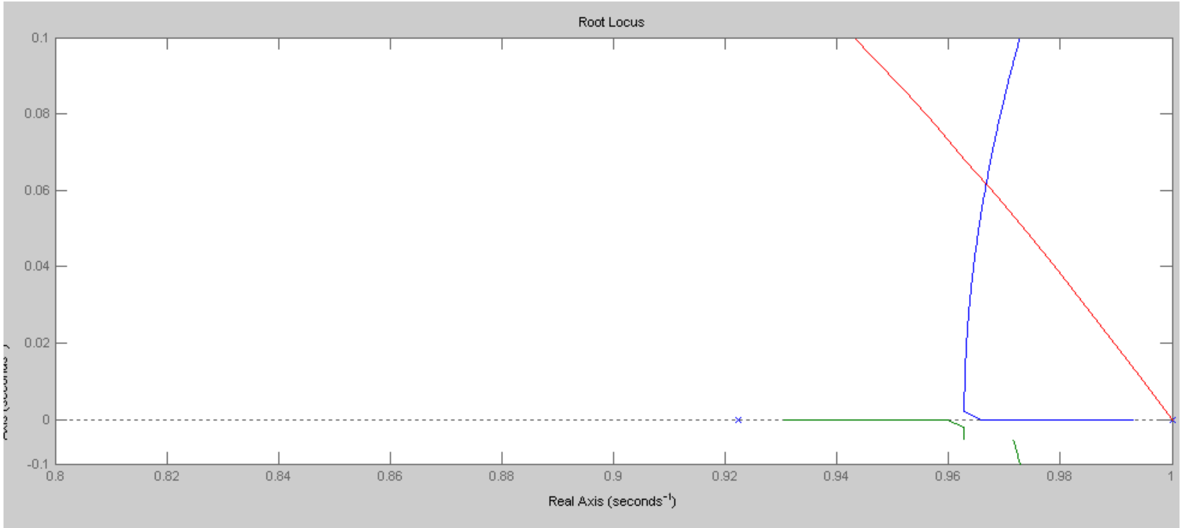
$$G(z) \approx \left(\frac{0.0135z^2}{(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801)} \right)$$

Add K(z)

$$GK = \left(\frac{0.0135z^3}{(z-1)(z-0.9223)(z-0.5945)(z-0.2568)(z-0.0801)} \right)$$

Draw the root locus along with the damping line

```
k = logspace(-2,2,1000)';
rlocus(GzKz,k);
hold on;
s = (-1 + j*2) * [0:0.01:10]';
z = exp(s*T);
plot(real(z),imag(z),'r');
```



Result: z = 0.9667 + j0.0616

Option #2: Numerical Solution.

The plant + compensator + sample & hold is

$$G \cdot K \cdot H = \left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)} \right) \cdot \left(\frac{kz}{z-1} \right) \cdot (e^{-sT/2})$$

Search along the damping line until the angles add up to 180 degrees

```
Kz = zpk(0,1,1,T);
T = 0.5;
s = -0.1 + j*0.2;
z = exp(s*T);
evalfr(Gs,s) * evalfr(Kz,z) * exp(-s*T/2)

  -4.3508 + 2.3219i

s = 0.9*s;
z = exp(s*T);
evalfr(Gs,s) * evalfr(Kz,z) * exp(-s*T/2)

  -5.4439 + 2.1500i
```

