## Homework \#11: ECE 461/661

Bode Plots. Nichols charts and gain compensation. Due Monday, November 23rd

## Bode Plots

1) Determine the system, $G(s)$, with the following gain vs. frequency


Draw in the asymptotes at multiples of $20 \mathrm{~dB} /$ decade (shown in orange)
Each corner is a pole

$$
G(s) \approx\left(\frac{k}{(s+0.9)(s+4.2)(s+8)(s+25)}\right)
$$

Pick k to match the gain at one point (DC)

$$
\begin{aligned}
& \left(\frac{k}{(s+0.9)(s+4.2)(s+8)(s+25)}\right)_{s=0}=9 d B=2.51 \\
& k=1899
\end{aligned}
$$

$$
G(s) \approx\left(\frac{1899}{(s+0.9)(s+4.2)(s+8)(s+25)}\right)
$$

2) Determine the system, $G(s)$, with the following gain vs. frequency


Draw in the straight line approximations

- There are two zeros at $\mathrm{s}=0$ (someplace left of 0.1 )
- There are two poles at $\mathrm{s}=0.7$
- There are two poles at $\mathrm{s}=20$

The gain at the corner tells you the damping ratio

$$
\begin{array}{lll}
\frac{1}{2 \zeta}=+5 d B=1.778 & \zeta=0.281 & \theta=73.67^{0} \\
\frac{1}{2 \zeta}=+9 d B=2.818 & \zeta=0.177 & \theta=79.78^{0} \\
G(s) \approx\left(\frac{k s^{2}}{\left(s+0.7 \angle \pm 73.67^{0}\right)\left(s+20 \angle \pm 79.78^{0}\right)}\right) &
\end{array}
$$

Pick k to match the gain at a point. As $\mathrm{s}=0.1$, the gain is -20 dB

$$
\begin{aligned}
& \left(\frac{k s^{2}}{\left(s+0.7 \angle \pm 73.67^{0}\right)\left(s+20 \angle \pm 79.78^{0}\right)}\right)_{s=0.1}=-20 d B=0.1 \\
& k=1960 \\
& G(s) \approx\left(\frac{1960 s^{2}}{\left(s+0.7 \angle \pm 73.67^{0}\right)\left(s+20 \angle \pm 79.78^{0}\right)}\right)
\end{aligned}
$$

## Nichols Charts

3) The gain vs. frequency of a system is measured

| w (rad/sec) | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gain (dB) | 4.63 | -0.21 | -4.45 | -8.17 | -11.51 | -14.55 |
| Phase (deg) | -67.6 | -107.0 | -133.67 | -154.08 | -170.63 | -184.51 |

## Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system $\mathbf{k}=+\mathbf{1 4 d B}$
- Determine the gain, k , that results in a maximum closed-loop gain of $\mathrm{Mm}=1.5 \mathbf{k}=\mathbf{1 . 8}$

```
>> w = [1,2,3,4,5,6]';
> GdB = [4.63,-0.21,-4.45,-8.17,-11.51,-14.55]';
>> Gdeg = [-67.6,-107,-133.67,-154.08,-170.63,-184.51]';
>> Gw = 10.^(GdB/20) .* exp(j*Gdeg*pi/180)
    0.6494 - 1.5755i
    -0.2854 - 0.9335i
    -0.4137 - 0.4333i
    -0.3511 - 0.1706i
    -0.2622 - 0.0433i
    -0.1867 + 0.0147i
>> Nichols2(Gw*[1,2],1.5);
>> Nichols2(Gw*[1,1.8],1.5);
>> Nichols2(Gw*[1,1.8],1.5);
```


4) Assume

$$
G(s)=\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right)
$$

Find G(jw) using Matlab (or similar program). With ths data,

- Draw a Nichols chart
- Determine the maximum gain that results in a stable system $\mathbf{k}=+\mathbf{2 8 d B}$
- Determine the gain, k , that results in a maximum closed-loop gain of $\mathrm{Mm}=1.5 \mathbf{k}=\mathbf{1 0}$

```
>> w = [0:0.1:10]';
>> s = j*W;
>>Gw = 1.4427 ./ ( (s+0.1617).*(s+1.04).*(s+2.719).*(s+5.05) );
>> Nichols2(Gw,1.5);
>> Nichols2(Gw*[1,2],1.5);
>> Nichols2(Gw*[1,4],1.5);
>> Nichols2(Gw*[1,5],1.5);
>> Nichols2(Gw*[1,7],1.5);
>> Nichols2(Gw*[1,8],1.5);
>> Nichols2(Gw*[1,9],1.5);
>> Nichols2(Gw*[1,10],1.5);
>>
```


5) Assume a 500ms delay is added

$$
G(s)=\left(\frac{1.4427}{(s+0.1617)(s+1.04)(s+2.719)(s+5.05)}\right) e^{-0.5 s}
$$

Find $G(j w)$ using Matlab (or similar program). With ths data,

- Draw a Nichols chart
- Determine the maximum gain that results in a stable system $\mathbf{k}=\boldsymbol{+} \mathbf{2 3 d B}$
- Determine the gain, k , that results in a maximum closed-loop gain of $\mathrm{Mm}=1.5 \mathbf{k}=\mathbf{6} .4$

```
>> w = [0:0.1:10]';
>> s = j*w;
>> Gw = 1.4427 ./ ( (s+0.1617).*(s+1.04).*(s+2.719).*(s+5.05) ) .*
exp(-0.5*s);
>> Nichols2(Gw*[1,10],1.5);
>> Nichols2(Gw*[1,6],1.5);
>> Nichols2(Gw*[1,6.2],1.5);
>> Nichols2(Gw*[1,6.3],1.5);
>> Nichols2(Gw*[1,6.4],1.5);
>>
```