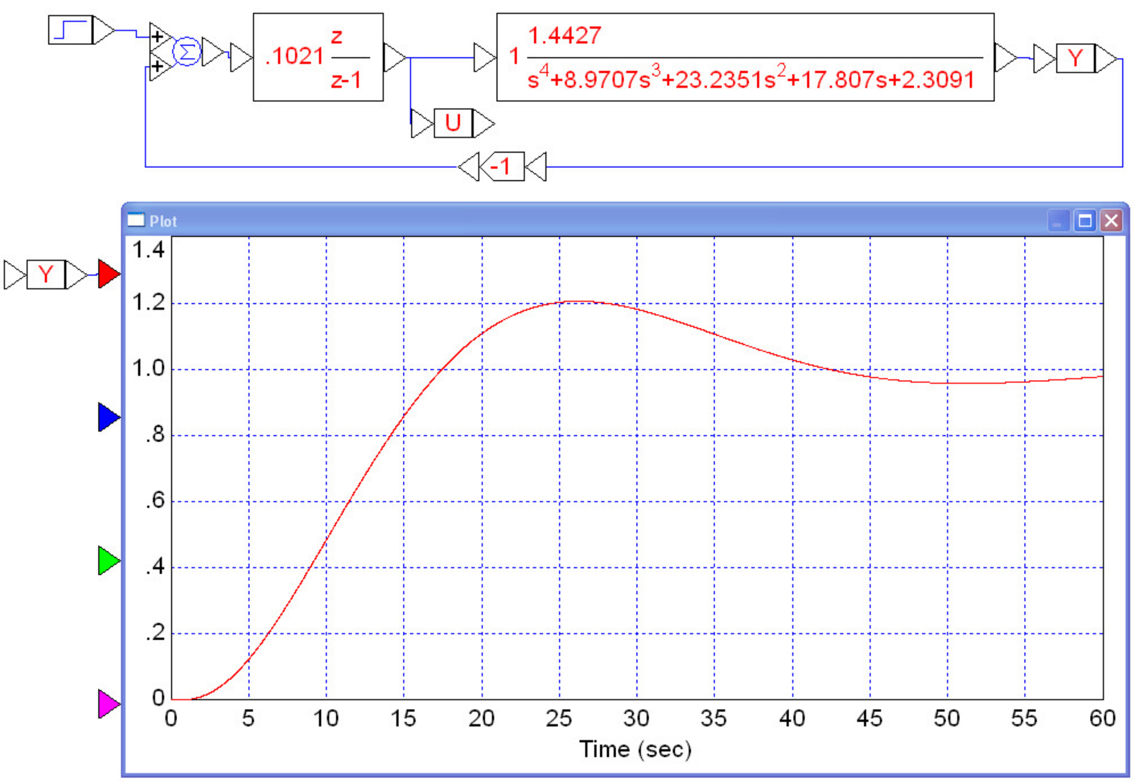
time passes....

```
s = 1.0001*s;
z = exp(s*T);
evalfr(Gs,s) * evalfr(Kz,z) * exp(-s*T/2)

  -9.7937 - 0.0001i

s =  -0.0630 + 0.1260i
z =   0.9671 + 0.0610i
```

K(z) is then

```
evalfr(Gs,s) * evalfr(Kz,z) * exp(-s*T/2)

ans =  -9.7937 - 0.0001i

k = 1 / abs(ans)

k =    0.1021

Kz = zpk(0,1,k, T)

0.1021 z
--------
 (z-1)

Sampling time (seconds): 0.5
```

Checking with VisSim



PI Control

2) Design a digital PI controller that results in 20% overshoot in the step response.

Use option #2 (numerical method). Choose the zero to cancel the pole at s = -0.1617

$$z = e^{sT} = 0.922$$

$$K(z) = k\left(\frac{z-0.922}{z-1}\right)$$

Search along the dampin line until the angles add up to 180 degrees

$$G(s) \cdot K(z) \cdot H(s) = 1\angle 180^0$$

$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) \cdot \left(\frac{k(z-0.922)}{z-1}\right) \cdot (e^{-sT/2})\right)_s = 1\angle 180^0$$

itterating results in

$$s = -0.2776 + j0.5553$$

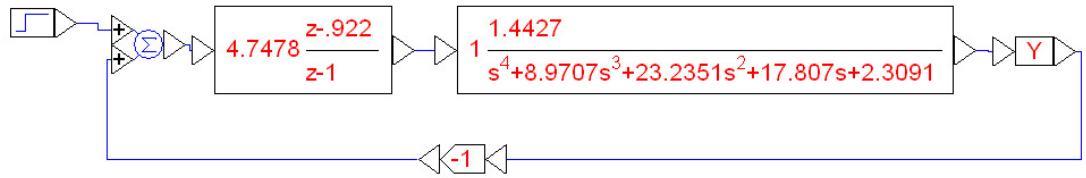
$$z = 0.8371 + j0.2386$$

$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) \cdot \left(\frac{k(z-0.922)}{z-1}\right) \cdot (e^{-sT/2})\right)_s = 0.2016\angle 180^0$$

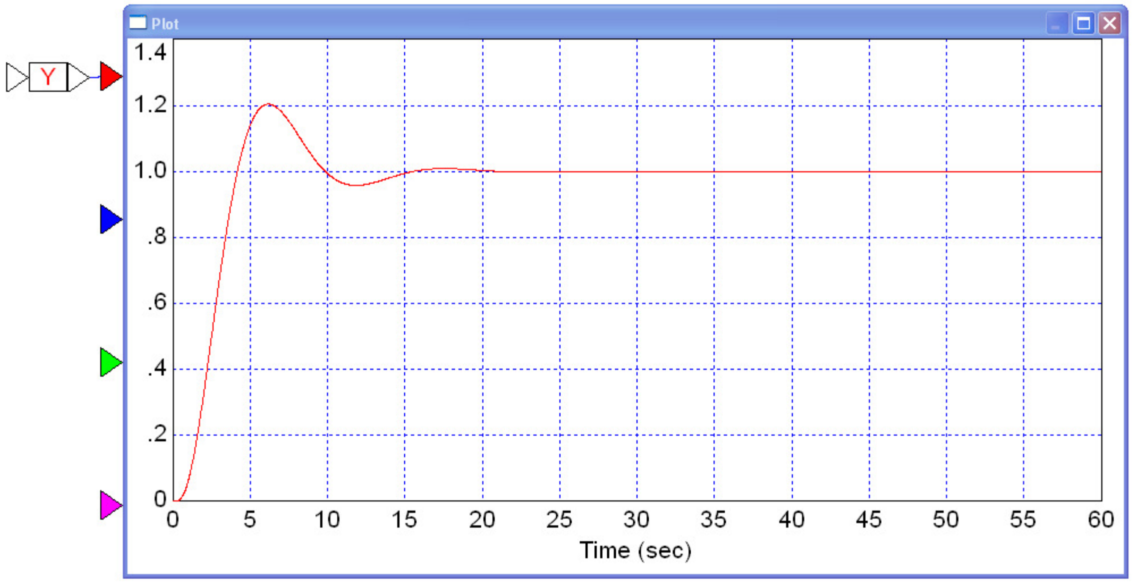
$$k = \frac{1}{0.2016} = 4.7478$$

and

$$\boxed{K(z) = 4.7478\left(\frac{z-0.922}{z-1}\right)}$$



+
+
4.7478
z-.922
z-1
1
1.4427
s^4+8.9707s^3+23.2351s^2+17.807s+2.3091
Y
-1



Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
5
10
15
20
25
30
35
40
45
50
55
60
Time (sec)

PID Control

3) Design a digital PID controller that results in 20% overshoot in the step response.

Let K(z) cancel the poles at s = -0.1617 and s = -1.04

$$K(z) = k\left(\frac{(z-0.922)(z-0.5945)}{z(z-1)}\right)$$

The plant + compensator + sample and hold is now

$$G(s)\cdot K(z)\cdot H(s) = 1\angle 180^0$$

$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)\cdot\left(\frac{k(z-0.922)(z-0.5945)}{z(z-1)}\right)\cdot\left(e^{-sT/2}\right)\right)_s = -1$$

Search along the damping line until the angles add up to 180 degrees

$$s = -0.4922 + j0.9843$$

$$z = 0.6891 + j0.3695$$

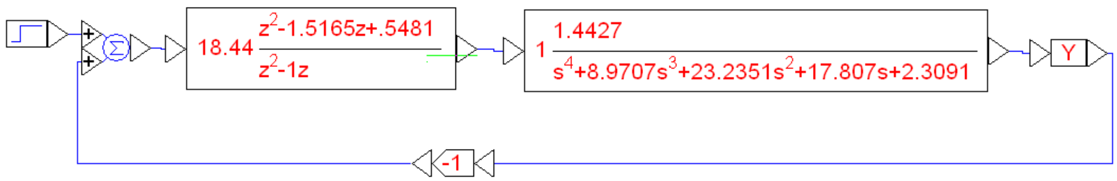
resulting in

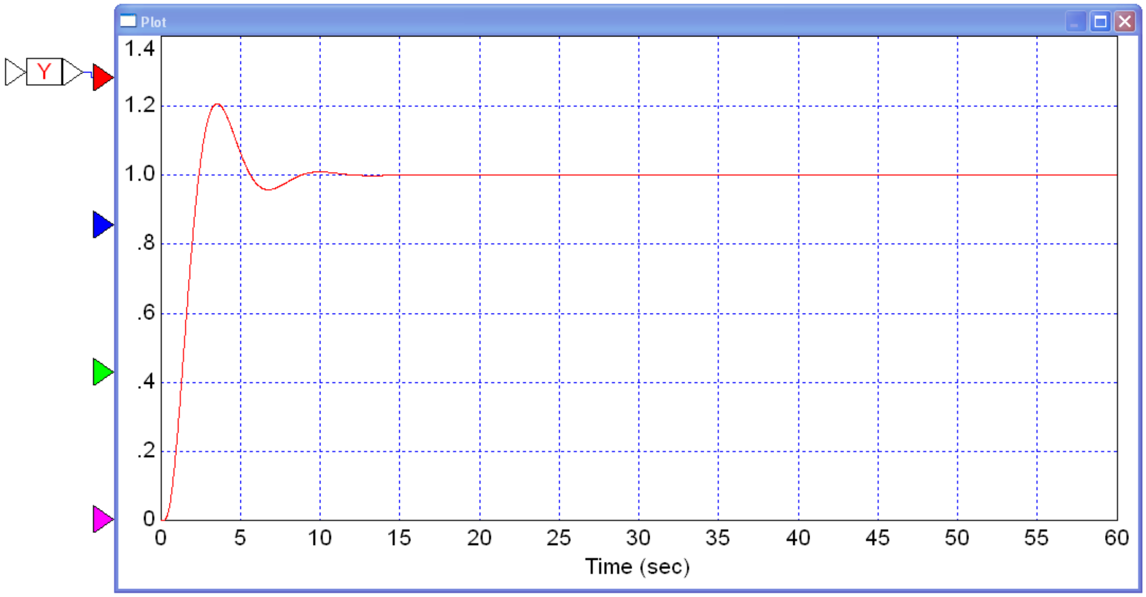
$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)\cdot\left(\frac{k(z-0.922)(z-0.5945)}{z(z-1)}\right)\cdot\left(e^{-sT/2}\right)\right)_s = 0.0542\angle 180^0$$

k is then

$$k = \frac{1}{0.0542} = 18.4478$$

$$\boxed{K(z) = 18.4478\left(\frac{(z-0.922)(z-0.5945)}{z(z-1)}\right)}$$


18.44
z2-1.5165z+.5481
z2-1z
1
1.4427
s4+8.9707s3+23.2351s2+17.807s+2.3091
Y
-1


Plot
Y
1.4
1.2
1.0
.8
.6
.4
.2
0
0
5
10
15
20
25
30
35
40
45
50
55
60
Time (sec)

Meeting Design Specs

4) Design a digital controller with T = 0.5 seconds that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 10 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Translation

- Make the system type 1
- Place the closed-loop dominant pole at s = -0.4 + j0.8
- Place the closed-loop dominant pole at z = 0.7541 + j0.3188

Let

$$K(z) = k\left(\frac{(z-0.922)(z-0.5945)}{(z-1)(z-a)}\right)$$

Evaluate what we know

$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) \cdot \left(\frac{(z-0.922)(z-0.5945)}{(z-1)}\right) \cdot (e^{-sT/2})\right)_s$$

$$= 0.0514\angle -144.64^0$$

The pole at z = a must contribute the remainder (-35.3567 degrees)

$$\angle(z-a) = 35.3567^0$$

$$a = 0.7571 - \frac{0.3188}{\tan\left(35.3567^0\right)} = 0.3078$$

and

$$K(z) = k\left(\frac{(z-0.922)(z-0.5945)}{(z-1)(z-0.3078)}\right)$$

Evaluating at the design point (s and z):

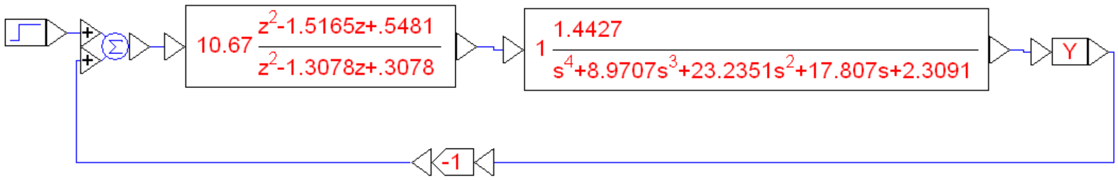
$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) \cdot \left(\frac{(z-0.922)(z-0.5945)}{(z-1)(z-0.3078)}\right) \cdot (e^{-sT/2})\right)_s$$

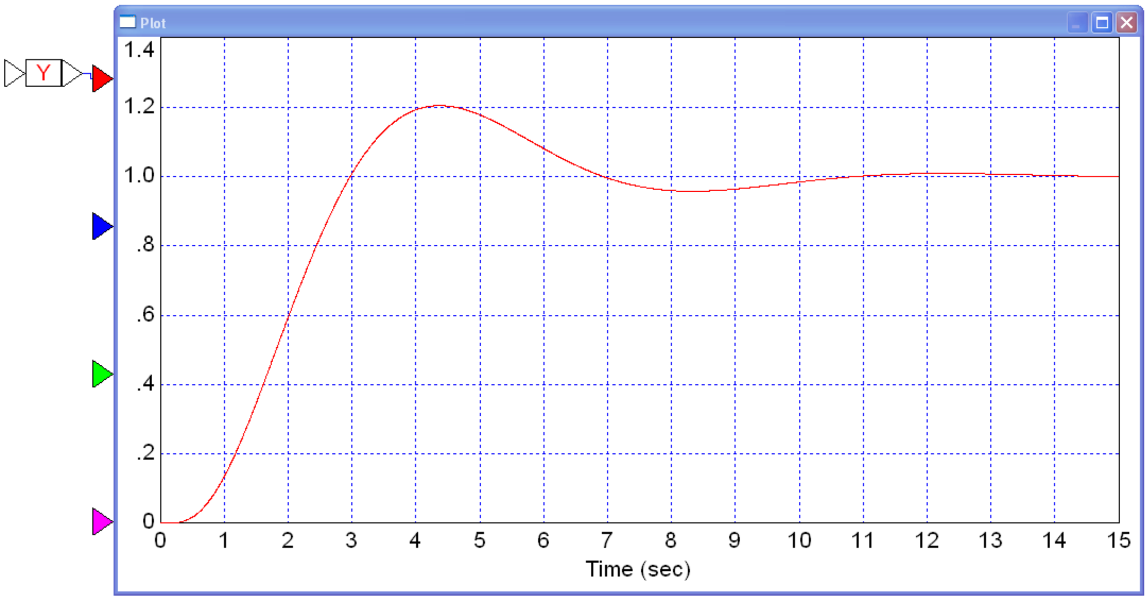
$$= 0.0937\angle 180^0$$

meaning

$$k = \frac{1}{0.0937} = 10.671$$

$$K(z) = 10.67\left(\frac{(z-0.922)(z-0.5945)}{(z-1)(z-0.3078)}\right)$$





5) Design a digital controller with T = 0.1 second that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 4 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Translation

- Make the system type 1
- Place the closed-loop dominant pole at s = -0.4 + j0.8
- Place the closed-loop dominant pole at z = 0.9577 + j0.0768

Let

$$K(s) = k\left(\frac{(s+0.1617)(s+1.04)}{s(s+b)}\right)$$

$$K(z) = k\left(\frac{(z-0.9940)(z-0.9012)}{(z-1)(z-a)}\right)$$

Evaluate what we know

$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)\cdot\left(\frac{(z-0.9940)(z-0.9012)}{(z-1)}\right)\cdot\left(e^{-sT/2}\right)\right)_s$$

$$= 0.137\angle -138.92^0$$

The pole at (z = a) must subtract 41.08 degrees to bring this to 180 degrees

$$\angle(z+a) = 41.08^0$$

$$a = 0.9577 - \frac{0.0768}{\tan\left(41.08^0\right)} = 0.869$$

so now

$$K(z) = k\left(\frac{(z-0.9940)(z-0.9012)}{(z-1)(z-0.8696)}\right)$$

Evaluate what we now know

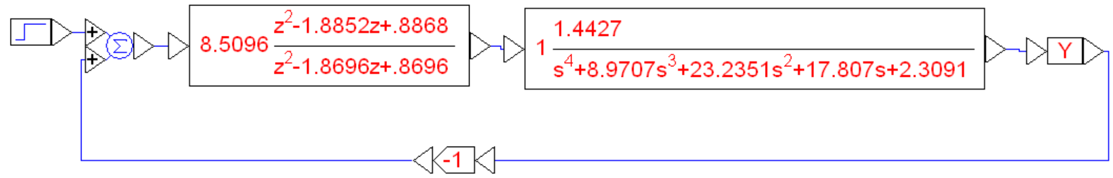
$$\left(\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)\cdot\left(\frac{(z-0.9940)(z-0.9012)}{(z-1)(z-0.8696)}\right)\cdot\left(e^{-sT/2}\right)\right)_s$$

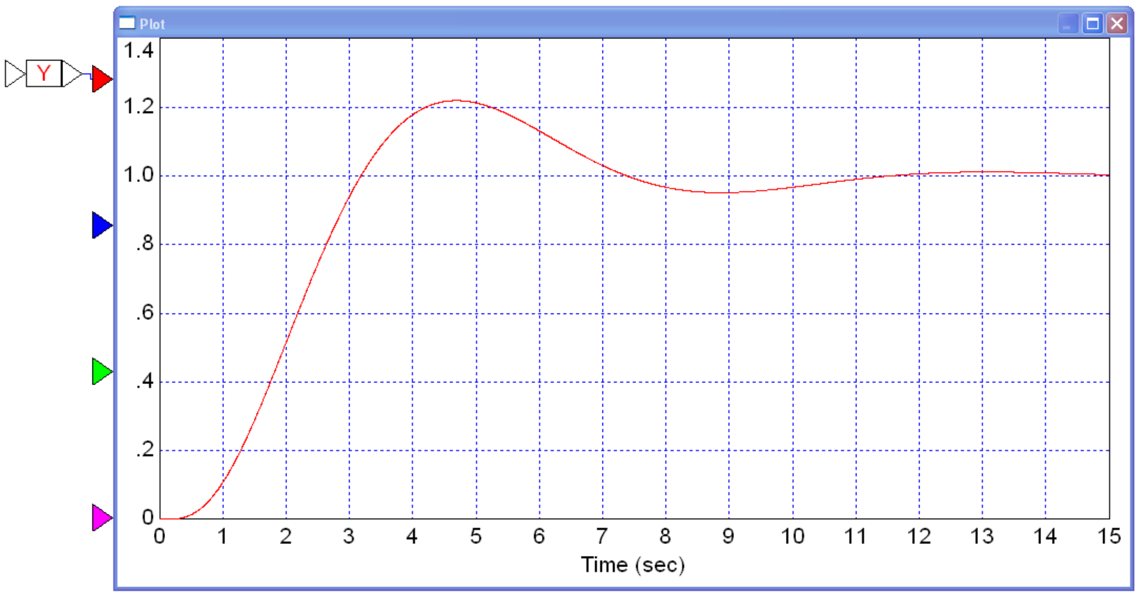
$$= 0.1175\angle 180^0$$

k is then

$$k = \frac{1}{0.1175} = 8.5096$$

$$K(z) = 8.5096\left(\frac{(z-0.9940)(z-0.9012)}{(z-1)(z-0.8696)}\right)$$





Note: If you change the sampling rate, it's a complete re-design.